

Monitoring and Manipulation: Asset Prices When Agents are Marked-to-Market*

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Abstract

Trading in security markets is delegated. We study the “efficient markets” paradigm in the context of such agency relations. Principal-investors want to monitor and compensate their agent-traders using market security prices in “mark-to-market” contracts. This introduces an externality because security prices are informative only insofar as the agent-traders of other principal-investors have an incentive to produce information and trade accordingly. If the market is dominated by such delegated traders, then these traders can attempt to manipulate the market price by jointly shirking and buying or selling in the same direction. In this way, traders provide market “proof” that they have worked hard and deserve high compensation. We show that even if agent-traders can coordinate perfectly, there is no equilibrium in which all principal-investors ignore market prices. The extent of “market efficiency,” indexed by the delegated traders’ propensity of joint shirking, is endogenized.

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1. Introduction

The intellectual triumph of the concept of “efficient markets” over the last 25 years has led to this paradigm becoming widespread, governing the compensation of professional security traders and money managers, and providing foundations for accounting and risk management. “Marking-to-market” refers to the widespread practice of measuring and monitoring portfolio managers’ and security traders’ performance and risk-taking propensity based on market prices. As a risk management practice, marking-to-market professional traders’ positions received fresh impetus after the collapse of Enron, and other derivatives scandals.¹ Moreover, accounting measurements of profit and loss are also increasingly based on security market prices, as the best measure of “fair value.”² The logic of efficient markets is clear: If market prices are the best measure of value, because they “contain all relevant information,” then these prices should be used for compensation, accounting, and risk management purposes. Hence, the term “market discipline.”

An important step in the intellectual foundation of efficient markets was the solution to the problem of how information came to be embedded into security prices if markets are efficient, somehow already containing all relevant information. Grossman and Stiglitz (1980) addressed this issue by introducing the notion of “noise” (a noisy supply curve for the asset, in their case). This allowed some traders to produce costly information and trade profitably on their findings, with the information being (at least weakly) impounded in the price. See Dow and Gorton (2006) for a review. In this paper, we revisit the issue of market efficiency in the context of delegated trading, focusing on the use of security prices in private contracts with delegated traders. Principals hire traders to produce and trade on private information. We show that when market prices are used to try to control the actions of agent-traders, agent-traders can manipulate prices resulting in less informative prices and ultimately making mark-to-market contracts undesirable. The efficiency of market prices is not exogenous to the risk management practices adopted by principal-investors. In fact, markets become less efficient.

In the context of a security market in which some traders are trading on behalf of investing principals, principals must design compensation contracts to control the trading behavior of the professional traders (or money managers) they hire. In the standard

¹ Derivatives-related scandals include the failure of Orange County, California in the early 1990s, the failure of Metallgesellschaft in 1994, the failure of Barings PLC in 1995, the collapse of Long Term Capital Management in 1998, Enron in 2001, the failure of China Aviation Oil in 2005, and the most recent failure of Société Générale in 2008.

² Indeed, the sudden collapse of Enron in late 2001 gave further impetus to the confluence of the efficient markets theory of security prices and the accounting profession's notion of “fair value.” Accountants are increasingly embracing the notion that “fair value” for accounting purposes is best measured using security market prices. Enron was accused of mis-marking complex derivatives using models. The issue of whether “fair value” is best represented by market prices is most controversial and problematic in derivatives markets, but is present in all security markets. The relevant rule is largely stated in the Federal Accounting Standards Board SFAS 157, “Fair Value Measurements.”

agency paradigm, an agent's contractual compensation is only a function of the final value or price, not any interim information. A mark-to-market contract is different, as it involves interim price information from security markets, information that arrives between the contract initiation and the final payoff. Since interim market security prices are observable and contractible, these prices may be used to value the traders' positions as part of the optimal contract. When the interim market information is exogenously given, contracts ignoring it are suboptimal. However, when the information production is endogenized, contract choices that are based on the informative market price also have an impact on market prices. We provide conditions under which "marking-to-market" is part of an optimal contract. This confirms the logic of using market prices as a measure of a trader's performance (in addition to the final payoff).³

Another difference from the standard agency problem is that in our setting the compensation of agent-traders depends on a public signal that is jointly affected by the agent-traders' actions prior to the final date. In particular, agent-traders trade in a securities market and the interim market price, which is (at least somewhat) informative due to the actions of other agent-traders, is used by each principal to monitor his agent-trader's behavior. The problem is that the trading behavior affects the security prices, which in turn, affects the mark-to-market values of the traders' positions, affecting their compensation. This externality is at the heart of the contracting problems. In fact, traders have an incentive to coordinate to manipulate the price, meaning that they can jointly shirk (not producing any information) and trade in the same direction, leading to prices which will result in high compensation under the mark-to-market contracts. Hence, the tension between monitoring agent-traders based on the security market prices and the manipulation that can occur under these contracts, and this endogenously determines the asset price and market efficiency.

The possible manipulation of security prices by agent-traders means that the principals' choice of contract for their traders at the initial date is also intertwined. Principals can always choose the "standard contract," that is, one which does not include interim market price information, but rather is just based on final outcomes. Under a standard contract, an agent-trader can be induced to produce information and trade in a direction consistent with the principal's objective. But, adopting the standard contract means that the price will be (at least weakly) informative, allowing other principals to free ride on this information using mark-to-market contracts for their agent-traders. The equilibrium then must be one in which the principals follow mixed strategies in contracts, sometimes adopting mark-to-market contracts and sometimes adopting standard contracts.

³ In the managerial contracting literature, it has been demonstrated that the market price of the firm should be included in the optimal contract for the manager as long as it contains information not reflected by the firm's current or future profit data; see, for example, Holmstrom and Tirole (1993). Our paper goes beyond this result; we study the externality effect of these mark-to-market contracts when there are multiple principal-agent pairs in the economy. In particular, we show that the agents have an incentive to coordinate to manipulate the market price to get a higher payoff.

This introduction of noise, interpreted as randomness in principals' discretion over compensation, makes it harder for the agent-traders to manipulate their bonuses.

The agency relation we study is pervasive in security and derivatives trading. Professional traders work at hedge funds, mutual funds, money management firms, and banks. In over-the-counter markets, like those for government securities, foreign currency, corporate bonds, residential and commercial mortgage-backed securities, other asset-backed securities, and derivative securities (comprising interest rate, equity, foreign currency, credit, commodity, energy and other derivatives), the entire market is based on traders working for others. These are very large and important markets. For example, the notional amount of interest rate derivatives in June 2007 was \$346.9 trillion and the amount of credit default swaps outstanding in June 2007 was \$42.6 trillion (see Bank for International Settlements (2007)).⁴

The competition between principals in choosing contracts for their competing agents that we analyze is similar to some previous literature in Industrial Organization. Sklivas (1987) and Fershtman and Judd (1987) both study this type of contracting problem in a duopoly setting with Cournot or Bertrand competition. In Sklivas (1987), the manager's payoff is assumed to be a linear combination of the firm's revenue and profit, while in Fershtman and Judd (1987), the manager's payoff is assumed to be linear in the firm's performance relative to its competitor. In our setting, we do not restrict the format of the agent's compensation; rather we endogenously determine the optimal contract. In a similar financial market setting, Gümbel (2005) studies the asset allocation problem in a multiple principal-agent environment, but the contract is restricted to be based on relative performance, similar to Fershtman and Judd (1987).

Two other papers study issues to do with marking-to-market, though not in an optimal contracting context. Plantin, Sapra, and Shin (2005) study the effects of an assumed mark-to-market regime when bank managers are assumed to have short horizons. Asset prices are assumed to be negatively related to asset sales, i.e., the price goes down when banks sell assets. Allen and Carletti (2006) show that when asset prices sometimes reflect liquidity factors, rather than fundamentals, a mark-to-market regime can induce contagion. Our paper is quite different from these in several respects. First, we do not assume that marking-to-market is optimal. We determine when mark-to-market contracts between principals and their traders are optimal. Second, we do not assume that there are effects on security prices due to supply and demand, causing prices to deviate from fundamentals. We isolate the effects of mark-to-market contracts in a standard trading environment; there are no exogenous liquidity factors that cause the price to deviate from fundamentals and there are no other assumed frictions, e.g., short horizons.

⁴ Even in the public equity market, most trading is delegated to professional portfolio managers. In 2004, U.S. households directly held less than 40% of corporate equities, while they held about 90% in 1950 and 70% in 1970 (see "Flow of Funds" issued by the Federal Reserve Board). Also according to the survey results released by the Investment Company Institute (ICI) and the Securities Industry Association (SIA), in 2002, 89% of the investors invested in mutual funds and 58% of the investors relied on professional financial advisers when making investment decisions.

Finally, our paper also belongs to the strand of the literature that examines the impact of agency problems on asset pricing. For example, Allen and Gorton (1993) show that when there is asymmetric information between investors and portfolio managers, portfolio managers have an incentive to churn; their trades are not motivated by changes in information about liquidity needs or risk sharing but rather by a desire to profit at the expense of the investors that hire them. As a result, assets can trade at prices that do not reflect their fundamentals and bubbles can exist. Dow and Gorton (1994) and Goldman and Slezak (2003) examine the pricing impact of an interaction between agency problems and time-horizon and show that managers' incentives can be distorted by an information externality. In our paper, while the principal would only care about the final price without hiring a trader, a trader's horizon is endogenously shorter since the optimal contract links his payoff to the interim price. Dow and Gorton (1997) study a model in which investors optimally contract with portfolio managers who may have stock-picking abilities, and portfolio managers trade optimally given the incentives provided by this contract. Because investors cannot distinguish "actively doing nothing" from "simply doing nothing," some managers trade with no proper reason (noise trade). Noise trade causes high levels of turnover.⁵

The paper proceeds as follows. In Section 2 we set up the model and state some assumptions about the security payoffs and the noisy signals about fundamentals. The analysis then proceeds in three steps. We first solve for the optimal standard contract in Section 3. It is the benchmark case, yet a suboptimal solution. Then we solve for the optimal mark-to-market contract and compare it with the standard contract. We show that the optimal mark-to-market consists of a step function based on the interim security price, which is the proxy for the trader's private information. In order to prevent the trader from shirking, the optimal contract requires that the trader's trading decision (i.e., buy or sell) be consistent with the change in the security price. The optimal mark-to-market contract dominates the standard contract because the additional information derived from the interim market price enables the principal to lower the cost of monitoring the agent.

In Section 4 we analyze the case of many principal-agent pairs. When there is more than one trader in the market, additional equilibria emerge. In these equilibria, traders either sell the security upon receiving private good news or buy the security upon receiving bad news--the opposite of what they would do in the absence of the externality and actions that are not in the principal-investors' interests. The interaction of the agency problem and the externality gives rise to these seemingly irrational equilibria. We consider how the principals compete in their choice of contract, mark-to-market or not, when they hire agent-traders. The externality caused by the feedback from traders' trading activity results in a nontrivial contract choice by principal-investors at the initial date. Although coordinated market manipulation by traders may force some principal-investors to offer the standard contract at some times, there is no equilibrium in which principal-investors only offer standard contract.

⁵ There is a large literature on delegated portfolio management, which focuses on issues of the manager's compensation structure. E.g., Bhattacharya and Pfleiderer (1985), Brennan (1993), Admati and Pfleiderer (1997), Cuoco and Kaniel (2000), and Ou-Yang (2003). For the most part, the issue in these studies concerns the choice of a "benchmark" to use to evaluate the manager's performance.

We conclude in Section 5.

2. Model Setup

Our focus is on an agency problem in which a principal-investor hires an agent-trader to trade on his behalf in a security market. Like the standard agency problem, the principal must design a contract to induce the agent-trader to make an effort to improve the value of the realization of the “project,” in this case a trading position.⁶ In our setting the principal observes an interim price signal which can provide some information about the performance of the agent-trader. Here, in addition to an initial effort choice, the trader can make another choice later, namely, he can buy or sell in the interim securities market. While the effort choice is hidden, as in standard agency problems, the trading action/position is observable and contractible. Thus, the strategy space of the agent-trader is larger than in the standard problem, introducing the issue of “risk management” in a way which is not present in the usual problem. The risk management issue concerns how the principal can control the agent-trader’s hidden action using his observable actions in combination with market conditions in the interim securities market. We first investigate this in the simplest setting in this section. We will call the agent-trader the “professional trader” or simply the “trader,” as he is hired by the principal.

The security market has five types of (risk neutral) participants: a direct investor, an indirect investor, a professional trader, liquidity (or noise) traders, and a market maker. The direct investor invests for himself; he has no need to hire an agent-trader.⁷ Both the professional trader and the direct investor have the technology to become imperfectly informed about the value of the security at a final date. The direct investor has money to invest in the security market, but the trader does not. However, the professional trader can be hired by the indirect investor (the principal), who has the money to invest, but no access to the technology to become (privately) informed. After the trader is hired, he can choose not to make an effort to acquire information; not making an effort yields a shirking benefit of $\kappa > 0$. For ease of exposition, we will refer to the security as “stock.”

There are three dates in this economy:

- Date 0: The trader and the principal sign a contract which authorizes the trader to trade on behalf of the principal and which specifies the payoffs to the trader under each contingency observable to both parties to the contract.

⁶ See Salanié (2005) for an explication of the standard contracting problem with unobservable effort choice.

⁷ The existence of a direct investor makes the price informative even when the agent trader shirks (i.e., does not produce information) and simply traders randomly. “Liquidity traders” or “noise traders” are agents who play a special role in financial economics. They trade for exogenous reasons and, on average, lose money, allowing other agents to produce costly information and still make a profit. See Dow and Gorton (2006).

- Date 1: Both the direct investor and the trader (if he does not shirk) receive a signal about the terminal value of the stock. Then each decides whether to submit a buy order or a sell order. For simplicity, investors can trade a maximum of x shares. The liquidity traders trade an amount $\delta \sim N(0, \sigma^2)$. The market maker sets the price based on the total order flow by forming the conditional expectation of the security value, given knowledge of the model, as is standard in financial economics following Kyle (1985).
- Date 2: The liquidation value of the stock is realized, and the principal pays his trader according to the contract.

