

Knightian Uncertainty and Interbank Lending

Matthew Pritsker*

September 3, 2009

Abstract

The collapse of the housing price bubble during 2007 and 2008 was accompanied by high interbank lending spreads, and a partial collapse in interbank lending. This paper models how Knightian uncertainty over risk exposures to the collapsed assets affects interbank lending spreads, and may have contributed to the collapse of interbank lending. Our main finding is that institutional aspects of the Fed Funds market in the U.S. help to make the performance of that market in terms of spreads and activity robust to Knightian uncertainty over risk exposures in many economic circumstances. However, in some circumstances the market may occasionally collapse — and when it does private incentives may be insufficient to restart the market. In some circumstances when markets collapse, government inspection of banks accompanied by the release of information about risk exposures can improve welfare by internalizing an externality associated with reducing economy-wide uncertainty during a crisis. Our results also show that policies that create better publicly available information on aggregate exposures of core banks within the financial system may also help reduce collapses due to uncertainty ex-ante. The success of “transparency initiatives” in restarting markets depends on the financial architecture of bank linkages. This suggests that public policy aimed at resuscitating markets ex-post should focus on both the initiatives and the optimal design of the ex-ante financial architecture.

Keywords: Banking, Risk, Knightian Uncertainty

JEL Classification Numbers: G21, G32

*Board of governors of the Federal Reserve System. The views expressed in this paper are those of the author but not necessarily those of the Board of Governors of the Federal Reserve System or other members of its staff. This work was begun while the author was visiting NYU Stern School of Business. The author thanks without implicating Thomas Sargent, Anthony Saunders, Viral Acharya, Tanju Yorulmazer, Douglas Gale, Rangarajan Sundaram, Karan Bhanot, Wenying Jiangli, Andrei Kirilenko, and seminar participants at the Federal Reserve Bank of New York and the Federal Reserve Board for useful conversations and comments. Contact info: The Federal Reserve Board, Mail Stop 91, Washington DC 20551. Phone: (202) 452-3534. Email:mpritsker@frb.gov.

1 Introduction

The U.S. housing price boom of 2000-2006 was accompanied by the proliferation of housing backed collateral in the form of mortgage-debt that was repackaged, and resold to investors and banks, both in the US, and around the world. In the wake of the collapse of housing prices, a global financial crisis ensued in which major institutions failed, financial market volatility rose to historically unprecedented levels, interbank spreads were sharply elevated, interbank credit extension was sharply reduced, and governments needed to take many actions to address the crisis.¹

In this paper I argue that an important aspect of the crisis was severely heightened uncertainty that resulted from the interaction of a riskier economic environment and a build-up of structural economic uncertainty, which means incomplete knowledge of the structure of the economy. In the present crisis this was exemplified by incomplete knowledge of the risk exposures of financial institutions and instruments to various sources of risk in general, and housing risk in particular. Intuitively, precise knowledge of risk exposures is not important when risk is low, but can become important when risk is elevated. When housing prices declined and housing risk became elevated during the recent crisis, it was unclear who had taken large losses and who was still exposed. This made counterparty risk evaluation problematic, contributed to market dysfunction, and exacerbated the crisis.

To help address this and potential future crisis, this paper proposes policy steps that reduce uncertainty. Because uncertainty reduction is a public good, these actions will be underprovided by the private sector both ex-post, i.e. during a crisis, as well as ex ante. The paper shows that during a crisis the government may be needed to step-in and reduce uncertainty. A variant of bank stress tests, such as those used by the US in the crisis of 2007-2009, is recommended. In addition, we show that uncertainty reduction before a crisis is also beneficial. We present a proposal for disclosure of bank risk limits, and the release of information on the aggregated across banks risk exposures of systemically important banks, known as core banks hereafter. This proposal has the advantage that it keeps the risk exposures of individual banks private, but by revealing information on risk limits and risk aggregates helps market participants place bounds on the financial health of the average systemically important bank. In the context of stylized examples, we show these measures reduce the likelihood of financial crises, and reduce the costs of restarting markets if a

¹For details on money market spreads in the U.S., Europe, and Japan, see Heider et al. (2009), and Taylor and Williams (2009).

financial crisis should occur. These points are illustrated in the context of the interbank market, but apply more generally.

Before proceeding further it is useful to clarify the distinction between risk, uncertainty, and structural uncertainty as they are used here. A decision maker faces risk if the outcomes from his decisions are random. He faces uncertainty if the outcomes are random and he does not know the probabilities of the outcomes. Uncertainty, as it is used here is also referred to as Knightian uncertainty. An important source of uncertainty is incomplete knowledge of the structure the economic environment, which is referred to above as structural uncertainty.

The reasons why uncertainty can be harmful, and the relationship between risk, uncertainty, and structural uncertainty are best illustrated in a canonical banking example. Consider a loan officer who can make a loan in one of two communities, A or B . In community A there are two types of borrowers, H and L . Both types are indistinguishable but H -types are high-risk and default with probability p^H and L -types are low risk and default with probability p^L , where $p^H > p^L$. In community A the loan officer knows the proportions of H and L types are $\pi^H(A)$ and $1 - \pi^H(A)$. If the loan officer lends in community A , the probability that his borrower is an H type is just its proportion in the population, $\pi^H(A)$. Community B is just like community A , except that the loan officer has limited information on the proportion of H and L -type borrowers that takes the form $\pi^H(B) \in [\underline{\pi}^H(B), \bar{\pi}^H(B)]$. This means he has a range for the proportion of high and low risk borrowers, but does not have beliefs that are sufficiently well formed that they can be described by a probability distribution. This lack of well formed beliefs about $\pi^H(B)$ is a source of structural uncertainty in community B .