The liquidation value of the stock has the following distribution:

$$v = \begin{cases} v_H & \text{with probability } 1/2 \\ v_L & \text{with probability } 1/2, \end{cases} \quad (1)$$

and we assume $v_H > v_L$.⁸

For simplicity, the direct investor and the trader (if he does not shirk) receive the same private signal, which can be either s_H or s_L , and the signal is correlated with the true liquidation value of the stock as follows:

$$v = \begin{cases} v_H & \text{with probability } \theta > 1/2 \text{ if } v = v_H \\ v_L & \text{with probability } \theta > 1/2 \text{ if } v = v_L. \end{cases} \quad (2)$$

Let π_s denote the unconditional probability of receiving a high signal, then we have:

$$\pi_s = \frac{1}{2}\theta + \frac{1}{2}(1 - \theta) = \frac{1}{2}. \quad (3)$$

The probability distribution of v conditional on the signal s can be written as:

$$\begin{aligned} \text{prob}[v_H | s_H] &= \text{prob}[v_L | s_L] = \theta \\ \text{prob}[v_L | s_H] &= \text{prob}[v_H | s_L] = 1 - \theta. \end{aligned} \quad (4)$$

When both the direct investor and the agent-trader trade on privately produced information, the distribution of the market order flow z is a mixture of two Normal distributions:

$$z \sim \frac{1}{2}N(2x, \sigma^2) + \frac{1}{2}N(-2x, \sigma^2). \quad (5)$$

Let $\phi^+(\cdot)$ denote the probability density function for the distribution $N(2x, \sigma^2)$, $\phi^-(\cdot)$ denote the probability density function for the distribution $N(-2x, \sigma^2)$, and $\phi^0(\cdot)$ denote the probability density function for the distribution $N(0, \sigma^2)$. So:

⁸ The assumption of equal probability for v_H and v_L is not crucial for our results.

$$\begin{aligned}
\phi^+(z) &= \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{\frac{-(z-2x)^2}{2\sigma^2}\right\} \\
\phi^-(z) &= \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{\frac{-(z+2x)^2}{2\sigma^2}\right\} \\
\phi^0(z) &= \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{\frac{-z^2}{2\sigma^2}\right\}.
\end{aligned} \tag{6}$$

It is easy to show that $\phi^+(z)/\phi^0(z)$ is increasing in z and $\phi^-(z)/\phi^0(z)$ is decreasing in z . The monotonicity of the likelihood ratio $\phi^+(z)/\phi^0(z)$ tells us that the higher the order flow z , the more likely it is from the distribution $N(2x, \sigma^2)$. A similar argument applies for the likelihood ratio $\phi^-(z)/\phi^0(z)$. This property of the monotone likelihood ratio is important for characterizing the optimal contract.

Let p_H denote the expected liquidation value at date 3 conditional on the signal s_H being received at date 2 by the trader and the direct investor; p_L is defined similarly conditional on the signal s_L being received. Then:

$$\begin{aligned}
p_H &= E[v | s_H] = \theta v_H + (1-\theta)v_L \\
p_L &= E[v | s_L] = \theta v_L + (1-\theta)v_H.
\end{aligned} \tag{7}$$

The market maker will set the price to be the expected liquidation value conditional on the order flow; see Kyle (1985). Therefore, if both the direct investor and the agent-trader trade truthfully based on the produced information, the price can be expressed as a function of the order flow z as follows:

$$p(z) = \frac{\phi^+(z)p_H + \phi^-(z)p_L}{\phi^+(z) + \phi^-(z)}. \tag{8}$$

We can show $dp(z)/dz > 0$, $\lim_{z \rightarrow \infty} p(z) = p_H$, and $\lim_{z \rightarrow -\infty} p(z) = p_L$. The market maker can only observe the total order, z , but not the identity of traders or their orders. From the magnitude of the total order flow, he infers the news received by the informed agents. The larger the order size, the more likely it is that the good news was received, and thus the market maker sets a higher price. In the limit, when the market maker receives an extremely large buy order, he is pretty sure that good news was received by the informed agents and the price is set to p_H , the expected value conditional on good news; when the market maker receives an extremely large sell order, he is pretty sure that bad news was received by the informed agents and the price is set to p_L , the expected value conditional on bad news.

According to equation (8), the interim market price is always between p_H and p_L . Therefore, it is in the best interests of the direct investor to buy x shares given a good signal and sell x shares given a bad signal. In the next section, we will discuss the

equilibrium in which the agent-trader is hired with incentive compatible contracts that induce him to produce information and trade on it, thus the interim price defined in (8) is consistent with the market maker's rational expectation.⁹ Later on, we extend the model to the case with multiple principal-agent pairs in the economy, where the agent-traders might shirk in equilibrium and the interim price would change accordingly.

In Section 3 below, we will study two types of contracts. The first type of contract only depends on the final value or price, not any interim information. This is the typical contract that is studied in the standard agency paradigm and we call it *Standard Contract*. Since it does not employ all information available in the market, it is suboptimal. The second type of contract stipulates that the payoff to the agent depends on his trading position¹⁰ (buy or sell), λ , the final liquidation value of the security, v , and in addition, the interim market price, p . We call it *Mark-to-Market Contract*.

3. Cost Saving of A Mark-to-Market Contract

3.1 Standard Contract

As a benchmark case, we first solve for the optimal Standard Contract before we study contracts that are marked-to-market.

- *Assumption 1: There is an upper bound \bar{w} for wage payment to the agent-trader.*

The role of this assumption is to guarantee a bounded optimal solution. This will become clear as we proceed; the interpretation of \bar{w} will also be discussed later.

Let us first characterize the optimal Standard Contract. Define W^S as follows:

$$W^S \equiv \{w^S(\lambda, v) \mid 0 \leq w^S(\lambda, v) \leq \bar{w} \text{ for any } \lambda \text{ and } v\}. \quad (9)$$

W^S is the set of all feasible standard non-mark-to-market contracts.

A Standard Contract $w^S(\lambda, v)$, characterized by $\{w_{bH}, w_{bL}, w_{sH}, w_{sL}\}$, is incentive compatible if it satisfies:

⁹ It is easy to see that as long as the indirect investor hires a trader in equilibrium, the trader will produce information trade truthfully, otherwise he will not be hired at the first place.

¹⁰ We assume agents cannot observe each other's security position, or alternatively we can assume that this information is not verifiable or contractible. A complete contract specifies the payoff to the trader for every possible trading position between $-x$ and x . To simplify our discussion, we restrict our attention to buying or selling x shares. We can actually show that, for the many types of equilibrium that we study, the optimal contract pays zero for any position other than buying or selling x shares.

$$(IC.S1) \quad \begin{aligned} \theta w_{bH} + (1-\theta)w_{bL} &\geq \theta w_{sH} + (1-\theta)w_{sL} \\ (1-\theta)w_{sH} + \theta w_{sL} &\geq (1-\theta)w_{bH} + \theta w_{bL}, \end{aligned} \quad (10)$$

and

$$(IC.S2) \quad \begin{aligned} &\frac{1}{2}[\theta w_{bH} + (1-\theta)w_{bL} + (1-\theta)w_{sH} + \theta w_{sL}] \\ &\geq \max\left\{\frac{1}{2}(w_{bH} + w_{bL}) + \kappa, \frac{1}{2}(w_{sH} + w_{sL}) + \kappa\right\}. \end{aligned} \quad (11)$$

(IC.S2) can be rewritten as:

$$(IC.S2) \quad \begin{aligned} \frac{1}{2}[\theta w_{bH} + (1-\theta)w_{bL}] &\geq \frac{1}{2}[\theta w_{sH} + (1-\theta)w_{sL}] + \kappa \\ \frac{1}{2}[(1-\theta)w_{sH} + \theta w_{sL}] &\geq \frac{1}{2}[(1-\theta)w_{bH} + \theta w_{bL}] + \kappa, \end{aligned} \quad (12)$$

and it is clear that (IC.S2) implies (IC.S1).

(IC.S1) says that, when the trader receives a good signal, he will be better off submitting a buy order, and when he receives a bad signal, he will be better off submitting a sell order. (IC.S2) says that the trader will be better off by acquiring information and trading in the best interests of the principal instead of shirking (getting κ) and randomly buying or selling the stock.

The expected payoff to the trader can be written as:

$$A^S = \frac{1}{2}[\theta w_{bH} + (1-\theta)w_{sH} + \theta w_{sL} + (1-\theta)w_{bL}], \quad (13)$$

and the optimal Standard Contract $w^S(\lambda, \nu)$ solves the following programming problem:

$$\min_{w \in W_s} A^S \text{ subject to the IC conditions in (12)}. \quad (14)$$

The constraints in (12) have to be binding; otherwise the principal investor can always lower the trader's compensation without invalidating the IC conditions. The following proposition states that the optimal compensation contract is simple: the trader will either get a positive payoff or zero, depending on how his position aligns with the final liquidation value.

Proposition 1 (Optimal Standard Contract) *Assume $\bar{w} \geq 2\kappa/(2\theta-1)$. Then the optimal contract $w^S(\lambda, \nu) = \{w_{bH}, w_{bL}, w_{sH}, w_{sL}\}$ solving (14) takes the following form:*

$$w_{bH} = w_{sL} = \frac{2\kappa}{2\theta-1}, \text{ and } w_{sH} = w_{bL} = 0. \quad (15)$$

Proof: See Appendix. ■

When all informed participants trade truthfully, the expected payoff derived from informed trading is:

$$\begin{aligned} V^S &= \frac{1}{2}x \left[\int_{-\infty}^{\infty} [p_H - p(z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p(z) - p_L]\phi^-(z)dz \right] \\ &= x(p_H - p_L) \int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz. \end{aligned} \quad (16)$$

The expected wage payment to the agent trader is:

$$A^S = \frac{2\theta\kappa}{2\theta - 1}. \quad (17)$$

In order for a principal investor to hire an agent-trader with a Standard Contract, two conditions have to be satisfied. First, the principal investor needs to have enough resources. In other words,

$$\bar{w} \geq \frac{2\kappa}{2\theta - 1}. \quad (18)$$

Second, the principal investor's participation condition has to be satisfied:

$$V^S - A^S \geq 0 \quad \Rightarrow \quad x(p_H - p_L) \int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz \geq \frac{2\theta\kappa}{2\theta - 1}. \quad (19)$$

Henceforth we make the following assumption:

- *Assumption 2:* $x(p_H - p_L) \int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz \geq (1 + \frac{1}{2\theta - 1})\kappa$ and $\bar{w} \geq \frac{2\kappa}{2\theta - 1}$.

The above assumption speaks to the two roles that a small κ plays: (i) A small κ ensures that the expected value of an incentive compatible wage payment is low enough such that a principal is willing to hire an agent trader; and (ii) A small κ ensures that there exists an incentive compatible contract given the value of \bar{w} . The agent-trader's expected wage payment is equal to $2\theta\kappa/(2\theta - 1)$, which is greater than the cost he incurred to collect information, κ . The difference is $\kappa/(2\theta - 1)$, which can be interpreted as the information rent due to the agency problem.

Since the Standard Contract does not incorporate the market information into monitoring, it is suboptimal to a Mark-to-Market Contract, which uses the information

contained in the interim price, thus lower the monitoring cost. We will discuss this in more detail in the next subsection.

3.2 Mark-to-Market Contract

To simplify the notation, we define an agent-trader's expected payoff as following:

$$A_{\lambda\eta}^{\chi} = \int_{-\infty}^{\infty} w(p(z), \lambda, v_{\eta}) \phi^{\chi}(z) dz, \quad (20)$$

where $\chi = +, -, \text{ or } 0$; $\lambda = b \text{ or } s$; and $\eta = H \text{ or } L$. Basically, $A_{\lambda\eta}^{\chi}$ is the expected payoff to the trader when his trading strategy is λ (buy or sell x), the informed order flow is χ , and the realized liquidation value is v_{η} .

A Mark-to-Market Contract $w(p, \lambda, v)$ is incentive compatible if it satisfies:

$$(IC.1) \quad \begin{aligned} \theta A_{bH}^+ + (1-\theta)A_{bL}^+ &\geq \theta A_{sH}^0 + (1-\theta)A_{sL}^0 \\ (1-\theta)A_{sH}^- + \theta A_{sL}^- &\geq (1-\theta)A_{bH}^0 + \theta A_{bL}^0, \end{aligned} \quad (21)$$

and

$$(IC.2) \quad \begin{aligned} &\frac{1}{2} [\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\ &\geq \max \left\{ \frac{1}{2} [\theta A_{bH}^+ + (1-\theta)A_{bH}^0 + (1-\theta)A_{bL}^+ + \theta A_{bL}^0] + \kappa, \right. \\ &\quad \left. \frac{1}{2} [\theta A_{sH}^0 + (1-\theta)A_{sH}^- + (1-\theta)A_{sL}^0 + \theta A_{sL}^-] + \kappa \right\}. \end{aligned} \quad (22)$$

Note that (IC.2) can be rewritten as:

$$(IC.2) \quad \begin{aligned} \frac{1}{2} [\theta A_{bH}^+ + (1-\theta)A_{bL}^+] &\geq \frac{1}{2} [\theta A_{sH}^0 + (1-\theta)A_{sL}^0] + \kappa \\ \frac{1}{2} [(1-\theta)A_{sH}^- + \theta A_{sL}^-] &\geq \frac{1}{2} [(1-\theta)A_{bH}^0 + \theta A_{bL}^0] + \kappa, \end{aligned} \quad (23)$$

so it is clear that (IC.2) implies (IC.1). Intuitively, if a trader collects information at a cost ex-ante, then he will make a full use of it ex-post.

The expected payoff to the trader can be written as:

$$A = \frac{1}{2} [\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-], \quad (24)$$

and the optimal contract $w^*(p, \lambda, v)$ solves the following programming problem:

$$\min_{w \in W} A \text{ subject to the IC conditions in (23).} \quad (25)$$

Before we proceed to study the properties of the optimal contract $w^*(p, \lambda, v)$, we state the following lemma.