The loan officer's knowledge implies that lending in community A only involves risk since the loan officer knows the probability a loan in A will default is given by

$$PD(A) = \pi^H(A)p^H + [1 - \pi^H(A)]p^L. \quad (1)$$

Conversely, in community B , because the loan officer does not know or have a probability distribution for $\pi^H(B)$, he does not know $PD(B)$. Nevertheless, using equation 1, for every possible $\pi^H(B) \in [\underline{\pi}^H(B), \bar{\pi}^H(B)]$ he can calculate a probability of default. Therefore, he believes there is a range of plausible values for $PD(B)$ given by:

$$\begin{aligned} PD(B) &\in [\underline{\pi}^H(B)p^H + (1 - \underline{\pi}^H(B))p^L, \bar{\pi}^H(B)p^H + (1 - \bar{\pi}^H(B))p^L] \\ &= [\underline{PD}(B), \overline{PD}(B)]. \end{aligned} \quad (2)$$

This means, if the loan officer was asked to assess the probability a loan will default in community B , he might state he does not know, but believes it could range from 1 to 3 percent. The fact he cites a range is the consequence of his structural uncertainty and how it interacts with the the risk of the two borrower types. Even though the loan officer does not know $PD(B)$, if the upper bound of his interval for it is low enough, and the spread he can charge in community B is high enough, he may rationally decide to extend the loan.

If the loan officer sets his spread based on the high end of his range for $PD(B)$, he is choosing it in an uncertainty averse fashion, which guarantees his loan spread will cover the highest possible default probability. This may not be too harmful to the borrowers in community B most of the time, but may be problematic if the spreads grow high and borrowers are cut off. Gilboa and Schmeidler (1989) provide an axiomatic foundation for uncertainty averse behavior when agents uncertainty about probabilities is modeled by assuming that agents hold multiple prior distributions over state-variables. All of the analysis in the paper are an application of their framework within a very simple class of multiple priors.²

The same problems that uncertainty and uncertainty averse behavior can cause for community B can also occur in the interbank market because a creditor bank's loan in the interbank market closely resembles a loan to community B . For example, if the borrowing bank has two kinds of loans in its portfolio, then its probability of defaulting on the interbank loan depends on the riskiness of each type of loan in its portfolio, and their proportions as given by the bank's portfolio weights. The creditor bank may know the risk of each type of loan, but have incomplete knowledge about the portfolio weights. Therefore, like the loan officer, the creditor bank faces uncertainty when lending.

When one bank extends a loan to another, the level of uncertainty that the creditor bank has about the portfolio holdings of the borrowing bank is an endogenous part of the transaction since the creditor bank could seek more information before transacting; or the borrowing bank could provide better information to improve the terms of the loan. Many interbank transactions typically take place in the face of substantial uncertainty. This is especially true in the core of the Fed Funds interbank market, where the largest banks have hundreds of billions to trillions of dollars of assets on their own balance sheets, and often extend unsecured loans to each other on very short notice (minutes to hours) through an anonymous brokered market that leaves little time for lending banks to learn about borrowing banks risk exposures.

²The example can also be understood as an application of multiple priors in which each of the multiple priors assigns probability one to a single value of $\pi^H(B)$ and 0 to the other values.

In this paper, I show that the institutional arrangements in the Fed Funds market make it possible for the market to support lending when there is a significant amount of uncertainty. I also provide an explanation for why in the recent financial crisis interbank market lending, especially for longer term loans, dried up, and the spreads on interbank loans climbed to unprecedented levels.

When markets collapse because of uncertainty about risk exposures, banks have incentives to reduce uncertainty about their own conditions in order to restart markets, but in some states of nature the private costs of reducing uncertainty exceed the private benefits—and as a result the markets will not restart on their own. In some circumstances a government policy that sequentially audits individual banks and releases information about their health and their positions reduces the economy wide level of uncertainty, restarts markets, and raises welfare. In addition to these ex-post steps, measures that improve the quality of information on aggregate exposures among core banks can help prevent collapses ex ante. The efficacy of such policies depends on the architecture of the financial system—who is connected to whom, and who knows about it. Therefore, the broader financial architecture is important for formulating policies to mitigate the effects of structural economic uncertainty.

This paper is a contribution to the literature on Knightian Uncertainty, and also to the growing literature on the financial crisis of 2007 - 2009. If depositors in a bank are uncertain about the probability that a bank will default, but believe it may be high; they may take the precautionary step of withdrawing their deposits. Easley and O’Hara (2009) interpret deposit insurance as a step to prevent this uncertainty averse behavior among small depositors not by eliminating the uncertainty, but rather by insuring that depositors will be repaid even if the bank fails.³ Not all depositors can receive deposit insurance because if they did, then depositor withdrawals could not serve as a device to discipline bank’s risk-taking. This paper shows the government has a different role to play by directly reducing uncertainty rather than insuring against it.

The most closely related research on the crisis is Gorton (2008), Gorton (2009), and Metrick and Gorton (2009).⁴ An important theme in all three papers is that the value of AAA tranches that were used as collateral in interbank transactions became informationally

³Other recent related work is Caballero and Krishnamurthy (2008), who model uncertainty during a stylized crisis, and examine government policies to address the crisis. However, the context of their work is entirely different — since it is related to uncertainty over the timing of shocks.

⁴Within the banking literature, the most closely related recent paper is Heider et al. (2009), which focuses on breakdown in the interbank market due to counterparty credit risk. The contribution of this paper is its specific focus on breakdowns that occur due to uncertainty over risk exposures, and market institutions and government actions that can be taken to reduce the effects of the uncertainty.

sensitive to the portfolio composition of the assets backing the tranches when the value of some of those assets declined.⁵ The heightened information sensitivity combined with information asymmetry regarding portfolio composition is interpreted as having caused spreads to widen and the market for AAA tranches to partially collapse because of adverse selection.

This paper departs from Gorton and coauthors in that their approach attributes a part of the crisis to heightened information sensitivity and asymmetric information, while I attribute it to information sensitivity and uncertainty averse behavior. In other words, in my story markets collapsed because many participants did not understand (i.e. were uncertain about) the economic environment, and based on their worst fears required huge spreads in order to take positions. In their story, huge spreads were required because market participants were worried that someone understood the environment better than they did. Neither explanation is mutually exclusive. Future empirical analysis is needed to distinguish among these explanations, and to study whether there are environments when one explanation is more important than the other.

The remainder of the paper consists of 5 sections. Section 2 illustrates the key ideas of the paper in the context of a canonical example. Section 3 presents our model of the economy and studies interbank loan spreads and market breakdown in the context of stripped down model of the interbank market. Section 4 presents our general model and uses it to analyze steps the government can take to fix markets that breakdown, and institutional features of the Federal Funds market that help to prevent breakdown. It also discusses government policy when agents are uncertainty averse versus when they are expected utility maximizers with well formed beliefs. Section 5 discusses the role of financial architecture in government efforts to prevent breakdown. A final section concludes.

2 A Canonical Example

This section completes the canonical example from the introduction. An added twist is that the default probabilities of the high and low risk loans, p^H and p^L depend on macro-economic conditions, F that are known at the time a loan is extended as follows:

⁵Gorton and his coauthors terminology defines the sensitivity of asset value to a state variable as the informational sensitivity to the variable.

$$\begin{aligned}
p^H &= \alpha^H + \beta^H F, \\
p^L &= \alpha^L + \beta^L F,
\end{aligned}$$

with $\alpha^H > \alpha^L > 0$ and $\beta^H > \beta^L > 0$.

F takes the value 0, when times are good, and is equal to the random variable $f > 0$ when conditions are poor, such as during a recession.

Recall that if the loan officer sets his spread in community B in an uncertainty averse fashion, then he will set his spread based on the high end of the range for the probability of default, given in equation 2. If loan officers are risk neutral, uncertainty averse, and compete to make loans in community B , their loan spread over the risk free rate, $S(B)$, will be given by $S(B) = \overline{PD}(B)LGD(B)$, where $LGD(B)$ is the loss given default on loans in community B . Using this spread, the loan officers expected return on the loan will be the same as the risk free rate when taking expectations when using their worst beliefs about default probabilities in community B . Letting $PD(B)$ be the true probability of default in community B , the spread can be decomposed into a default premium and into an additional premium for uncertainty:⁶:

$$\begin{aligned}
S(B) &= \overline{PD}(B)LGD(B) \\
&= PD(B)LGD(B) + [\overline{PD}(B) - PD(B)]LGD(B) \\
&= \text{Default Premium} + \text{Uncertainty Premium}.
\end{aligned}$$

The uncertainty premium can be further decomposed as the product of an uncertainty component and a risk component:

$$\begin{aligned}
\text{Uncertainty Premium} &= [\overline{\pi}^H(B) - \pi^H(B)] \times [(p^H - p^L) \times LGD(B)] \\
&= [\text{Uncertainty Component}] \times [\text{Risk Component}],
\end{aligned}$$

The decomposition of the uncertainty premium shows that structural uncertainty and risk interact in determining loan spreads. In particular the state variables about which there is structural uncertainty affect the uncertainty premia paid in B , and the amount by which they are affected depends on the risk of the loans. Since the risk of the loans vary

⁶The default premium is for expected losses with $PD(B)$ known. The spread does not contain a risk premium because loan officers are risk neutral.

between good and bad times so does the uncertainty premium:

$$\text{Uncertainty Premium} = \begin{cases} [\bar{\pi}^H(B) - \pi^H(B)]LGD(B)(\alpha^H - \alpha^L) & \text{Good times} \\ [\bar{\pi}^H(B) - \pi^H(B)]LGD(B)[(\alpha^H - \alpha^L) + (\beta^H - \beta^L)f] & \text{Bad Times} \end{cases} \quad (3)$$

During good economic times such as economic expansions if α^H is not much different from α^L , then the uncertainty premium that is due to structural uncertainty will be low, and borrowers in community B may not care much about it.

During bad times, the uncertainty premium can become extremely large when f is high. The consequence is borrowers in community B either pay very high spreads given their true average risk, or in the extreme borrowers may be cutoff causing borrowing to collapse.

When borrowing collapses, the provision of information on $\pi^H(B)$ may help it restart. But, private incentives to provide the information may prove insufficient. Thus, in some cases government action may be needed to reduce the uncertainty and resuscitate lending. The analysis that follows expands on on these ideas, but applies them in the context of the interbank market.

3 Model

Our basic framework is of a stylized competitive economy that has M economic sectors, and $2N$ large banks that make loans to those sectors for the purposes of long term investments and for short-term liquidity needs. There are three dates, 0, 1, and 2. At date 0 the banks raise funds from depositors and equity holders and invest those funds in long-term loans that mature at date 2. At date 1, news arrives about the performance of the macro-economy. In addition each bank experiences either a funding shock that provides it with more deposits, or a lending shock that provides it with opportunities to make short-term loans, but not both. Following the news and shocks, banks use the interbank market to channel funds from those banks that have excess funds to those banks that have excess lending opportunities. The heart of the paper analyzes the institutions in the Federal Funds market that facilitate this transfer of funds at time 1 in the presence of uncertainty. To close the model, at date 2, the returns on banks loan portfolios are realized, and banks pay back their stakeholders

if fully solvent—or default and make partial payments if not solvent.

3.1 Date 0

At date 0, each large bank $i = 1, \dots, 2N$ is endowed with a fixed amount of equity funding E_i , and then chooses to raise deposit funding D_i , in order to fund a long-term asset portfolio of size A_i ($A_i = D_i + E_i$). The long-term asset portfolio consists of investments in M sector-portfolios. Each sector portfolio consists of loans to a positive measure of small borrowers in each of the M sectors of the economy. The gross return per dollar received at date 2 for \$ 1 invested in the m 'th sector portfolio at date 0 is R_m . The return on the vector of sector portfolios is denoted by the $M \times 1$ vector $R = (R_1, \dots, R_M)'$. Bank i 's portfolio weights are denoted by the M -vector ω_i with individual elements $\omega_{i,m}$.