Lemma 1 *In any optimal incentive compatible contract solving (25), the constraints in (23) are binding.*

Proof: If the first inequality is not binding, then we can lower $w(p, b, v)$ a bit, and both inequalities will still hold as long as the change in $w(p, b, v)$ is small. Similarly, if the second inequality is not binding, then we can lower $w(p, s, v)$ a bit while not violating the inequalities. ■

The result in the above lemma will be used to prove the following proposition, in which we show that the optimal compensation contract is simple; the trader will either receive a positive fixed wage, \bar{w} , or zero, depending on how his trading position (λ) aligns with the interim price (p) and the final liquidation value (v).

Proposition 2 (Optimal Mark-to-Market Contract) *Given the pricing function in (8), the optimal contract $w^*(p, \lambda, v)$ solving (25) is characterized by four cutoff values, denoted as $z^* = \{z_{bH}^*, z_{bL}^*, z_{sH}^*, z_{sL}^*\}$ such that: (i) for any $p < p(z_{b\eta}^*)$, $w^*(p, b, v_\eta) = 0$, and for any $p \geq p(z_{b\eta}^*)$, $w^*(p, b, v_\eta) = \bar{w}$; for any $p > p(z_{s\eta}^*)$, $w^*(p, s, v_\eta) = 0$, and for any $p \leq p(z_{s\eta}^*)$, $w^*(p, s, v_\eta) = \bar{w}$. (ii) $z_{\lambda\eta}^*$ satisfy the following conditions:*

$$z_{bH}^* - z_{bL}^* = z_{sH}^* - z_{sL}^* = \frac{\sigma^2}{x} \ln\left(\frac{1-\theta}{\theta}\right), \quad (26)$$

$$\begin{aligned} \frac{1}{2}\bar{w}\{\theta[1-\Phi^+(z_{bH}^*)] + (1-\theta)[1-\Phi^+(z_{bL}^*)]\} &= \frac{1}{2}\bar{w}[\theta\Phi^0(z_{sH}^*) + (1-\theta)\Phi^0(z_{sL}^*)] + \kappa \\ \frac{1}{2}\bar{w}[(1-\theta)\Phi^-(z_{sH}^*) + \theta\Phi^-(z_{sL}^*)] &= \frac{1}{2}\bar{w}\{(1-\theta)[1-\Phi^0(z_{bH}^*)] + \theta[1-\Phi^0(z_{bL}^*)]\} + \kappa, \end{aligned} \quad (27)$$

and

$$z_{bH}^* - z_{sH}^* = z_{bL}^* - z_{sL}^* > 2x, \quad (28)$$

where $\Phi^x(\cdot)$ is the cumulative distribution functions corresponding with the probability distribution functions $\phi^x(\cdot)$ defined in (6).

Proof: See Appendix. ■

Proposition 2 says that the optimal contract is a marked-to-market one that depends on both the interim price and the final security market price, as well as the interim trading position of the trader. Because the direct investor always trades truthfully, there is information in the interim price. This information can be used by the principal to provide the trader with an incentive to acquire information and trade accordingly. The optimal contract punishes the trader when both the interim market price and the final liquidation value are against the direction of his trade at the interim date.

The four equations in Proposition 2, equations (26) – (27), are four equations with four unknowns, z_{bH}^* , z_{bL}^* , z_{sH}^* and z_{sL}^* , which characterize the optimal contract. As the pricing function in (8) is monotone in z , it is perhaps easiest to think of the cutoff values in z as corresponding to prices. Then, the proposition says that the optimal contract consists of a step function (this is the result of monotone likelihood ratio property of the normal distribution as we discussed earlier) in terms of the security price p for each pair of λ and v .

Figure 1 portrays Proposition 2 in terms of the z -cut-offs. The cut-off order flows depend on the trader's interim trading position (buy or sell) and on the final liquidation value (H or L). Since prices and order flow cut-offs are isomorphic, one can think in terms of price directly.

So, for example, if the trader buys the stock when the final value is H and the price at date 1, p , was lower than a certain level, $p(z_{bH}^*)$, he will receive a zero payoff regardless of whether the realized liquidation value is high or low. If p is higher than $p(z_{bL}^*)$, he will receive \bar{w} regardless of the liquidation value. In between these two price cut-offs, his payoff will be depend on the final liquidation value, and he will receive zero if the liquidation value is v_L , and \bar{w} , otherwise. Similarly, if the trader sells the stock and the security price at date 1, p , is higher than a certain level, $p(z_{sL}^*)$, he will receive a zero payoff; if p is lower than $p(z_{sH}^*)$, he will receive \bar{w} for sure. In between, he will receive zero if the liquidation value is v_H , \bar{w} otherwise. We can interpret these results as a “mark-to-market” contract for the trader.

In terms of the trader's expected compensation, Proposition 2 implies that $A_{bH}^+ > A_{bL}^+$ and $A_{sL}^- > A_{sH}^-$, direct results of (26). So the trader receives a lower payoff if the realized liquidation value contradicts the trader's prior trading decision, on average. In other words, he gets a lower payoff if he bought the stock, but its final value was L . Also, notice that the difference between z_{bH}^* and z_{bL}^* (or z_{sH}^* and z_{sL}^*) is greater when the signal is more accurate, that is, when the value of θ is greater. Intuitively, the trader gets a harsher punishment if the probability of receiving a wrong signal is lower.

If there were no upper bound on the contract payment, the optimal contract would be to pay an infinitely high amount to the trader when he bought the stock, but only when the price was close to p_H (that is, when the order flow is infinitely large) or when he sold and

the price was close to p_L . The existence of \bar{w} guarantees the boundedness of the optimal contract. One way to interpret the upper bound \bar{w} is to imagine that the trader is risk averse instead of risk neutral. Then paying a very high wage but only with a small probability is not optimal for the principal since the risk premium required by the trader will make the expected payment very high. With a risk-averse trader, the existence of \bar{w} could be justified and endogenized. We do not pursue this here.

We have already discussed the trader's incentive constraints above, and in order to demonstrate that the optimal contract characterized above can be sustained in a Nash equilibrium, we need to check two more things: (i) there is no other contract that dominates the contract characterized in Proposition 2, and (ii) the payoff to the principal-investor is higher than his best alternative payoff without hiring an agent-trader.

For (i), we have shown that the optimal contract is the best one among all incentive compatible contracts, and we only need to properly specify the trader's response when a non-incentive compatible contract is offered to make sure the principal will not offer any of those. For example, we can assume that the trader will shirk and always sell when such a contract is offered.

For (ii), we need to compare the payoff to the principal with the payoff in the case in which no trader is hired. The expected net payoff of the investment under informed trading is:

$$\begin{aligned} V &= \frac{1}{2}x \left[\int_{-\infty}^{\infty} [p_H - p(z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p(z) - p_L]\phi^-(z)dz \right] \\ &= x(p_H - p_L) \int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz. \end{aligned} \quad (29)$$

The quantity V is the joint expected payoff to the principal and the trader (also the expected payoff to the direct investor), that is, $V = V_P + A$, with V_P and A denoting the payoffs to the principal and the trader, respectively.

If the principal does not hire a trader, but rather trades for himself, his payoff could be one of the following three payoffs: V_b , the payoff from uninformed buying at date 1; V_s , the payoff from uninformed selling; or V_0 , the payoff from doing nothing. Obviously we need to have $V_P \geq \max\{V_b, V_s, V_0\}$. We have:

$$\begin{aligned} V_b &= \frac{1}{2}x \left\{ \int_{-\infty}^{\infty} [p_H - p(z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p_L - p(z)]\phi^0(z)dz \right\} \\ V_s &= \frac{1}{2}x \left\{ \int_{-\infty}^{\infty} [p(z) - p_H]\phi^0(z)dz + \int_{-\infty}^{\infty} [p(z) - p_L]\phi^-(z)dz \right\} \\ V_0 &= 0. \end{aligned} \quad (30)$$

It is easy to show $V_b < 0$ and $V_s < 0$.¹¹ Therefore, $V_P \geq \max\{V_b, V_s, V_0\}$ is equivalent to $V_P \geq 0$.

Proposition 3 (Existence and Uniqueness of the Optimal Mark-to-Market Contract) Under Assumption 2,¹² there exists a unique optimal Mark-to-Market Contract is used by the principal-investor with the pricing function defined in (8).

Proof: See Appendix. ■

The Standard Contract is a feasible choice because it also satisfies (IC.2) in (23), but it is dominated by the Mark-to-Market contract because the latter tightens monitoring through using the market price. The gross payoffs from the informed trading are exactly the same; however, the monitoring cost is smaller with the Mark-to-Market contract.

To conclude this section, we discuss how the payoffs to the market participants change with model parameter values. First, let us write out the expected payoff to the liquidity trader:

$$V_N = \frac{1}{2} \int_{-\infty}^{\infty} z \{ [p_H - p(z + 2x)] + [p_L - p(z - 2x)] \} \phi^0(z) dz. \quad (31)$$

With $\phi^0(z) = \phi^+(z + 2x) = \phi^-(z - 2x)$, some algebra leads us to $V_N + 2V = 0$, which confirms the zero-sum game property of this type of model. Recall from (29) that V is the expected value of trading to an informed investor, either the direct investor or the principal-agent pair.

The next lemma helps us understand how the cutoff values in z associated with the optimal Mark-to-Market Contract change with the cost of information production, κ , and the maximum wage, \bar{w} .

Lemma 2 Let $z^* = \{z_{bH}^*, z_{bL}^*, z_{sH}^*, z_{sL}^*\}$ be the solution for an optimal Mark-to-Market Contract. Then $\frac{\partial z_{bH}^*}{\partial \kappa} = \frac{\partial z_{bL}^*}{\partial \kappa} < 0$, $\frac{\partial z_{sH}^*}{\partial \kappa} = \frac{\partial z_{sL}^*}{\partial \kappa} > 0$, $\frac{\partial z_{bH}^*}{\partial \bar{w}} = \frac{\partial z_{bL}^*}{\partial \bar{w}} > 0$, and $\frac{\partial z_{sH}^*}{\partial \bar{w}} = \frac{\partial z_{sL}^*}{\partial \bar{w}} < 0$.

Proof: See Appendix. ■

Lemma 2 can be understood with reference to Figure 1. Given the relationship between z^* and A defined in (24), the lemma above says that the principal has to pay the trader

¹¹ We use the following two inequalities to show $V_b < 0$ and $V_s < 0$:

$$\int_{-\infty}^{\infty} [p_L - p(z)] \phi^0(z) dz < \int_{-\infty}^{\infty} [p_L - p(z)] \phi^-(z) dz$$

$$\int_{-\infty}^{\infty} [p(z) - p_H] \phi^0(z) dz < \int_{-\infty}^{\infty} [p(z) - p_H] \phi^+(z) dz.$$

¹² The existence of an Optimal Mark-to-Market contract requires a weaker condition than Assumption 2, which also guarantees the incentive compatibility and the principal's profitability of a Standard Contract.

more when the cost of information production increases. When the cost of information production rises, the z^* cut-off values move so that the trader receives \bar{w} over a wider range of outcomes to cover the higher cost of information production. But, the compensation cannot be so high that the principal does not want to hire a trader. The second part of the lemma relates the compensation, \bar{w} , to the cut-offs. If \bar{w} rises, then the cut-offs have to move to reduce the range of prices over which the trader gets compensated –since when he does get compensated, the compensation is higher.

The joint expected payoff to the principal and the trader is not affected by the parameter values of κ or \bar{w} , which affect the contractual payoff distribution between the principal and his trader, but are affected by σ , the size of noise trading, and θ , the precision of information production. We summarize the payoff dependency in the proposition below.

Proposition 4 (Comparative Statics) *With the optimal mark-to-market contract, (i) the joint expected payoff to the principal and the trader is increasing with σ and θ , and the expected payoff to the liquidity traders is decreasing with σ and θ ; (ii) the expected net payoff to the trader is increasing with σ , increasing with κ , decreasing with \bar{w} , and decreasing with θ ; the expected payoff to the principal is decreasing with κ , increasing with \bar{w} , and increasing with θ .*

Proof: See Appendix. ■

The first result is the same as Kyle (1985), where a privately informed trader always benefits from a higher σ , while a liquidity trader is worse off with a higher σ . An informed trader can benefit from a higher σ (i.e., a wider dispersion of liquidity trade) because the monitoring role of the price is less effective since the price is less informative; when σ is smaller, the principal can offer a contract that is more sensitive to the price and thus reduce the payoff to his trader. However, it is not clear how the value of σ affects the expected payoff to the principal. On the one hand, a higher σ leads to a higher expected wage payment to the trader, on the other hand, the joint payoff, V , also increases with σ .

With regard to θ , a higher precision in information production allows the information producers to take a greater advantage of the liquidity traders, thus improving their payoffs.

The results on κ and \bar{w} can be interpreted as follows. When the (opportunity) cost of information production, κ , is higher, the principal has to pay the trader more. When we increase \bar{w} , the optimal contract with the original lower w is still feasible, but we know from Proposition 1 that there is still slack with respect to the conditions for the optimal contract with the higher \bar{w} , i.e., it can be improved upon. Thus, it results in a lower payoff to the trader with a higher \bar{w} . Intuitively, when \bar{w} is higher, the contract in the form of “ \bar{w} or nothing” allows the principal to pay more when the market price is more extreme, thus generating a stronger incentive for the trader to produce information.

To summarize, if there is an informative, verifiable, signal – namely, the interim security market price-- in addition to the final value, then, perhaps not surprisingly, it is

optimal to include it as part of the optimal contract, as long as doing so is not so expensive for the principal. This is the essential logic of the mark-to-market contract. The direct trader has no agency problem and always produces information and trades optimally. Although his information is private, his order is going to move the market price set by the rational market maker. The principal makes use of this behavior of the direct trader to monitor his agent-trader. He free rides on the information in the price that is due to the direct trader.

The existence of the direct investor guarantees that the market price is informative, at least to some extent. However, the idea that the interim market price is “efficient” does not rule out the possibility that traders will trade in suboptimal ways, and that they may alter the information reflected by the market price. In case this happens, the principals need to incorporate it into the optimal contracts, and may even use contracts that are not marked-to-market. In the next section, we begin exploring these issues in the trading context.