The subsection that follows provides a derivation of the probability distribution of R . It can be skipped on first-reading. The key results in the subsection is that conditional on information known at date 0, I_0 :

$$R|I_0 \sim N(\mu, \Sigma)$$

and conditional on I_1 , information learned at date 1:

$$R|I_1 \sim N(\mu(1), \Sigma(1))$$

where $\mu = \alpha$; $\mu(1) = \alpha + \beta f(1)$; $f(1)$ is a $K \times 1$ vector of factors which represents news about the macro-economy that is learned at date 1; and β is an $M \times K$ matrix of factor sensitivities which indicates how different sectors load on the factors.

This specification of the return distribution makes two contributions. First, although the bank makes individual loans in each sector, the returns on its loan portfolio is Gaussian. This makes the model amenable to mean-variance portfolio analysis as well as to the standard tools of information economics. We use the tractability provided by Gaussianity in the analysis that follows. Second, the parameters of the distribution function of the loan portfolio, depend on loan characteristics that aggregate-up from the micro-level. For example, loan interest rates in each sector affect the sensitivity of each sector-portfolio to macro-economic risks, and affect the covariance matrix of all loans returns. This is an interesting subject in itself,

but is not explored in this paper.

3.2 Lending Opportunities

In each sector m there is a continuum of infinitesimal potential long-term borrowers indexed by $\eta_m \in [0, \bar{\eta}_m]$. In this expression, η_m denotes borrower η in sector m , and $\bar{\eta}_m$ is the measure of potential borrowers in sector m . The sectors may vary in size — so that for some sectors, such as housing, $\bar{\eta}_m$ is very large, while for other sectors, $\bar{\eta}_m$ is small. The distinction in sector sizes will not be important for the analysis in the paper until section 4.4. Each borrower η_m requires 1 dollar of bank financing for a project that returns r_{η_m} in period 2, where:

$$r_{\eta_m} = \theta_m + \gamma_m F + \epsilon_m + u_{\eta_m}. \quad (4)$$

The return depends on a sector-specific constant, θ_m , and three independently distributed components. F is normally distributed K-vector that represents news about the macroeconomy. $\epsilon_m \sim \mathcal{N}[0, \sigma^2(\epsilon_m)]$ represents news about sector m , and is distributed independently of news in sectors $m' \neq m$. Finally, u_{η_m} is a borrower-specific component that is independent across borrowers and distributed uniformly on $[0, \bar{u}_m]$.⁷

The macroeconomic factor can be further decomposed into components $f(1)$ and $f(2)$:

$$F = f(1) + f(2), \quad (5)$$

where $f(1)$ is the best forecast of F conditional on public information I_1 that arrives at date 1 ($f(1) = E[F|I_1]$), and $f(2)$ represents the error in the forecast which is learned at date 2.

Because $f(1)$ and $f(2)$ are innovations in beliefs about F , they have mean 0, and are uncorrelated; for tractability they are assumed to be normally distributed:

$$\begin{pmatrix} f(1) \\ f(2) \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_f(1) & 0 \\ 0 & \Sigma_f(2) \end{pmatrix} \right]. \quad (6)$$

Each loan that is extended to a borrower in sector m promises to pay back contractually agreed upon principal plus interest X_m at maturity, but in the case of default only produces

⁷This decomposition is a modification of the decomposition in Kodres and Pritsker (2002).

the non-stochastic recovery value RGD_m .⁸ The loan defaults if the rate of return on the entrepreneurs project is less than X_m . For the purposes of this paper, X_m is a fixed parameter that is determined by the financial intermediation process for long-term loans. I abstract from its determination because it is not essential for the analysis of the interbank market.

We will make the following additional approximating assumption about the distributions:

Assumption 1 For all values of F and ϵ_m ,

$$0 < X_m - (\theta_m + \gamma_m F + \epsilon_m) < \bar{u}_m$$

To a first approximation, the assumption can be understood as requiring that the variability in borrowers returns due to the macro factors F and sector specific risk ϵ_m are small relative to the variability due to the borrowers idiosyncratic risk since $[0, \bar{u}_m]$ represent upper and lower bounds of the idiosyncratic return risk. This assumption cannot literally be true because F and ϵ_m are Gaussian random variables, but in the appendix we show that the returns on loans can be calibrated so that the probability the assumption is violated is approximately 10^{-7} . For simplicity and tractability we assume that it is true. Under the condition that it is true,

$$\begin{aligned} Prob(r_{\eta_m} < X_m \mid \epsilon_m, F) &= Prob(\theta_m + \gamma_m F + \epsilon_m + u_{\eta_m} < X_m), \\ &= Prob\left(\frac{u_{\eta_m}}{\bar{u}_m} < \frac{X_m - \theta_m - \gamma_m F - \epsilon_m}{\bar{u}_m}\right), \\ &= \frac{X_m - \theta_m - \gamma_m F - \epsilon_m}{\bar{u}_m}. \end{aligned} \quad (7)$$

From this probability, conditional on F and ϵ_m it then follows by the law of large numbers that for any positive mass of loans in sector m , the fraction that default in period 2 is given by the expression on the right hand side of equation 7. Because F and ϵ_m are normally distributed conditional on the information sets I_0 and I_1 , it then follows that the fraction of loans from sector m that will default in period 2 is also normally distributed conditional on I_0 and I_1 .

⁸ RGD_m stands for the recovery given default in sector m . It is not restricted to depend on X_m .