4. Competition in Contracts with Multiple Principal-Investors

4.1 Externality Effect of Marking-to-Market

In the previous section, the optimal Mark-to-Market Contract is signed based upon three variables: the realized value (or final price), the interim security market price, and the trader’s trading position. The interim price is formed in response to the trading positions of all the market participants, some of whom produce information. Because the interim price is informative, it can be used to tighten the monitoring of the trader. Due to a positive correlation (in our case, a correlation of one) of the signals received by different agents, a hard-working trader likely should have received a high signal when the interim price is pushed up by the buying orders from other informed investors, and a low signal when the interim price is pushed down by the selling orders from other informed investors. By examining this consistency, the principal makes sure that the trader’s good performance is the result of hard-work, rather than the outcome of good luck.

If the informativeness of the interim market price can act as a monitoring device, then we have to address the question: where does the information come from? The assumption of the existence of a direct investor provides a trivial answer: it comes from the direct investor who trades for himself and is not subject to the moral hazard problem. What if the market is mainly dominated by delegated traders? In this case, the information has to come from other delegated traders. In other words, the information injected into the market by some traders is used to monitor other traders and vice versa. If traders understand the mutual monitoring feature of the interim price, then they can act strategically to jointly undo the monitoring effects of the price.

Lemma 3 *Suppose there are two delegated traders (each hired by a principal) who each received the optimal Mark-to-Market Contract we solved for in Proposition 2 of Section 3. Then if one trader shirks and buys (sells) x shares, it is the other trader’s best response*

to follow. Joint shirking gives traders higher payoffs than what they receive by not shirking.

Proof: See Appendix. ■

Roughly, when there are multiple delegated traders, the optimal contract pays a high wage to a trader when his trading position is consistent with the market price, that is, the information impounded in the price due to the trading by some traders is used to monitor the other traders. However, when traders understand this mechanism, they have an incentive to jointly shirk and trade in the same direction, moving the price, if they can find a way to coordinate their trading. By so doing, they can “manipulate” the market such that the interim price is likely to move in line with their trading position, enabling them to take advantage of the Mark-to-Market Contract. Of course, if principal-investors anticipate this, then the optimal contract should reflect the possibility of this joint moral hazard problem.¹³ If the probability of market manipulation is small, investors are still better off by offering the Mark-to-Market Contract. Otherwise, investors may consider other contracts to disrupt the traders’ coordinated manipulation. In particular, a Standard Contract might be appealing to an investor-principal as it eliminates the incentive of coordinated price manipulation. In the next two subsections, we formally study the equilibrium in which the traders coordinate to shirk and trade in the same direction due to the externality effect of a Mark-to-Market Contract.

4.2 Pure Strategy Manipulation Equilibrium

To proceed, let us assume that there are two delegated traders in the market,¹⁴ and we introduce an irrelevant noise signal, r , which is, with probability q , observed by the traders before they make their effort to acquire information. The signal may be thought of as a news story, a rumor, a technical trading signal, a superstitious event, and so on. Traders regularly communicate, these days by Bloomberg mail and e-mail, previously by phone. The social structure of markets, including the culture and communication, is essentially subsumed by our assumption of an irrelevant signal, but that background makes the idea quite plausible. See, e.g., Abolafia (1996) and Cetina and Bruegger (2002).

The noise signal is independent of the fundamentals: the relevant signal s and the liquidation value v . It should be ignored. But, it serves as a coordination device. Further we assume that the irrelevant signal has two values: 0 or 1, and they are observed with

¹³ As we can show, the result in Lemma 3 still holds when the market price and the optimal contract reflect the possibility of joint shirking.

¹⁴ We take out the direct investor from the model mainly for two reasons: (i) With only two informed traders in the market, it is easy to compare the results in this section with those in the last section; (ii) This emphasizes the idea that the agent-traders can coordinate to shirk and herd in trading only when they constitute a substantial fraction of market participants and their trades could possibly move the market.

equal probability.¹⁵ Upon observing the irrelevant signal, the traders both buy x shares if the signal is 1 or sell x shares if the signal is 0, without acquiring the information.¹⁶

In this subsection, we will discuss the equilibrium in which only mark-to-market contracts are offered. As we have discussed earlier, when mark-to-market contracts are offered to agent-traders in the economy, they have an incentive to use the noise signal as a coordination device and trade the asset in the same direction without producing any information and, in this way, they can manipulate the price to get a higher payoff. We call this type of equilibrium a *Manipulation Equilibrium*, which we formally define below:

Definition 1 (A Pure Strategy Manipulation Equilibrium) *A Pure Strategy Manipulation Equilibrium is a Nash equilibrium in which a trader produces information and trades truthfully only when he does not observe the irrelevant signal. Upon observing the irrelevant signal, the traders both shirk and buy x shares if the signal is 1, and they both sell x shares if the signal is 0.*

In the above definition, the agent-traders coordinate on the irrelevant signal, r , to manipulate the market price, and the probability of this noise signal, q , can be interpreted as the “propensity of shirking”.

In a Pure Strategy Manipulation Equilibrium, the pricing function is:

$$p^q(z) = \frac{(1-q)[\phi^+(z)p_H + \phi^-(z)p_L]}{\phi^+(z) + \phi^-(z)} + \frac{1}{2}q(p_H + p_L). \quad (32)$$

Given the equilibrium pricing function $p^q(z)$, and with $A_{\lambda\eta}^x$ similarly defined as in (9), the optimal Mark-to-Market Contract $w^M(p, \lambda, \nu)$, which allows for coordinated price manipulation by agent-traders upon observing the noise signal, must satisfy the following incentive constraints:

When informed, a trader is willing to trade truthfully when the other trader is doing so:

$$(IC.M1) \quad \begin{aligned} \theta A_{bH}^+ + (1-\theta)A_{bL}^+ &\geq \theta A_{sH}^0 + (1-\theta)A_{sL}^0 \\ (1-\theta)A_{sH}^- + \theta A_{sL}^- &\geq (1-\theta)A_{bH}^0 + \theta A_{bL}^0. \end{aligned} \quad (33)$$

In the absence of the irrelevant signal, a trader has an incentive to produce information when the other trader is doing so:

¹⁵ This equal probability assumption is purely for simplification, and our analysis below can be generalized to any probability specification between (including) zero and one.

¹⁶ Of course, the traders can buy (or sell) on both signal values, and that is equivalent to assuming that the probability of $r = 1$ (or 0) is equal to one.

$$\begin{aligned}
& \frac{1}{2} [\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
\text{(IC.M2)} \quad & \geq \max \left\{ \frac{1}{2} [\theta A_{bH}^+ + (1-\theta)A_{bH}^0 + (1-\theta)A_{bL}^+ + \theta A_{bL}^0] + \kappa, \quad (34) \right. \\
& \left. \frac{1}{2} [\theta A_{sH}^0 + (1-\theta)A_{sH}^- + (1-\theta)A_{sL}^0 + \theta A_{sL}^-] + \kappa \right\},
\end{aligned}$$

which can be rewritten as:

$$\begin{aligned}
\text{(IC.M2)} \quad & \frac{1}{2} [\theta A_{bH}^+ + (1-\theta)A_{bL}^+] \geq \frac{1}{2} [\theta A_{sH}^0 + (1-\theta)A_{sL}^0] + \kappa \quad (35) \\
& \frac{1}{2} [(1-\theta)A_{sH}^- + \theta A_{sL}^-] \geq \frac{1}{2} [(1-\theta)A_{bH}^0 + \theta A_{bL}^0] + \kappa,
\end{aligned}$$

Without information, a trader is willing to buy/sell when the other trader is doing so:

$$\begin{aligned}
\text{(IC.M3)} \quad & \frac{1}{2} (A_{bH}^+ + A_{bL}^+) \geq \frac{1}{2} (A_{sH}^0 + A_{sL}^0) \quad (36) \\
& \frac{1}{2} (A_{sH}^- + A_{sL}^-) \geq \frac{1}{2} (A_{bH}^0 + A_{bL}^0).
\end{aligned}$$

Upon observing the noise signal, a trader has an incentive to shirk and buy/sell (instead of producing information and trading truthfully) if the other trader is doing so:

$$\begin{aligned}
\text{(IC.M4)} \quad & \frac{1}{2} (A_{bH}^+ + A_{bL}^+) + \kappa \geq \frac{1}{2} [\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^0 + \theta A_{sL}^0] \quad (37) \\
& \frac{1}{2} (A_{sH}^- + A_{sL}^-) + \kappa \geq \frac{1}{2} [\theta A_{bH}^0 + (1-\theta)A_{bL}^0 + (1-\theta)A_{sH}^- + \theta A_{sL}^-]
\end{aligned}$$

If an incentive compatible mark-to-market contract is offered to both agent-traders, the expected payoff to a trader is:

$$\begin{aligned}
A^M &= \frac{1}{2} (1-q) [\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
& \quad + \frac{1}{4} q (A_{bH}^+ + A_{bL}^+ + A_{sH}^- + A_{sL}^-), \quad (38)
\end{aligned}$$

and the optimal contracting problem can be written as:

$$\min_{w \in W} A^M \text{ subject to the IC conditions in (33) - (37)}. \quad (39)$$

We solve the optimal contracting problem (39) by first solving a weakened problem with only IC condition (IC.M2). Then we show that the optimal solution for the weakened problem satisfies (IC.M1), (IC.M3), and (IC.M4).

By dropping (IC.M1), (IC.M3), and (IC.M4) from the full programming problem in (38), we can write an alternative simplified programming problem:

$$\min_{w \in W} A^M \text{ subject to the IC conditions in (35)}. \quad (40)$$

This problem is similar to the one in (25), and can be characterized in a similar way. First, as analogously to Lemma 1 and Proposition 2, we can show that the constraints in (35) are binding for the optimal solution to (40), and the solution is characterized by four cutoff values $z^* = \{z_{bH}^*, z_{bL}^*, z_{sH}^*, z_{sL}^*\}$. Define:

$$\begin{aligned} \mu_1 &\equiv \frac{1}{1-\theta} \frac{[(1-q)(1-\theta) + \frac{1}{2}q]\phi^+(z_{bL}^*)\phi^-(z_{sL}^*) + [(1-q)\theta + \frac{1}{2}q]\phi^0(z_{bL}^*)\phi^-(z_{sL}^*)}{\phi^0(z_{bL}^*)\phi^0(z_{sL}^*) - \phi^+(z_{bL}^*)\phi^-(z_{sL}^*)} \\ \mu_2 &\equiv \frac{1}{1-\theta} \frac{[(1-q)(1-\theta) + \frac{1}{2}q]\phi^+(z_{bH}^*)\phi^-(z_{sH}^*) + [(1-q)\theta + \frac{1}{2}q]\phi^+(z_{bH}^*)\phi^0(z_{sH}^*)}{\phi^0(z_{bH}^*)\phi^0(z_{sH}^*) - \phi^+(z_{bH}^*)\phi^-(z_{sH}^*)}. \end{aligned} \quad (41)$$

In the following proposition, we first characterize the solution to (40), then show that it is also the solution to (39).

Proposition 5 (Characterization of the Optimal Contract) *In a Pure Strategy Manipulation Equilibrium, (i) the optimal contract $w^*(p, \lambda, v)$ takes the following form: when $\lambda = b$, there exists $z_{b\eta}^*$ such that, for any $p < p(z_{b\eta}^*)$, $w^*(p, b, v_\eta) = 0$, and for any $p \geq p(z_{b\eta}^*)$, $w^*(p, b, v_\eta) = \bar{w}$; when $\lambda = s$, there exists $z_{s\eta}^*$ such that, for any $p > p(z_{s\eta}^*)$, $w^*(p, s, v_\eta) = 0$, and for any $p \leq p(z_{s\eta}^*)$, $w^*(p, s, v_\eta) = \bar{w}$. Moreover, (ii) $z_{\lambda\eta}^*$ satisfy the following conditions:*

$$\begin{aligned} z_{bH}^* - z_{bL}^* &= \frac{\sigma^2}{2x} \left[\ln\left(\frac{1-\theta}{\theta}\right) + \ln\left(\frac{2(1-\theta)[(1-q) + \mu_1] + q}{2\theta[(1-q) + \mu_1] + q}\right) \right] \\ z_{sH}^* - z_{sL}^* &= \frac{\sigma^2}{2x} \left[\ln\left(\frac{1-\theta}{\theta}\right) + \ln\left(\frac{2(1-\theta)[(1-q) + \mu_2] + q}{2\theta[(1-q) + \mu_2] + q}\right) \right], \end{aligned} \quad (42)$$

$$\begin{aligned} \frac{1}{2}\bar{w}\{\theta[1 - \Phi^+(z_{bH}^*)] + (1-\theta)[1 - \Phi^+(z_{bL}^*)]\} &= \frac{1}{2}\bar{w}[\theta\Phi^0(z_{sH}^*) + (1-\theta)\Phi^0(z_{sL}^*)] + \kappa \\ \frac{1}{2}\bar{w}[(1-\theta)\Phi^-(z_{sH}^*) + \theta\Phi^-(z_{sL}^*)] &= \frac{1}{2}\bar{w}\{(1-\theta)[1 - \Phi^0(z_{bH}^*)] + \theta[1 - \Phi^0(z_{bL}^*)]\} + \kappa, \end{aligned} \quad (43)$$

and

$$z_{bH}^* - z_{sH}^* = z_{bL}^* - z_{sL}^* > 2x. \quad (44)$$

Proof: See Appendix. ■

As before, the four equations (42) and (43) allow for the determination of the four z-cutoff points. Immediately, we can check that when $q = 0$, the above proposition is reduced to Proposition 2. Proposition 5 also implies that $A_{b\eta}^0 < A_{b\eta}^+$ and $A_{s\eta}^0 < A_{s\eta}^-$, which says that under the optimal contract, when the trader buys/sells the stock, he will be better off if the other trader is doing the same. This generates the externality effect, which is at the root of the existence of an equilibrium with manipulation.