Let a sector portfolio denote a portfolio of loans to a positive mass of borrowers in sector m . Because X_m and the recovery given default are non-stochastic, the return per dollar to the sector-portfolio, denoted R_m , is proportional to the fraction of borrowers that default, and is also normally distributed. Additionally, the vector of returns per dollar for loans in each sector, denoted by $R = (R_1, \dots, R_M)'$, is jointly normally distributed. This result, and the details of the conditional distribution functions is stated formally below:

Proposition 1 *Under assumption 1, R , the gross return on an M -vector of sector portfolios has the following distribution conditional on I_0 and I_1 :*

$$R|I_0 \sim \mathcal{N}[\mu, \Sigma] \quad (8)$$

$$R|I_1 \sim \mathcal{N}[\mu(1), \Sigma(1)] \quad (9)$$

where,

$$\begin{aligned} \mu &= \alpha, \\ \mu(1) &= \alpha + \beta f(1), \\ \alpha &= \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_M \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta'_1 \\ \vdots \\ \beta'_M \end{pmatrix} \\ \alpha_m &= \left[X_m + (RGD_m - X_m) \left(\frac{X_m - \theta_m}{\bar{u}_m} \right) \right], \\ \beta_m &= \left[\frac{(X_m - RGD_m)\gamma_m}{\bar{u}_m} \right], \\ \Sigma &= \beta[\Sigma_f(1) + \Sigma_f(2)]\beta' + \begin{bmatrix} \sigma^2(\epsilon_1) & & \\ & \ddots & \\ & & \sigma^2(\epsilon_M) \end{bmatrix}, \\ \Sigma[1] &= \beta[\Sigma_f(2)]\beta' + \begin{bmatrix} \sigma^2(\epsilon_1) & & \\ & \ddots & \\ & & \sigma^2(\epsilon_M) \end{bmatrix}. \end{aligned}$$

Proof: See the appendix.

The proposition is a straightforward application of the law of large numbers and closely follows Vasicek's(2002) development of asymptotic portfolios. However, in Vasicek, the returns on asymptotic portfolios are a nonlinear function of the factors, which is unwieldy for

theoretical modeling with even a single factor. By contrast, the derivation here presents a linear result, which is useful for many theoretical modeling applications, including the ones considered here.

3.3 The Banks' Optimization Problem and Portfolio Holdings

At date 0, each bank i chooses its optimal capital structure and portfolio weights in long-term loans ω_i to maximize its value to its shareholders. Bank i 's liabilities consist of shareholder equity, that is in fixed supply E_i , and insured deposits D_i , whose level is chosen by the bank. Including deposit insurance premium payments, for each dollar of deposits the bank pays interest rate R^D that we assume is fixed and does not depend on the banks portfolio weights.⁹ In the banks date 0 optimization it ignores the short-term lending opportunities that arise at date 1. This is rational because because the lending and funding shocks are exactly offsetting. Therefore, the loans at date 1 will be funded provided the interbank market functions well, which is very likely.¹⁰

For portfolio weights ω_i , and deposits D_i , the expected present value of Bank i to its equity holders at date 0 is $V_i(0)$ given by

$$\begin{aligned} V_i(0) &= \mathbb{E} \left\{ \frac{\max[(E_i + D_i)\omega'_i R - D_i R^D, 0]}{(1 + R^f)^2} \right\} \\ &= \mathbb{E} \left\{ E_i \frac{\max[R^D + (1 + L_i)(\omega'_i R - R^D), 0]}{(1 + R^f)^2} \right\} \end{aligned} \quad (10)$$

where L_i is bank i 's leverage ratio given by $L_i = D_i/E_i$, and R^f is the gross risk-free rate.

Bank i chooses its optimal portfolio to maximize 10 subject to two regulatory constraints. The first places an upper bound on the banks leverage:

$$L_i \leq \bar{L}. \quad (11)$$

The second places an upper bound on the bank's probability of default conditional on time

⁹This is without loss of generality because the banks as modeled here satisfy other regulatory constraints that keep their premiums small.

¹⁰An additional reason for not hedging in this environment is that it is very costly because hedging involves giving up long-term lending opportunities in exchange for short-term opportunities that are likely to be less lucrative.

0 information $PD_i(\omega_i, 0)$, and thus resembles a constraint on the firms credit value at risk:

$$PD_i(\omega_i, 0) \leq \bar{K}. \quad (12)$$

The bank will default and become insolvent at date 2, if the returns on its loan portfolio are insufficient to cover its deposits. A little bit of algebra then shows the probability of default is:

$$PD_i(\omega_i, 0) = \Phi \left(\frac{\frac{L_i}{1+L_i} R^D - \omega_i' \mu}{\sqrt{\omega_i' \Sigma \omega_i}} \right) \quad (13)$$

These constraints are an essential part of the banks maximization problem because its deposits are fully insured. Given deposit insurance, without constraints the bank's objective function is maximized by choosing infinite leverage, and a loan portfolio that has infinite variance.¹¹

With the constraints, tedious algebra shows that given the optimal leverage choice L_i , the bank's optimal asset portfolio has the maximal mean return and maximal standard deviation that satisfies the credit value at risk constraint. Using mean-variance portfolio theory, this implies the optimal portfolio is on the risky asset mean-variance efficient frontier, and is such that the value at risk constraint binds. Algebra shows that that for fixed leverage, the set of portfolios for which the credit value at risk constraint binds is an upward sloping line in mean-standard deviation space. Therefore, the optimal portfolio is where this line intersects the efficient frontier as illustrated in Figure 1. This portfolio is labelled $\bar{\omega}$ because it has the maximal mean and variance that can be achieved.

To model structural uncertainty, I assume that other bank's know bank i 's leverage because it is easily observable from i 's balance sheet. However, because of internal limitations within bank i due to limited underwriting capability or other constraints on investment opportunities they believe bank i cannot achieve its optimal portfolio, but instead chooses another portfolio on the efficient frontier that satisfies the constraints. More formally, let $\underline{\omega}$ represent the lowest mean efficient portfolio that other banks believe i could choose. Then, banks believe i 's portfolio is somewhere on the efficient frontier between $\underline{\omega}$ and $\bar{\omega}$; but other banks do not know more about bank i 's portfolio than this information.¹² Because mean-variance efficient portfolios are convex combinations of each other (provided there are no

¹¹In a fully dynamic setting if the bank has franchise value, then this would help to constrain its risktaking somewhat.