Similar to the last section, in order to demonstrate that the optimal contract characterized above can be sustained in a Nash equilibrium, we need to check that the payoff to the principal is higher than his best alternative payoff if he does not hire a trader.

In a Pure Strategy Manipulation Equilibrium, the expected net payoff of the investment under informed trading is:

$$\begin{aligned} V^M &= \frac{1}{2}(1-q)x \left\{ \int_{-\infty}^{\infty} [p_H - p^q(z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p^q(z) - p_L]\phi^-(z)dz \right\} \\ &+ \frac{1}{2}qx \left\{ \int_{-\infty}^{\infty} \left[\frac{p_H + p_L}{2} - p^q(z) \right] \phi^+(z)dz + \int_{-\infty}^{\infty} \left[p^q(z) - \frac{p_H + p_L}{2} \right] \phi^-(z)dz \right\}. \end{aligned} \quad (45)$$

Again, let V_P denote the payoffs to the principal. We need to have $V_P \geq \max\{V_b, V_s, V_0, V_P^S\}$, where V_b is the payoff from uninformed buying at date 1; V_s , the payoff from uninformed selling; V_0 , the payoff from doing nothing, and V_P^S , the off-equilibrium payoff from offering an optimal standard contract. We have:

$$\begin{aligned} V_b &= \frac{1}{2}(1-q)x \left\{ \int_{-\infty}^{\infty} [p_H - p^q(z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p_L - p^q(z)]\phi^0(z)dz \right\} \\ &+ \frac{1}{2}qx \left\{ \int_{-\infty}^{\infty} \left[\frac{p_H + p_L}{2} - p^q(z) \right] \phi^+(z)dz + \int_{-\infty}^{\infty} \left[\frac{p_H + p_L}{2} - p^q(z) \right] \phi^0(z)dz \right\} \\ &= \frac{1}{2}x \left\{ \int_{-\infty}^{\infty} [p_H - p^q(z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p_L - p^q(z)]\phi^0(z)dz \right\} \\ V_s &= \frac{1}{2}(1-q)x \left\{ \int_{-\infty}^{\infty} [p^q(z) - p_H]\phi^0(z)dz + \int_{-\infty}^{\infty} [p^q(z) - p_L]\phi^-(z)dz \right\} \\ &+ \frac{1}{2}qx \left\{ \int_{-\infty}^{\infty} \left[p^q(z) - \frac{p_H + p_L}{2} \right] \phi^0(z)dz + \int_{-\infty}^{\infty} \left[p^q(z) - \frac{p_H + p_L}{2} \right] \phi^-(z)dz \right\} \\ &= \frac{1}{2}x \left\{ \int_{-\infty}^{\infty} [p^q(z) - p_H]\phi^0(z)dz + \int_{-\infty}^{\infty} [p^q(z) - p_L]\phi^-(z)dz \right\} \\ V_0 &= 0. \end{aligned} \quad (46)$$

We can again show $V_b < 0$ and $V_s < 0$.¹⁷

On the other hand, off the equilibrium path, if one principal offers a standard contract while the other principal is offering a mark-to-market contract, the expected joint payoff to the principal and the trader can be written as:

$$V^S = \frac{1}{2} x \left\{ \int_{-\infty}^{\infty} [p_H - p^q(z)] \phi^+(z) dz + \int_{-\infty}^{\infty} [p^q(z) - p_L] \phi^-(z) dz \right\} \quad (47)$$

We have $V_p^S = V^S - A^S$, with A^S defined in (13) and (17).

Therefore, $V_p \geq \max\{V_b, V_s, V_0, V_p^S\}$ is equivalent to $V^M - A^M \geq \max\{0, V^S - A^S\}$ with V^M defined in (45), and A^M defined in (38). We have the following relationship between V^M and V^S and between A^M and A^S .

Lemma 4 *In a Pure Strategy Manipulation Equilibrium, we have $V^S = V^M + \frac{1}{2} qx(p_H - p_L) > V^M$ and $A^S > A^M$.*

Proof: See Appendix. ■

While $V^S > V^M$ reflects the benefit of using a standard contract, $A^S > A^M$ reflects the cost of using a standard contract. On one hand, a standard contract precludes the possibility of market manipulation and provides the agent-trader with an incentive to acquire information; on the other hand, such a contract results in more costly monitoring due to its failure to fully utilize the market price information to monitor the agent-trader. A Pure Strategy Manipulation Equilibrium exists when the cost of a standard contract dominates its benefit

Define the principal's payoff from using a mark-to-market contract as $V_p^M = V^M - A^M$, with V^M and A^M defined in (45) and (38), respectively; Define the principal's payoff from using a standard contract as $V_p^S = V^S - A^S$, with V^S and A^S defined in (47) and (17), respectively; Define the difference as $\Delta V_p = V_p^M - V_p^S$. The next lemma contains some technical results, which will be used for our existence proof.

Lemma 5 *V_p^M is decreasing with q , V_p^S is increasing with q and ΔV_p is decreasing q .*

Proof: See Appendix. ■

¹⁷ Similar, we use the following two inequalities to show $V_b < 0$ and $V_s < 0$:

$$\int_{-\infty}^{\infty} [p_L - p^q(z)] \phi^0(z) dz < \int_{-\infty}^{\infty} [p_L - p^q(z)] \phi^-(z) dz$$

$$\int_{-\infty}^{\infty} [p^q(z) - p_H] \phi^0(z) dz < \int_{-\infty}^{\infty} [p^q(z) - p_H] \phi^+(z) dz,$$

The above lemma says that, with a higher propensity of shirking, a principal is worse off when a mark-to-market contract is used, while on the other hand he is better off if he uses a standard contract. Obviously, a mark-to-market contract is used only when q is not too large, when the propensity cost of a standard contract dominates its benefit.

Proposition 6 (Existence) *Under Assumption 2, there exists a $\bar{q} \in (0,1)$ such that a Pure Strategy Manipulation Equilibrium exists if and only if $0 \leq q \leq \bar{q}$.*

Proof: See Appendix. ■

Given a small κ , a small q has three roles: (i) It guarantees that the space of incentive compatible contracts is not empty when the price is less informative due to coordinated manipulation and it is harder to monitor the behavior of an agent; (ii) It ensures that the principal has a sufficient incentive to hire an agent trader even though the agents in the economy coordinate to manipulate the price with some probability; (iii) It ensures that the benefit of using a standard contract in comparison with a mark-to-market contract is not large enough to cover the extra cost of using a standard contract.

From the above discussion, we can infer that for a given cost of information production, κ , there exists a maximum level of the shirking propensity, \bar{q} , below which a Pure Strategy Manipulation Equilibrium may exist. Moreover, we have $\lim_{\kappa \rightarrow 0} \bar{q}(\kappa) = 0$ because as κ goes to zero, the extra wage payment from using a standard contract goes to zero, without q going to zero. So, a standard contract will dominate a mark-to-market contract. However, a standard contract can never sustain in a pure strategy equilibrium even in those parameter regions where a Pure Strategy Manipulation Equilibrium does not exist.¹⁸ Intuitively, when one principal-investor offers a standard contract, his agent-trader will trade truthfully, but then the other principal-investor will be better off by offering a mark-to-market contract as there is no risk of price manipulation. In the next subsection, we analyze a mixed strategy equilibrium, in which both principal-investors mix between a mark-to-market contract and a standard contract.

4.3. A Mixed Strategy Manipulation Equilibrium

From the above analysis, we saw that employing the interim security price to monitor an agent-trader's behavior could be costly: it could potentially reduce the benefit of hiring a trader since the traders can jointly shirk with probability q based on some coordination device. When q gets large, there are two effects: First, it is possible that the gain from using the market price as a monitoring device is cancelled out by the cost of hiring a trader due to the traders' joint manipulation of the market price. Second, maybe more interestingly, the benefit of a standard contract becomes larger than its cost.

¹⁸ Here we restrict our attention to subgame perfect equilibrium. It is possible that a standard contract can sustain in an equilibrium with threats.

With the second effect, the mark-to-market contract cannot sustain in a pure strategy equilibrium. In this case, principal-investors may choose to hire agent-traders with a standard contract. As we have already discussed, a standard contract cannot sustain in a pure strategy equilibrium, so a mixed strategy equilibrium may exist. Intuitively, the benefit of a standard contract grows as the propensity of shirking, q , increases, and when q is too large, a Pure Strategy Manipulation Equilibrium breaks down. However, mixing between a mark-to-market contract and a standard contract might mitigate the second effect as the benefit of a standard contract decreases as the probability of the other principal using a standard contract increase, until a principal is indifferent between the two types of contract.

More specifically, assume that traders can observe each other's contracts, thus they can coordinate to shirk when they both receive a mark-to-market contract. If at least one trader receives a standard contract, they have no choice but to produce information and trade truthfully because the standard contract does not depend on the interim price. However, when both traders receive a mark-to-market contract, they can coordinate on the irrelevant signal to shirk. Therefore, the principal can mix between a standard contract and a mark-to-market contract to reduce the overall propensity of shirking.

Definition 2 (A Mixed Strategy Manipulation Equilibrium) *A Mixed Strategy Manipulation Equilibrium is a Nash equilibrium in which principals and traders adopt the following strategies:*

- (i) *Both principals independently offer a mark-to-market contract with probability m and a standard contract with probability $1-m$;*
- (ii) *If both traders receive a mark-to-market contract, then they shirk upon observing, with probability q , an irrelevant noise signal r ; otherwise they collect information and trade truthfully.*

With the equilibrium strategies described above, shirking will occur with probability $\hat{q} = m^2 q$, when both traders receive the mark-to-market contract and the irrelevant signal appears. The market maker will set the stock pricing function as:¹⁹

$$p^m(z) = \frac{(1 - \hat{q})[\phi^+(z)p_H + \phi^-(z)p_L]}{\phi^+(z) + \phi^-(z)} + \frac{1}{2}\hat{q}(p_H + p_L). \quad (48)$$

Now let us discuss how m is determined in a Mixed Strategy Manipulation Equilibrium described above. When a mark-to-market contract is offered, the expected joint payoff to the principal and the trader can be written as:

¹⁹ We assume that the market maker cannot observe the contracts received by the traders and does not know when manipulation exactly occurs. If the market maker can observe the contracts, then he can set the price contingent on the principals' contract choices at the interim date. Our results would still hold, but the analysis would be more complicated.

$$\begin{aligned}
V^M &= \frac{1}{2}(1-mq)x \left\{ \int_{-\infty}^{\infty} [p_H - p^m(z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p^m(z) - p_L]\phi^-(z)dz \right\} \\
&+ \frac{1}{2}mqx \left\{ \int_{-\infty}^{\infty} \left[\frac{p_H + p_L}{2} - p^m(z) \right] \phi^+(z)dz + \int_{-\infty}^{\infty} \left[p^m(z) - \frac{p_H + p_L}{2} \right] \phi^-(z)dz \right\} \quad (49) \\
&= \frac{1}{2}x \left\{ \int_{-\infty}^{\infty} [p_H - p^m(z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p^m(z) - p_L]\phi^-(z)dz \right\} - \frac{1}{2}mqx(p_H - p_L).
\end{aligned}$$

The expected wage payment to the trader can be written as:

$$\begin{aligned}
A^M &= \frac{1}{2}(1-mq) \left[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^- \right] \\
&+ \frac{1}{4}mq \left(A_{bH}^+ + A_{bL}^+ + A_{sH}^- + A_{sL}^- \right), \quad (50)
\end{aligned}$$

and the expected payoff to the principal is $V_p^M = V^M - A^M$.

Similar to a Pure Strategy Manipulation Equilibrium, the optimal mark-to-market contract solves the following problem:

$$\min_{w \in W} A^M \text{ subject to the IC conditions in (33) - (37)}. \quad (51)$$

The incentive constraints remain the same as in a Pure Strategy Manipulation Equilibrium.

On the other hand, when a standard contract is offered, the expected joint payoff to the principal and the trader can be written as:

$$\begin{aligned}
V^S &= \frac{1}{2}x \left\{ \int_{-\infty}^{\infty} [p_H - p^m(z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p^m(z) - p_L]\phi^-(z)dz \right\} \\
&= V_3^m + \frac{1}{2}mqx(p_H - p_L) > V_3^m. \quad (52)
\end{aligned}$$

The expected payoff to the trader is:

$$A^S = \frac{1}{2}[\theta w_{bH} + (1-\theta)w_{sH} + \theta w_{sL} + (1-\theta)w_{bL}], \quad (53)$$

and the expected payoff to the principal is $V_p^S = V^S - A^S$.

Define the principal's payoff from using a mark-to-market contract as $V_p^M = V^M - A^M$, with V^M and A^M defined in (49) and (50), respectively; Define the principal's payoff from using a standard contract as $V_p^S = V^S - A^S$, with V^S and A^S defined in (52) and (53), respectively; Define the difference as $\Delta V_p = V_p^M - V_p^S$. We have the following lemma.

Lemma 6 V_P^S is increasing with m and ΔV_P is decreasing m .

Proof: See Appendix. ■

In equilibrium, m is such that a principal is indifferent between offering a standard contract and a mark-to-market contract, that is $\Delta V_P = 0$. The next proposition gives a sufficient condition for the existence of m .

Proposition 7 (Existence) Under Assumption 2, when $\bar{q} \leq q \leq 1$, with \bar{q} being the maximum value of q such that a Pure Strategy Manipulation Equilibrium exists, there exists a unique Mixed Strategy Manipulation Equilibrium in which a principal offers a Mark-to-Market Contract with probability $m^*(q)$ and a Standard Contract with probability $1 - m^*(q)$ with $m^*(q) \in (0,1)$. In addition, $m^*(q)$ is decreasing in q .

Proof: See Appendix. ■

A small κ here ensures that the benefit from using a standard contract exceeds the cost when the propensity of manipulation, now measured by m^2q (with fixed q), is relatively large. When the other principal is mixing between a standard contract and a mark-to-market contract, using a standard contract generates a larger joint payoff for a principal investor and his agent trader as it excludes manipulation, and the difference is independent of the cost of information production, κ , as we can see from (53). At the same time, with a standard contract, the expected wage payment to the agent is higher than using a mark-to-market contract and the difference goes to zero as κ goes to zero. A small κ here guarantees that, if a standard contract is used, the payoff to the principal net of the wage payment to his agent is larger than using a mark-to-market contract when m is large enough.

The mixed strategy mitigates the agency problem due to the Mark-to-Market Contract. For a large q , the agency problem is so large that no agency-traders will be hired if they tend to shirk upon noise signals; however, mixing between a standard (non-mark-to-market) contract and a mark-to-market contract reduces the propensity for shirking from q to m^2q . In other words, the principals' mixed strategy reduces the traders' manipulation. This speaks directly to the fact that the existence of a Mixed Strategy Manipulation Equilibrium does not require a small q , which is in contrast to the result in Proposition 5, where we do need a small q to guarantee the existence of a Pure Strategy Manipulation Equilibrium.

In the mixed strategy of Proposition 7, the principal-investor sometimes bases his trader's compensation on marking-to-market and sometimes not. At his discretion, the principal decides on the contract form. In reality, traders' compensation is discretionary in this way, and principals do not make such stark choices. Principals recognize that compensation should be based on performance, without explicitly committing to one contract form or the other. For example, in one study of four financial firms:

Details of incentives differed between the four firms, but the following were common. First, contingent pay was a very high proportion of total pay. Second, although there were acknowledged ‘desk (i.e., market) effects’, contingent pay was based on individual performance. Third, the precise performance bases used by managers for evaluation were kept deliberately unclear to preserve managerial discretion; market and internal behavioral criteria were mixed. Fourth, managers retained discretion over contingent pay, and bonuses were not known until announcement at year end. (Fenton-O’Creevy, et al. (2004), p. 190.)

How the principal, in effect, commits to one form of compensation or the other, is an interesting question. Presumably, the repeated interaction between the principal and the agent plays a role. This is a subject for further research.

Finally, notice that the mixed strategy means that market efficiency is also changing, it varies through time as a function of the principals’ contract choice.

4.4 The Payoff Distribution and Market Efficiency

We now discuss how the payoffs to the market participants change with the propensity of shirking, q . First, let us write out the expected payoff to the liquidity trader:

$$V_N = \frac{1}{2} \int_{-\infty}^{\infty} z \{ [p_H - p^q(z + 2x)] + [p_L - p^q(z - 2x)] \} \phi^0(z) dz, \quad (54)$$

where $p^q(z)$ is the pricing function in a Pure Strategy Manipulation Equilibrium defined in (31).

The next proposition demonstrates how the noise-trading propensity affects the payoffs to the market participants.

Proposition 8 (Comparative Statics) *In a Pure Strategy Manipulation Equilibrium, if it exists, (i) the trader’s expected payoff net of the information production cost is increasing with the propensity of shirking, q , and the expected payoff to the principal is decreasing with q , while (ii) the exogenous liquidity traders benefit from a higher q .*

Proof: See Appendix. ■

In summary, the externality of the mark-to-market contract generates incentives for traders to coordinate on irrelevant signals. Traders’ coordinated manipulation adds additional noise into the asset price, and price informativeness is reduced. At the same time, this behavior reduces the payoff to each principal. When the manipulation problem is too severe, the principals might actually use non-mark-to-market contracts to prevent traders from shirking. We discuss both issues below.

Proposition 9 (Reduced Market Efficiency) *Let q be the propensity of market manipulation in a Pure Strategy Manipulation Equilibrium and $(m^*)^2q$ be the propensity of market manipulation in a Mixed Strategy Equilibrium. Price volatility is decreasing in the propensity of market manipulation. Also, when $z < 0$ ($z < 0$), the market price is increasing (decreasing) in the propensity of market manipulation.*

Proof: See Appendix. ■

Kyle (1985) defines market depth as the order flow required to move prices by one unit. Proposition 9 says that the market maker does not move prices by as much in response to a given order flow because he recognizes that sometimes the trading by the agent-traders is not based on fundamental information. When there is market manipulation, the traders sometimes coordinate on irrelevant signals. Since the order flow is less informative about the fundamental value of the security, the price is less volatile compared to the price volatility in the case where there is no manipulation. The second part of the proposition shows that when the order flow is negative, the price drops less dramatically when the possibility of market manipulation is larger and vice versa when the order flow is positive. Again, the point is that the market maker understands that the traders may be responding to the irrelevant signal, so the order flow is not as informative.

5. Discussion

Market prices are widely viewed as the most accurate measure of value, as security markets are usually taken to be “efficient” in the sense that the prices aggregate diverse information. So, these prices can be used contractually as accounting measures, for risk management, and for compensation. In this paper, we focused on delegated trading, due to the separation of capital ownership and capital management, and we studied its impact on market efficiency. Delegated trading is predominant in practice. Most trade in securities, and almost all trade in derivatives, occurs in markets dominated by professional traders and money managers who have been hired to trade on behalf of others. In the context of delegated trading, market efficiency is endogenized as a function of principals’ contractual attempts to oversee their delegated traders.

Our setting is richer than the usual agency problem because agent-traders can take actions after they have received their private information, namely, they can trade. The ability to trade introduces risk management issues that principal-investors want to monitor and control. If market prices are “efficient” in the sense that the prices aggregate diverse information then principals can use the interim security price as part of the optimal contract. These prices then can be used to try to control the behavior of portfolio managers and traders, through the practice of mark-to-market compensation contracts. We show conditions under which “marking-to-market” is part of an optimal contract, confirming this logic.

When markets are dominated by delegated traders, monitoring becomes more complicated. Each principal-investor attempts to control his agent-trader by relying on the informativeness of a price, where the informativeness of the price depends on the behavior of other principals' agents. Traders realize that if there is a way to coordinate their trades, they can ignore fundamentals (which requires expending efforts to find out) and trade in the same direction, manipulating their compensation by influencing the security's price. This behavior makes the prices used to mark their positions less informative, reducing price volatility.

Principals recognize the possibility of manipulation of prices by their traders and this means that their choice of contract for their traders at the initial date is also intertwined. Principals can always choose the "standard contract," i.e., one which does not include interim market price information, but rather is just based on final outcomes. Under a standard contract, agent-traders can be induced to produce information and trade in a direction consistent with the principal's objective. But, adopting the standard contract means that the price will be (at least weakly) informative, allowing other principals to free ride on this information using mark-to-market contracts. The equilibrium then must be one in which the principals follow mixed strategies in contracts, sometimes adopting mark-to-market contracts and sometimes adopting standard contracts. Principals essentially coordinate to share responsibility for ensuring that the security market price is informative. A principal choosing a standard contract is playing this role for the other principal choosing the mark-to-market contract.

When principals use their discretion over compensation to mix between standard contracts and mark-to-market contracts, the level of market efficiency is endogenized. The propensity of joint shirking by traders, which can be used to quantify the level of market efficiency, depends on the likelihood of the irrelevant coordination signal occurring and on the probability of all principals using mark-to-market contracts, which is endogenously determined in equilibrium. In other words, the traders rely on irrelevant noise to coordinate their manipulation, and in response the principals, to mitigate the effects of traders trying to shirk, introduce uncertainty. The irrelevant noise signal that traders coordinate on does not always occur, and the response of principals to mix their contract choice, means that the degree of market efficiency, determined endogenously, is stochastic.

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Appendix

Proof of Proposition 1: Solving the binding constraints in (IC.S2) gives us:

$$w_{bH} - w_{sH} = \frac{2\kappa}{2\theta - 1}, \text{ and } w_{sL} - w_{bL} = \frac{2\kappa}{2\theta - 1}.$$

The results are immediate. ■

Proof of Proposition 2: We will only show the proof for (i); the proof for part (ii) follows a similar argument. We will prove (i) by contradiction. Without loss of generality, assume that there exist $z_1 \leq z_2 - \delta$ for some $\delta > 0$, such that, with some $\varepsilon > 0$, $w^*(p(z), b, v_\eta) \geq \varepsilon$ for any $z \in [z_1 - \delta/2, z_1 + \delta/2]$, and $w^*(p(z), b, v_\eta) \leq \bar{w} - \varepsilon \phi^+(z_1)/\phi^+(z_2)$ for any $z \in [z_2 - \delta/2, z_2 + \delta/2]$. Now construct a new wage schedule $\hat{w}^*(p(z), b, v_\eta)$ as follows:

$$\hat{w}^*(p(z), b, v_\eta) = \begin{cases} w^*(p(z), b, v_\eta) - \varepsilon & \text{if } z \in [z_1 - \delta/2, z_1 + \delta/2] \\ w^*(p(z), b, v_\eta) + \varepsilon \phi^+(z_1)/\phi^+(z_2) & \text{if } z \in [z_2 - \delta/2, z_2 + \delta/2] \\ w^*(p(z), b, v_\eta) & \text{otherwise.} \end{cases}$$

Basically, we cut a piece of wage from the neighborhood of z_2 and paste another piece into the neighborhood of z_1 . With the new constructed wage schedule $\hat{w}^*(p(z), b, v_\eta)$, the resulting $\hat{A}_{b\eta}^+$ remains approximately the same, while the resulting $\hat{A}_{b\eta}^0$ will be strictly smaller since $\phi^0(z_1)/\phi^0(z_2) > \phi^+(z_1)/\phi^+(z_2)$. Therefore, the second inequality of the IC conditions in (23) will become a strict inequality, which means that the contract can be strictly improved according to Lemma 1, a contradiction.

For the second part of the results, we write out the Lagrangian for the optimization problem with non-positive multiplier μ_1 and μ_2 , and get the first order conditions:

$$\begin{aligned} \theta \phi^+(z_{bH}^*) + \mu_1 \theta \phi^+(z_{bH}^*) - \mu_2 (1 - \theta) \phi^0(z_{bH}^*) &= 0 \\ (1 - \theta) \phi^+(z_{bL}^*) + \mu_1 (1 - \theta) \phi^+(z_{bL}^*) - \mu_2 \theta \phi^0(z_{bL}^*) &= 0 \\ (1 - \theta) \phi^-(z_{sH}^*) + \mu_2 (1 - \theta) \phi^-(z_{sH}^*) - \mu_1 \theta \phi^0(z_{sH}^*) &= 0 \\ \theta \phi^-(z_{sL}^*) + \mu_2 \theta \phi^-(z_{sL}^*) - \mu_1 (1 - \theta) \phi^0(z_{sL}^*) &= 0. \end{aligned}$$

Eliminating μ_1 and μ_2 , we get:

$$\frac{\phi^+(z_{bH}^*)}{\phi^0(z_{bH}^*)} = \frac{(1 - \theta)^2 \phi^+(z_{bL}^*)}{\theta^2 \phi^0(z_{bL}^*)} \quad \text{and} \quad \frac{\phi^0(z_{sH}^*)}{\phi^-(z_{sH}^*)} = \frac{(1 - \theta)^2 \phi^0(z_{sL}^*)}{\theta^2 \phi^-(z_{sL}^*)},$$

which implies:

$$z_{bH}^* - z_{bL}^* = z_{sH}^* - z_{sL}^* = \frac{\sigma^2}{x} \ln \left(\frac{1 - \theta}{\theta} \right) < 0.$$

The next two equations are the binding constraints. The last two inequalities are from non-positive Lagrange multipliers. We have:

$$\mu_1 = \frac{1}{1-\theta} \frac{(1-\theta)\phi^+(z_{bL}^*)\phi^-(z_{sL}^*) + \theta\phi^0(z_{bL}^*)\phi^-(z_{sL}^*)}{\phi^0(z_{bL}^*)\phi^0(z_{sL}^*) - \phi^+(z_{bL}^*)\phi^-(z_{sL}^*)} < 0$$

$$\mu_2 = \frac{1}{1-\theta} \frac{(1-\theta)\phi^+(z_{bH}^*)\phi^-(z_{sH}^*) + \theta\phi^+(z_{bH}^*)\phi^0(z_{sH}^*)}{\phi^0(z_{bH}^*)\phi^0(z_{sH}^*) - \phi^+(z_{bH}^*)\phi^-(z_{sH}^*)} < 0,$$

which implies $\phi^0(z_{bL}^*)\phi^0(z_{sL}^*) < \phi^+(z_{bL}^*)\phi^-(z_{sL}^*)$ and $\phi^0(z_{bH}^*)\phi^0(z_{sH}^*) < \phi^+(z_{bH}^*)\phi^-(z_{sH}^*)$. The results are immediate. ■

Proof of Proposition 3: First, existence. An immediate implication of Proposition 2 is that the optimal Mark-to-Market contract beats the optimal Standard Contract characterized in Proposition 1. To see this, with Assumption 2, let us construct a set of cutoff values $z^S = \{z_{bH}^S, z_{bL}^S, z_{sH}^S, z_{sL}^S\}$ such that the corresponding contract in the form of step function as described in Proposition 2 generate the following payoffs to the trader:

$$A_{bH}^+ = w_{bH} = \frac{2\kappa}{2\theta-1}, A_{bL}^+ = w_{bL} = 0;$$

$$A_{sL}^- = w_{sL} = \frac{2\kappa}{2\theta-1}, A_{sH}^- = w_{sH} = 0.$$

It is easy to check that with these cutoff values, the incentive constraints (IC.2) in (23) are satisfied but not binding. Therefore, we know: (i) The set of incentive compatible Mark-to-Market contracts is not empty, which guarantee a solution for the optimization problem in (25); and (ii) z^S can be improved according to Lemma 1. To improve $z^S = \{z_{bH}^S, z_{bL}^S, z_{sH}^S, z_{sL}^S\}$, we only need to increase z_{bH} or decrease z_{sL} , both of which lower the payoff to the trader while increasing the payoff to the principal. Notice that a Mark-to-Market contract characterized by z^S generates the same payoff for the principal as a Standard Contract, which is positive under Assumption 2. Therefore, the optimal Mark-to-Market Contract can generate a higher positive payoff for the principal than the optimal Standard Contract.