¹²There are other ways to motivate structural uncertainty and bounds on i 's portfolio weights. The main results in the paper only require that i 's portfolios is in a closed, bounded, convex set.

restrictions on the size of long or short positions), other banks structural uncertainty about i can be expressed as:

$$\omega_i \in \mathcal{C}(\underline{\omega}, \bar{\omega}) \quad (14)$$

where, $\mathcal{C}(\underline{\omega}, \bar{\omega})$ is the set of portfolios that are convex combinations of $\underline{\omega}_i$ and $\bar{\omega}_i$.¹³

3.4 Date 1

At date 1 banks receive macroeconomic news $f(1)$, and receive stochastic lending and funding shocks. The banks that receive stochastic lending shocks will be indexed by i , and those that receive a funding shock will be indexed by j . Each stochastic lending shock is a new demand for short-term loans from a mass of infinitesimal short-term borrowers that collectively have measure 1. Each short-term borrower needs a loan of $1ds$ but can only borrow if the rate on the loan is below their reservation value \bar{R}_i . To keep this part of the analysis simple, the short-term borrowers will default with probability one unless screened by bank i , but loans to them will be risk-free if screened. In addition, the mass of short-term loans available to each bank is small enough relative to the scale of the banks long-term loan portfolio that whether these loans are made or not has no effect on any bank's probability of default at time 2.

Each bank j that receives a funding shock has access to new funds that have measure 1, and that pay an interest rate R^D which is less than the riskfree rate. These funds can be invested at the risk-free rate until period 2, or loaned out in the interbank market to a bank i that needs the funds. I assume the tier of the interbank market in which banks i and j trade together operates competitively. This implies that the expected return on short-term interbank lending must be as good as lending at the riskfree rate. This implies $R_{i,j}$ the rate on a short-term interbank loan between any bank i and j satisfies:

$$R_{i,j} = R^f + \widehat{PD}_i LGD_i, \quad (15)$$

where R^f is the riskfree rate, \widehat{PD}_i is the perceived probability that bank i will default at date 2 given information that is available at date 1, and LGD_i is the loss given default

¹³Mathematically,

$$\mathcal{C}(\underline{\omega}, \bar{\omega}) = \{\omega : \omega = \theta \underline{\omega} + (1 - \theta) \bar{\omega}, \quad \theta \in [0, 1]\}.$$

experienced by bank j if bank i defaults. For simplicity, we assume this quantity is the same for all banks j .

Bank i will borrow in the interbank market provided that his borrowing cost is low enough that he can make profitable short-term loans with the funds. This requires that $\bar{R}_l \geq R_{i,j}$, which from equation (15) implies

$$\bar{R}_l - R^f \geq \widehat{PD}_i LGD_i \quad (16)$$

In other words, because funding for the mass of new short-term loans has to essentially be intermediated through the interbank market, the new loans must pay a spread over the riskfree rate that is at least as great as the spread paid by banks in the interbank market. The size of the spread that the new borrowers will have to pay depends on how beliefs about \widehat{PD}_i , i 's probability of default are formed. This in turn depends on the trading institutions within the interbank market. There are two main ways in which trade takes place in the market. The first is bilateral in which one bank directly arranges a loan with another; the second is when loans are arranged in an anonymous brokered market. I treat the first of these institutions below, and the second in section 4.

3.5 The Pricing of Interbank Debt with Bilateral Exchange

In this section, I model the bilateral exchange tier of the interbank market in which there is a bank i that needs a loan from bank j , and borrows directly. Like the loan officer in the canonical example, bank j is risk neutral, but uncertainty averse. Bank j knows bank i 's leverage but has structural uncertainty about bank i 's portfolio weights, but knows they are on the mean-variance efficient frontier, as in equation 14.

Because of j 's structural uncertainty over ω_i , he holds a range of beliefs for i 's probability of default and charges a spread based on his worst case beliefs for PD_i . Formally, the interest rate that bank i will face in the interbank market at date 1 is as given in equation 15 with

$$\widehat{PD}_i = \max_{\omega_i \in \mathcal{C}(\underline{\omega}, \bar{\omega})} \Phi \left(\frac{\frac{L_i}{1+L_i} R^D - \omega'_i \mu(1)}{\sqrt{\omega'_i \Sigma(1) \omega_i}} \right). \quad (17)$$

If we let ω_i^* denote i 's true portfolio holdings, then i 's true probability of default is given by:

$$PD_i(\omega_i^*) = \Phi \left(\frac{\frac{L_i}{1+L_i} R^D - \omega_i^{*\prime} \mu(1)}{\sqrt{\omega_i^{*\prime} \Sigma(1) \omega_i^*}} \right); \quad (18)$$

Like the canonical example, the spread that i faces can be decomposed into a premium for default and an additional premium for uncertainty:

$$R_i - R^f = \widehat{PD}_i LGD_i \quad (19)$$

$$= [PD_i(\omega_i^*) LGD_i] + [\widehat{PD}_i - PD_i(\omega_i^*)] LGD_i \quad (20)$$

$$= \text{Default Premium} + \text{Uncertainty Premium.}$$

In some conditions, such as those outlined in Case 1, the news that arrives at date 1 unambiguously shrinks the uncertainty premium:

Case 1: $\forall \omega_i \in \mathcal{C}(\underline{\omega}_i, \bar{\omega}_i)$

- (a) $\omega_i > 0$
- (b) $\frac{L_i}{1+L_i} R^D - \omega_i' \mu < 0$
- (c) $f(1)$ is such that $\beta_m f(1) > 0$ for all m .
- (d) $\Sigma[1] = \rho \Sigma$ for some $\rho \in (0, 1)$

The conditions for premia to shrink essentially require lower volatility, and require that the expected return on assets that the bank holds in positive amounts increase. These low volatility, positive news conditions are of the sort that prevailed ahead of the global financial turmoil that began in March 2007, and may help to explain how spreads could have gone down so much during those times.

Conversely as in case 2, if the bank could have a positive or negative exposure to some sector (such as sector 1 below), then if the news about the expected return in that sector is bad enough, i.e. bounded sufficiently far below 0, it will increase the uncertainty spread for bank i .

Case 2: (a) $\omega_i[1] \in [-A, A]$ for some $A > 0$.

(b) $\beta_1 f(1) < \psi < 0$.

If i 's potential exposure is large enough, its uncertainty premium may become too high for it to make loans to its short-term borrowers, causing interbank borrowing between i and j to break down. This generates a welfare loss because the best use of funds is to channel them to bank i 's borrowers.

These results are summarized in the following proposition.

Proposition 2 *Although there is Knightian uncertainty about bank i 's positions, under some economic conditions (such as case 1), favorable economic news can reduce the loan spread that bank i pays for borrowing in the interbank market even though there is Knightian uncertainty over bank i 's positions. Conversely, under other economic conditions, such as those in case 2, sufficiently unfavorable news about some sectors of the market can destroy bank i 's ability to finance its new lending opportunities in the interbank market, effectively causing the interbank market to break down.*

Proof: See the appendix.

The proposition shows that the uncertainty spread can be low in the right economic conditions. As shown in Figure 2, reproduced from Pritsker (2009), provided volatility is low enough, i 's uncertainty premium can remain small even when its leverage is high. However, if leverage is high, and volatility increases, uncertainty premia can increase very sharply, as they did during the crisis.

The magnitude of the uncertainty premium depends on the structure for transacting in the interbank market. An important distinction between bilateral borrowing and borrowing in an anonymous brokered market is that worst case beliefs about default probability in the latter are based on one bank, while in the latter case it is based on worst case beliefs are formed over the set of possible banks with whom a bank may transact. We turn to this topic in the next section.

4 Multiple Banks and the Interbank Market for Federal Funds

The analysis of the Federal Funds market is broken into five subsections. Section 4.1 describes the multi-tiered structure of the Federal Funds market; section 4.2 presents a model of the top tier of the market, which is also referred to as the center of the market. In the context of an example, section 4.3 studies how trade in the top-tier can breakdown and examines when a government policy that inspect banks and reveals information about their health and positions can improve welfare. Having illustrated how the top tier of the market may sometimes break down, section 4.4 examines the structure of the top-tier in more detail and shows that the top tier of the market has evolved to have a structure that substantially reduces the likelihood of a breakdown in this market due to Knightian uncertainty over risk exposures. The key that avoids breakdown is the information environment in the market which is determined by who can participate, and what they know about each other. Government can play a role in shaping the information about market conditions that is available, which is the subject of section 4.5.

4.1 The Interbank Market

This section describes the features of the Fed Funds market that are salient for our analysis. Formally, the interbank market is a market for overnight lending or for a longer-term in which banks trade with each other to fulfill liquidity needs, meet central bank reserve requirements, and finance new lending. Three facts about the market are important for our analysis:

1. Many loans are based on repeated relationships in which banks that have a tendency to borrow or lend to each other on a repeated basis face lower rates given other observable variables that are correlated with their credit risk [Furfine, (2001) for the U.S., Cocco et. al. (2009) for Portugal].
2. Many loans take place through an anonymous brokered market in which the borrowers identity is disclosed to a lender only after a match is established at an agreed upon rate:

“While borrowers and lenders may arrange trades directly with one another, larger more sophisticated market participants tend to arrange most of their trades through

brokers. A key feature of brokered trading is that trades are initiated anonymously between interested parties, as a borrower's identity is disclosed to a lender only after a match is established at an agreed-upon loan rate. After a match is established and the lender accepts to lend to the borrower (a decision usually conditioned on the presence of a predetermined credit line between the two parties), the trade is deemed 'executed' by the broker." [Bartolini et. al. (2008)]

3. The market has a multi-tiered structure in which small banks are net lenders to medium sized banks; medium-sized banks in turn are net lenders on average to large banks; while large banks primarily extend loans to each other [Allen et. al. (1989), Furfine (1999), and Bech and Atalay (2008)].

The first two facts about the market, that loans are based on relationship lending, and that loans are made in an anonymous brokered market, are nearly inconsistent with each other. The reconciliation comes from the third-fact that the Federal Funds market is multi-tiered which allows some participants to transact through an anonymous brokered market and others transact based on relationship lending as described by Bech and Atalay:

"There are two methods for buying and selling federal funds. Depository institutions can either trade directly with each other or use the services of a broker. ... In the direct trading segment, transactions commonly consist of sales by small-to-medium sized banks to larger banks and often take place on a recurring basis. The rate is set in reference to the prevailing rate in the brokered market. In the brokered segment, participation is mostly confined to larger banks acting on their own or a customer's behalf."

Bech and Atalay use network modeling techniques to describe the flow of funds in the market. They note that the center of the market consists of a set of large banks who tend to purchase fed funds from small banks, and then lend the funds among themselves, or to banks outside of the center.¹⁴ The banks in the center of the market are large banks that had on average about 400 billion dollars of assets on their balance sheets at the end of 2006. I model the interaction of the large banks in the center of the market in the next subsection.

¹⁴The center of the market in my terminology is known as the GSCC or giant strongly connected component of the network. Any bank in the GSCC can channel funds to other banks in the GSCC through either direct links (A lends to C) or indirect links (A lends to B who lends to C). Banks that only lend funds to banks in the GSCC, but cannot borrow from these banks are part of the GIN, or giant in-component. Banks that borrow from the GSCC, but do not lend are part of the GOUT, or giant out component.