Next, uniqueness. Assume that there exist two optimal contracts characterized by z^* and \hat{z}^* , and without loss of generality assume $z_{bH}^* > \hat{z}_{bH}^*$. From Equations in (26) and (27), we know that: $z_{bL}^* > \hat{z}_{bL}^*$, $z_{sH}^* < \hat{z}_{sH}^*$, and $z_{sL}^* < \hat{z}_{sL}^*$. However, this implies that z^* yields a strictly smaller expected payoff to the trader as measured in (25), which is a contradiction. ■

Proof of Lemma 2: We prove the results with respect to κ using the perturbation method; the results with respect to \bar{w} can be similarly shown. Define:

$$\begin{aligned}
\beta_{11} &\equiv \theta\phi^+(z_{bH}^*) + (1-\theta)\phi^+(z_{bL}^*) \\
\beta_{12} &\equiv \theta\phi^0(z_{sH}^*) + (1-\theta)\phi^0(z_{sL}^*) \\
\beta_{21} &\equiv (1-\theta)\phi^-(z_{sH}^*) + \theta\phi^-(z_{sL}^*) \\
\beta_{22} &\equiv (1-\theta)\phi^0(z_{bH}^*) + \theta\phi^0(z_{bL}^*)
\end{aligned}$$

By Proposition 1, since the difference between z_{bH}^* and z_{bL}^* or z_{sH}^* and z_{sL}^* does not change with κ , we know that $\Delta z_{bH}^* = \Delta z_{bL}^*$ and $\Delta z_{sH}^* = \Delta z_{sL}^*$, and we can derive the following perturbation equations from equations in (23):

$$\begin{aligned}
-\beta_{11}\Delta z_{bH}^* - \beta_{12}\Delta z_{sH}^* &= \Delta\kappa/\bar{w} \\
\beta_{21}\Delta z_{sH}^* + \beta_{22}\Delta z_{bH}^* &= \Delta\kappa/\bar{w}.
\end{aligned}$$

We can show:

$$\begin{aligned}
\frac{\Delta z_{bH}^*}{\Delta\kappa} &= \frac{\beta_{21} + \beta_{12}}{\bar{w}(-\beta_{11}\beta_{21} + \beta_{12}\beta_{22})} \\
\frac{\Delta z_{sH}^*}{\Delta\kappa} &= \frac{\beta_{11} + \beta_{22}}{\bar{w}(\beta_{11}\beta_{21} - \beta_{12}\beta_{22})}
\end{aligned}$$

We only need to show $\beta_{11}\beta_{21} - \beta_{12}\beta_{22} > 0$ to prove our results. We have:

$$\begin{aligned}
&\beta_{11}\beta_{21} - \beta_{12}\beta_{22} \\
&= [\theta\phi^+(z_{bH}^*) + (1-\theta)\phi^+(z_{bL}^*)][(1-\theta)\phi^-(z_{sH}^*) + \theta\phi^-(z_{sL}^*)] \\
&\quad - [\theta\phi^0(z_{sH}^*) + (1-\theta)\phi^0(z_{sL}^*)][(1-\theta)\phi^0(z_{bH}^*) + \theta\phi^0(z_{bL}^*)] \\
&= \theta(1-\theta)[\phi^+(z_{bH}^*)\phi^-(z_{sH}^*) - \phi^0(z_{bH}^*)\phi^0(z_{sH}^*)] \\
&\quad + \theta(1-\theta)[\phi^+(z_{bL}^*)\phi^-(z_{sL}^*) - \phi^0(z_{bL}^*)\phi^0(z_{sL}^*)] \\
&\quad + \theta^2[\phi^+(z_{bH}^*)\phi^-(z_{sL}^*) - \phi^0(z_{bL}^*)\phi^0(z_{sH}^*)] \\
&\quad + (1-\theta)^2[\phi^+(z_{bL}^*)\phi^-(z_{sH}^*) - \phi^0(z_{bH}^*)\phi^0(z_{sL}^*)].
\end{aligned}$$

The first two items are non-negative since we know $\phi^+(z_{bH}^*)/\phi^0(z_{bH}^*) > \phi^0(z_{sH}^*)/\phi^-(z_{sH}^*)$ and $\phi^+(z_{bL}^*)/\phi^0(z_{bL}^*) > \phi^0(z_{sL}^*)/\phi^-(z_{sL}^*)$ according to Proposition 2. For the third item (the fourth item can be shown non-negative in a similar way), using

$$\begin{aligned}
\phi^+(z_{bH}^*) &= \frac{(1-\theta)^2}{\theta^2} \frac{\phi^+(z_{bL}^*)\phi^0(z_{bH}^*)}{\phi^0(z_{bL}^*)} \quad \text{and} \quad \phi^0(z_{sH}^*) = \frac{(1-\theta)^2}{\theta^2} \frac{\phi^0(z_{sL}^*)\phi^-(z_{sH}^*)}{\phi^-(z_{sL}^*)}, \text{ it becomes:} \\
(1-\theta)^2 &\left[\frac{\phi^+(z_{bL}^*)\phi^0(z_{bH}^*)}{\phi^0(z_{bL}^*)} \phi^-(z_{sL}^*) - \frac{\phi^-(z_{sH}^*)\phi^0(z_{sL}^*)}{\phi^-(z_{sL}^*)} \phi^0(z_{bL}^*) \right] \\
&> (1-\theta)^2 \frac{\phi^0(z_{sL}^*)}{\phi^-(z_{sL}^*)} [\phi^0(z_{bH}^*)\phi^-(z_{sL}^*) - \phi^-(z_{sH}^*)\phi^0(z_{bL}^*)] > 0.
\end{aligned}$$

To get the first inequality, we use $\phi^+(z_{bL}^*)/\phi^0(z_{bL}^*) > \phi^0(z_{sL}^*)/\phi^-(z_{sL}^*)$; to get the second inequality, we use $z_{bH}^* - z_{bL}^* = z_{sH}^* - z_{sL}^* < 0$ and $z_{bH}^* - z_{sH}^* = z_{bL}^* - z_{sL}^* > 2x$ to show $\phi^0(z_{bH}^*)\phi^-(z_{sL}^*) - \phi^-(z_{sH}^*)\phi^0(z_{bL}^*) > 0$. ■

Proof of Proposition 4: With V defined in (29), to prove part (i), we only need to show

$\frac{\partial}{\partial \sigma} \int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz > 0$. We have:

$$\int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz \stackrel{\hat{z}=z-2x/\sigma}{=} \int_{-\infty}^{\infty} \frac{\exp(-\hat{z}^2/2)}{\exp[(4x/\sigma)(\hat{z} + 2x/\sigma)] + 1} d\hat{z}.$$

Taking derivative with respect to σ , we have:

$$\begin{aligned} & \frac{\partial}{\partial \sigma} \int_{-\infty}^{\infty} \frac{\exp(-z^2/2)}{\exp[(4x/\sigma)(z + 2x/\sigma)] + 1} dz \\ &= \frac{4x}{\sigma^2} \int_{-\infty}^{\infty} \frac{\exp[-(z + 4x/\sigma)^2/2](z + 4x/\sigma)}{\{\exp[-(4x/\sigma)(z + 2x/\sigma)] + 1\}^2} dz \\ & \stackrel{\hat{z}=z+4x/\sigma}{=} \frac{4x}{\sigma^2} \int_{-\infty}^{\infty} \frac{\exp(-\hat{z}^2/2)\hat{z}}{\{\exp[-(4x/\sigma)(\hat{z} - 2x/\sigma)] + 1\}^2} d\hat{z} \\ & > \frac{4x}{\sigma^2} \int_{-\infty}^{\infty} \exp(-\hat{z}^2/2)\hat{z} d\hat{z} = 0. \end{aligned}$$

The last inequality comes from the fact that $\{\exp[-(4x/\sigma)(\hat{z} - 2x/\sigma)] + 1\}^2$ is a positive function decreasing in z .

For θ , tedious algebra leads us to:

$$\frac{\partial V}{\partial \theta} = \int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{[\phi^+(z) + \phi^-(z)]^2} (v_H - v_L)[2\phi^+(z) + 2\phi^-(z)] dz > 0.$$

For (ii), with the trader's expected payoff A defined in (24), we can show:

$$\frac{\Delta A}{\Delta \kappa} = -\beta_{11} \bar{w} \frac{\Delta z_{bH}^*}{\Delta \kappa} + \beta_{21} \bar{w} \frac{\Delta z_{sH}^*}{\Delta \kappa} = \frac{\beta_{11}(\beta_{21} + \beta_{12}) + \beta_{21}(\beta_{11} + \beta_{22})}{\beta_{11}\beta_{21} + \beta_{12}\beta_{22}} > 1,$$

where β_{11} , β_{12} , β_{21} , and β_{22} are defined in the proof of Lemma 2.

Similarly, we can show $\Delta A / \Delta \bar{w} < 0$. In addition, because the sum of the trader's payoff and the principal's payoff does not depend on κ or \bar{w} , we must have the principal's payoff decrease in κ and increase in \bar{w} .

For the sign of $\Delta A / \Delta \theta < 0$, applying the Envelope Theorem to the optimization problem in (25), with $\mu_1 < 0$ and $\mu_2 < 0$ defined in proof for Proposition 2, we have:

$$\begin{aligned} \frac{\partial A}{\partial \theta} &= (A_{bH}^+ - A_{bL}^+ + A_{sL}^- - A_{sH}^-) + \mu_1 (A_{bH}^+ - A_{bL}^+ + A_{sL}^0 - A_{sH}^0) + \mu_2 (A_{sL}^- - A_{sH}^- + A_{bH}^0 - A_{bL}^0) \\ &= (1 + \mu_1)(A_{bH}^+ - A_{bL}^+) + (1 + \mu_2)(A_{sL}^- - A_{sH}^-) + \mu_1 (A_{sL}^0 - A_{sH}^0) + \mu_2 (A_{bH}^0 - A_{bL}^0). \end{aligned}$$

It is easy to verify that $1 + \mu_1 < 0$ and $1 + \mu_2 < 0$, and we conclude $\Delta A / \Delta \theta < 0$.

Finally, we prove that the trader's payoff is increasing in σ by construction. Given σ , pick an optimal contract $w(p(z), \lambda, v_\eta)$ characterized by the cutoff points $\{z_{bH}, z_{bL}, z_{sH}, z_{sL}\}$,

satisfying the IC conditions in (23). For $\hat{\sigma} < \sigma$, we construct the new wage functions $\hat{w}(p(z), \lambda, v_\eta)$, characterized by the cutoff points $\{\hat{z}_{bH}, \hat{z}_{bL}, \hat{z}_{sH}, \hat{z}_{sL}\}$, as follows:

$$\frac{\hat{z}_{bH} - 2x}{\hat{\sigma}} = \frac{z_{bH} - 2x}{\sigma}, \frac{\hat{z}_{bL} - 2x}{\hat{\sigma}} = \frac{z_{bL} - 2x}{\sigma}, \frac{\hat{z}_{sH} + 2x}{\hat{\sigma}} = \frac{z_{sH} + 2x}{\sigma}, \text{ and } \frac{\hat{z}_{sL} + 2x}{\hat{\sigma}} = \frac{z_{sL} + 2x}{\sigma}.$$

With the constructed $\hat{w}(p(z), \lambda, v_\eta)$, we can show that:

$$\hat{A}_{bH}^+ = A_{bH}^+, \hat{A}_{bL}^+ = A_{bL}^+, \hat{A}_{sH}^- = A_{sH}^-, \text{ and } \hat{A}_{sL}^- = A_{sL}^-.$$

Therefore, the left hand side values of the IC conditions in (23) remain the same. At the same time, we can show that:

$$\hat{A}_{bH}^0 < A_{bH}^0, \hat{A}_{bL}^0 < A_{bL}^0, \hat{A}_{sH}^0 < A_{sH}^0, \text{ and } \hat{A}_{sL}^0 < A_{sL}^0.$$

That is, the right hand side values of the IC conditions in (23) get smaller. We will only show $\hat{A}_{bH}^0 < A_{bH}^0$ as an example, the proofs for the other inequalities follow the same logic.

We have:

$$\begin{aligned} \hat{A}_{bH}^0 &= \bar{w} \left[1 - \int_0^{\hat{z}_{bH}} \frac{1}{\sqrt{2\pi\hat{\sigma}}} \exp\left(-\frac{\hat{z}^2}{2\hat{\sigma}^2}\right) d\hat{z} \right] \\ &\stackrel{(z-2x)/\sigma=(\hat{z}-2x)/\sigma}{=} \bar{w} \left\{ 1 - \int_0^{z_{bH}} \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2}\left(\frac{z}{\sigma} + 2x\left(\frac{1}{\hat{\sigma}} - \frac{1}{\sigma}\right)\right)^2\right] dz \right\} \\ &< \bar{w} \left\{ 1 - \int_0^{z_{bH}} \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{z^2}{2\sigma^2}\right) dz \right\} = A_{bH}^0. \end{aligned}$$

Therefore, the IC conditions become strict inequalities with our construction, and we can improve the contract. ■

Proof of Lemma 3: By working hard and trading according to the acquired information, the traders' expected payoff is: $[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + (1-\theta)A_{sL}^-]/2$. If they shirk jointly, they can both buy the stocks or both sell the stocks. If they both buy, their payoff is: $(A_{bH}^+ + A_{bL}^+)/2 + \kappa$. The difference between payoffs from joint shirking and both working is equal to:

$$\begin{aligned} &\frac{1}{2}[(1-\theta)A_{bH}^+ + \theta A_{bL}^+ - (1-\theta)A_{sH}^- - \theta A_{sL}^-] + \kappa \\ &= \frac{1}{2}[(1-\theta)A_{bH}^+ + \theta A_{bL}^+ - (1-\theta)A_{bH}^0 - \theta A_{bL}^0] > 0, \end{aligned}$$

where the first equality comes from the binding constraints in (IC.2) in (23). The case of selling can be proved similarly. ■

Proof of Proposition 5: We first ignore constrains (IC.M1), (IC.M3), and (IC.M4). With (IC.M2), the proof for (i) and (ii) of the first part is the same as for Proposition 1. For the second part, we write out the Lagrangian for the optimization problem with non-positive multiplier μ_1 and μ_2 , and get the first order conditions:

$$\begin{aligned}
& [(1-q)\theta + \frac{1}{2}q]\phi^+(z_{bH}^*) + \mu_1\theta\phi^+(z_{bH}^*) - \mu_2(1-\theta)\phi^0(z_{bH}^*) = 0 \\
& [(1-q)(1-\theta) + \frac{1}{2}q]\phi^+(z_{bL}^*) + \mu_1(1-\theta)\phi^+(z_{bL}^*) - \mu_2\theta\phi^0(z_{bL}^*) = 0 \\
& [(1-q)(1-\theta) + \frac{1}{2}q]\phi^-(z_{sH}^*) + \mu_2(1-\theta)\phi^-(z_{sH}^*) - \mu_1\theta\phi^0(z_{sH}^*) = 0 \\
& [(1-q)\theta + \frac{1}{2}q]\phi^-(z_{sL}^*) + \mu_2\theta\phi^-(z_{sL}^*) - \mu_1(1-\theta)\phi^0(z_{sL}^*) = 0.
\end{aligned}$$

Eliminating μ_1 and μ_2 , we get the first two equations in the proposition. The next two equations are the binding constraints in (IC.M2). The last two inequalities are from non-positivity of the Lagrange multipliers. We have:

$$\begin{aligned}
\mu_1 &= \frac{1}{1-\theta} \frac{[(1-q)(1-\theta) + \frac{1}{2}q]\phi^+(z_{bL}^*)\phi^-(z_{sL}^*) + [(1-q)\theta + \frac{1}{2}q]\phi^0(z_{bL}^*)\phi^-(z_{sL}^*)}{\phi^0(z_{bL}^*)\phi^0(z_{sL}^*) - \phi^+(z_{bL}^*)\phi^-(z_{sL}^*)} < 0 \\
\mu_2 &= \frac{1}{1-\theta} \frac{[(1-q)(1-\theta) + \frac{1}{2}q]\phi^+(z_{bH}^*)\phi^-(z_{sH}^*) + [(1-q)\theta + \frac{1}{2}q]\phi^+(z_{bH}^*)\phi^0(z_{sH}^*)}{\phi^0(z_{bH}^*)\phi^0(z_{sH}^*) - \phi^+(z_{bH}^*)\phi^-(z_{sH}^*)} < 0,
\end{aligned}$$

which implies $\phi^0(z_{bL}^*)\phi^0(z_{sL}^*) < \phi^+(z_{bL}^*)\phi^-(z_{sL}^*)$ and $\phi^0(z_{bH}^*)\phi^0(z_{sH}^*) < \phi^+(z_{bH}^*)\phi^-(z_{sH}^*)$. The results are immediate.

Now, we show that the solutions above also satisfy (IC.M1), (IC.M3), and (IC.M4). It is obvious that (IC.M2) implies (IC.M1), and we only need to check (IC.M3) and (IC.M4). First observe that, with (IC.M2) binding, (IC.M3) and (IC.M4) are equivalent. To see this, we have:

$$\begin{aligned}
& \frac{1}{2}(A_{bH}^+ + A_{bL}^+) + \kappa \geq \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^0 + \theta A_{sL}^0] \quad (\text{IC.M4}) \\
& = \frac{1}{2}[\theta A_{sH}^0 + (1-\theta)A_{sL}^0 + (1-\theta)A_{sH}^0 + \theta A_{sL}^0] + \kappa \quad (\text{IC.M2}) \\
& = \frac{1}{2}(A_{sH}^0 + A_{sL}^0) + \kappa. \quad (\text{IC.M3})
\end{aligned}$$

and

$$\begin{aligned}
& \frac{1}{2}(A_{sH}^- + A_{sL}^-) + \kappa \geq \frac{1}{2}[\theta A_{bH}^0 + (1-\theta)A_{bL}^0 + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \quad (\text{IC.M4}) \\
& = \frac{1}{2}[\theta A_{bH}^0 + (1-\theta)A_{bL}^0 + (1-\theta)A_{bH}^0 + \theta A_{bL}^0] + \kappa \quad (\text{IC.M2}) \\
& = \frac{1}{2}(A_{bH}^0 + A_{bL}^0) + \kappa. \quad (\text{IC.M3})
\end{aligned}$$

To prove (IC.M2) implies (IC.M4), using $A_{s\eta}^0 < A_{s\eta}^-$ and $A_{b\eta}^0 < A_{s\eta}^+$, we have:

$$\begin{aligned}
& \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^0 + \theta A_{sL}^0] \\
& < \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
& = \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{bH}^0 + \theta A_{bL}^0] + \kappa \quad (\text{IC.M2}) \\
& < \frac{1}{2}(A_{bH}^+ + A_{bL}^+) + \kappa.
\end{aligned}$$

and

$$\begin{aligned}
& \frac{1}{2}[\theta A_{bH}^0 + (1-\theta)A_{bL}^0 + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
& < \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
& = \frac{1}{2}[\theta A_{sH}^0 + (1-\theta)A_{sL}^0 + (1-\theta)A_{sH}^- + \theta A_{sL}^-] + \kappa \quad (\text{IC.M2}) \\
& < \frac{1}{2}(A_{sH}^- + A_{sL}^-) + \kappa.
\end{aligned}$$

■

Proof of Lemma 4: The first result is trivial if we compare (45) and (47). For the second result, we know that a standard contract also satisfies the incentive constraints (IC.M2), but it is not the solution to the optimal contracting problem (40). Therefore, we know that a standard contract does not yield the minimum wage payment, that is: $A^S > A^M$. ■

Proof of Lemma 5: Some manipulation with V^M gives us:

$$\begin{aligned}
V_P^M &= (1-q)x(p_H - p_L) \int \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz - \frac{1}{2}(1-q)[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
&\quad - \frac{1}{4}q(A_{bH}^+ + A_{bL}^+ + A_{sH}^- + A_{sL}^-).
\end{aligned}$$

We have:

$$\begin{aligned}
\frac{\partial V_P^M}{\partial q} &= -x(p_H - p_L) \int \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz + \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
&\quad - \frac{1}{4}(A_{bH}^+ + A_{bL}^+ + A_{sH}^- + A_{sL}^-).
\end{aligned}$$

Because:

$$\begin{aligned}
& \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^0 + \theta A_{sL}^0] \\
& < \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
& = \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{bH}^0 + \theta A_{bL}^0] + \kappa \quad (\text{IC.M2}) \\
& < \frac{1}{2}(A_{bH}^+ + A_{bL}^+) + \kappa,
\end{aligned}$$

and

$$\begin{aligned}
& \frac{1}{2}[\theta A_{bH}^0 + (1-\theta)A_{bL}^0 + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
& < \frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\
& = \frac{1}{2}[\theta A_{sH}^0 + (1-\theta)A_{sL}^0 + (1-\theta)A_{sH}^- + \theta A_{sL}^-] + \kappa \quad (\text{IC.M2}) \\
& < \frac{1}{2}(A_{sH}^- + A_{sL}^-) + \kappa,
\end{aligned}$$

we have:

$$\frac{\partial V_P^M}{\partial q} < -(V - \kappa) < 0,$$

where V is defined in (19), and $V > \kappa$ follows Assumption 2.

At the same time, because A^S is independent of q , we have:

$$\frac{\partial V_P^S}{\partial q} = \frac{\partial V^S}{\partial q} = \frac{1}{4}x(p_H - p_L) \int \frac{[\phi^+(z) - \phi^-(z)]^2}{\phi^+(z) + \phi^-(z)} dz > 0.$$

Then it follows that:

$$\frac{\partial \Delta V_P}{\partial q} = \frac{\partial \Delta V_P^M}{\partial q} - \frac{\partial \Delta V_P^S}{\partial q} < 0.$$

■

Proof of Proposition 6: First, we observe that the optimization problem in (40) converges to the one in (25) as q goes to zero, thus we get the existence of an optimal incentive compatible contract to (40) by continuity under Assumption 2.

Second, following the Proof of Lemma 5, we have $\frac{\partial \Delta V_P}{\partial q} = \frac{\partial \Delta V_P^M}{\partial q} - \frac{\partial \Delta V_P^S}{\partial q} < 0$. In addition, $\Delta V_P > 0$ when $q = 0$ and $\Delta V_P < 0$ when $q = 1$. Therefore there must exist a unique $\bar{q} \in (0,1)$ such that $\Delta V_P \geq 0$ if and only if $0 \leq q \leq \bar{q}$. Moreover, it is easy to see that under

Assumption 2, at \bar{q} , $V_p^M = V_p^S > 0$, which is also true for any $q \leq \bar{q}$ as $\frac{\partial V_p^M}{\partial q} < 0$ according to Lemma 5. ■

Proof of Lemma 6: Because A^S is independent of m , we have:

$$\frac{\partial V_p^S}{\partial m} = \frac{\partial V^S}{\partial m} = \frac{\partial V^S}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial m} = \frac{1}{4} xmq(p_H - p_L) \int \frac{[\phi^+(z) - \phi^-(z)]^2}{\phi^+(z) + \phi^-(z)} dz > 0.$$

At the same time, we have:

$$\Delta V_p = V_p^M - V_p^S = -\frac{1}{2} mqx(p_H - p_L) - (A^M + A^S).$$

We know that $\partial A^S / \partial m = 0$, and applying Envelope theorem to the optimization problem (52), we have:

$$\frac{\partial A^M}{\partial m} = -\frac{q}{2} [\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] + \frac{q}{4} (A_{bH}^+ + A_{bL}^+ + A_{sH}^- + A_{sL}^-).$$

Therefore, we have:

$$\begin{aligned} \frac{\partial \Delta V_p}{\partial m} &= \frac{q}{2} [\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] - \frac{q}{4} (A_{bH}^+ + A_{bL}^+ + A_{sH}^- + A_{sL}^-) - \frac{q}{2} x(p_H - p_L) \\ &< q \left[\kappa - \frac{1}{2} x(p_H - p_L) \right] < q \left[\kappa - x(p_H - p_L) \int_{-\infty}^{+\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz \right] = -q(V - \kappa) < 0, \end{aligned}$$

where V is defined in (29), and $V > \kappa$ under Assumption 2.

Proof of Proposition 7: First, existence. Under Assumption 2, we know from the proof of Proposition 6 that $V_p^M < V_p^S$ when $q > \bar{q}$, where V_p^M and V_p^S are the payoffs to the principals in a Pure Strategy Manipulation Equilibrium, and they coincide with the corresponding payoffs to the principals in a Mixed Strategy Manipulation Equilibrium with $m = 1$. To avoid being confused with their counterparts in a Pure Strategy Manipulation Equilibrium, we denoted the principals payoff in a mixed strategy equilibrium as $V_p^M(m)$ and $V_p^S(m)$. At the same time, when $m = 0$, we know $V_p^M(0) > V_p^S(0)$ because $V^S = V^M$ and $A^S > A^M$. Therefore, when $q > \bar{q}$, we have:

$$\begin{aligned} V_p^M(m=0) &> V_p^S(m=0) \\ V_p^M(m=1) &< V_p^S(m=1). \end{aligned}$$

By continuity, we know that there exists $m^* \in (0,1)$, such that $V_p^M(m^*) = V_p^S(m^*) > 0$. Here, $V_p^M(m^*) = V_p^S(m^*) > 0$ is because $V_p^S(m=0) > 0$ and $V_p^S(m)$ is increasing with m according to Lemma 6.

The uniqueness directly comes from $\frac{\partial \Delta V_p}{\partial m} < 0$, which is similar to $\frac{\partial \Delta V_p}{\partial q} < 0$ proved

in Lemma 6. Finally, we have $\frac{\partial m^*}{\partial q} = -\frac{\frac{\partial \Delta V_p}{\partial q}}{\frac{\partial \Delta V_p}{\partial m}} < 0$.

Proof of Proposition 8: The expected net payoff to the trader is:

$$\begin{aligned} V_T &= A^M - (1-q)\kappa \\ &= \frac{1}{2}(1-q)[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\ &\quad + \frac{1}{4}q(A_{bH}^+ + A_{bL}^+ + A_{sH}^- + A_{sL}^-) - (1-q)\kappa. \\ \frac{\partial V_T}{\partial q} &= -\frac{1}{2}[\theta A_{bH}^+ + (1-\theta)A_{bL}^+ + (1-\theta)A_{sH}^- + \theta A_{sL}^-] \\ &\quad + \frac{1}{4}(A_{bH}^+ + A_{bL}^+ + A_{sH}^- + A_{sL}^-) + \kappa. \end{aligned}$$

From the proof of Lemma 5, we know the above expression has a strictly positive value, which implies that the trader's expected net payoff is increasing with q . At the same time, as we have shown in Lemma 5, in a Pure Strategy Manipulation Equilibrium, the principal's expected payoff, defined as $V_p^M = V^M - A^M$, is decreasing with q .

We also have:

$$\frac{\partial V^M}{\partial q} = -x(p_H - p_L) \int \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz < 0,$$

which implies:

$$\frac{\partial V_N}{\partial q} = 2x(p_H - p_L) \int \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz > 0,$$

because in a Pure Strategy Manipulation Equilibrium we have $V_N + 2V^M = 0$. Therefore, the liquidity trader's expected payoff is increasing with q .

Proof of Proposition 9: We will only prove the results for a Pure Strategy Manipulation Equilibrium, and the results for a Mixed Strategy Manipulation Equilibrium can be similarly proved. For the first part, we have:

$$E[(p^q(z) - E[p^q(z)])^2] = (1-q)^2 \int_{-\infty}^{\infty} \frac{(p_H - p_L)^2 [\phi^+(z) - \phi^-(z)]^2}{8[\phi^+(z) + \phi^-(z)]} dz.$$

The result is immediate.

For the second part, we have:

$$p^q(z) - p(z) = \frac{-(p_H - p_L)q[\phi^+(z) - \phi^-(z)]}{2[\phi^+(z) + \phi^-(z)]}.$$

The result is immediate. ■

Figure 1: The Optimal Contract for the Trader