4.2 Banks at the Center

In this section we model the interaction of the multiple large banks that trade in the center of the Fed Funds market. We also refer to the center as the core of the market, and we refer to the large banks that trade in the center as core banks. For simplicity, we assume that at date 1, each bank's portfolio looks identical to outsiders, which means that all large banks publicly report the same balance sheet leverage. At the beginning of date 1, all banks learn the value of $f(1)$. The remaining events that occur at date 1 are broken up into M sub-periods, $m = 1, \dots, M$. In each subperiod, m , N randomly selected banks are hit by a lending shock that gives the bank the opportunity to make a short-term loan that matures at date 2. The other N banks are hit by positive funding shocks. As in the earlier analysis, following the shocks, banks that are hit with positive funding shocks can make interbank loans that mature at date 2, or they can invest the funds at the risk free rate until the same date. Banks with positive funding shocks are not allowed to hoard funds across subperiods; this assumption is made for simplicity and does not qualitatively change the results. After these decisions are taken, the sub-period ends, and the next sub-period begins.

Trade in the center of the interbank market is modeled as a random matching game. Each large bank that needs funds places a bid for the funds in the market. The other large banks that have funds to lend are then randomly matched with the bidding banks. If the lending bank consents to the transaction at the agreed upon rate, then a trade is consummated. For simplicity, if the trade is not consummated, then the borrower and lender do not get the opportunity to again trade again in the subperiod. The random matching game closely resembles anonymous brokered trading in the Fed Funds market.¹⁵ The game as specified here gives all of the bargaining power to the borrowing banks. Each borrowing bank captures maximal surplus by proposing to borrow at the lowest possible rate that could be acceptable to lending banks, which means proposing a spread equal to $\widehat{PD}_i LGD_i$. Since LGD_i is identical for all banks, this is equivalent to borrowing banks proposing an estimate of \widehat{PD}_i that is equal to borrowing bank's assessment of the worst case probability of default that lending banks could arrive at when making a loan in the anonymous brokered market. Since all borrowing banks will propose the same estimate of \widehat{PD}_i , randomly matching borrowing and lending banks is an equilibrium in the brokered market. A core borrowing bank may be able to get an even better rate by signalling that it holds a low risk portfolio, and then

¹⁵It is not clear precisely how much trading large banks in the top tier of the market conduct through brokers. Bartolini et. al. (2008) claim that the largest and most sophisticated participants in the market trade through brokers, while based on conversations with experts Ashcraft and Duffie (2007) suggest that less than 27% of trades by large banks is conducted this way.

transacting in the bilateral tier of the Fed Funds market. We assume that signalling is too costly for core borrowing banks to engage in, or put differently, borrowing banks have already provided information to lending banks about their portfolios (the bounds on the possible portfolio holdings), and that further signalling is prohibitively expensive.

When a bank lends in the anonymous brokered market, the bank it lends to is a random draw from the distribution of the set of all banks to whom it may extend a loan. Since lending banks do not know who is on the other side of the market, it could be any of the other $2N - 1$ core banks. In computing the worst case distribution of the credit quality of borrower banks, the lending banks must take into account everything they know about the characteristics of the borrowing banks. Since the borrower banks are ex-ante identical, they all have the same leverage L_i , and the same equity capital E_i , and assets A_i .¹⁶ Lender banks also know that each borrower banks portfolio weights lie in the range specified by equation 14, and we assume they know the aggregate supply of loans that all banks in the center of interbank market may have, denoted by the aggregate endowment vector Y_M , but they do not know how these exposures are divided among the banks. In later sections we study the consequences that coarser information about Y_M may have.

Now we turn to the spread that borrowing banks propose. Since lending banks are uncertainty averse, borrowing banks propose a spread that is equal to the worst case probability of default that a lending bank could have. Since each lending bank j has a probability $1/(2N - 1)$ of being randomly matched with one of the other $2N - 1$ banks its worst case beliefs about the probability that its borrowing bank i will default is the solution to:

$$\widehat{PD}_i = \max_{\omega_k, k=1, \dots, 2N} \frac{1}{2N - 1} \sum_{k \neq j}^{2N} PD_k(\omega_k) \quad (21)$$

such that,

$$\sum_{k=1}^{2N} \omega_k A_k = Y_M. \quad (22)$$

In addition for each bank k , $k = 1, \dots, 2N$,

$$\omega_k \in \mathcal{C}(\underline{\omega}, \bar{\omega}). \quad (23)$$

¹⁶If banks have different L_i , E_i and A_i this would complicate the analysis, but have little effect on the main results.

In the above maximization, the borrowing bank computes the worst probability beliefs that a lending bank could have about its counterparties while accounting for the common knowledge that all banks have. This common knowledge is the adding up constraint that beliefs about bank's individual loan portfolios must add up to the aggregate loan supply in each sector, which is known (equation 22); as well as the constraint that each bank's portfolios lie within an interval on the efficient frontier (equation 23).

When the borrowing bank computes for the worst case default probability that a lending bank could have, it solves the constrained problem while taking into account what the portfolio holdings of the lending bank may be, since from equation 22, the lending bank's portfolio constrain beliefs about the borrowing banks portfolios. Because equations 22 and 23 are satisfied by banks initial holdings, there is a solution for the worst case probability of default. To find the solution, we make one more mild auxiliary assumption:

Assumption 2 *For each bank $k = 1, \dots, 2N$, for every feasible ω_k , $PD_k(\omega_k) < 0.5$*

This is a very mild assumption except for very severe banking crises.

To solve for the worst-case beliefs, we note two facts. First, the constraint set is convex. Second, in the proof we show that the objective function is a convex function. When a convex function is maximized with respect to convex constraints, the solution is at the extremes. This means if $PD(\bar{\omega}) > PD(\underline{\omega})$ then the solution for worst case beliefs involves believing as many potential borrowing banks as possible hold portfolio $\bar{\omega}$, subject to the other constraints of the maximization. Importantly, when equation 22 holds as an equality, then the worst case beliefs involve no more than one bank holding a portfolio in the interior of $\mathcal{C}(\underline{\omega}, \bar{\omega})$ since if more than one did, by convexity \widehat{PD}_i could be increased by moving one portfolio up toward $\bar{\omega}$ and the other toward $\underline{\omega}$, which violates the condition that the original portfolio was an optimum. The formal solution for the worst case beliefs about the default probabilities of borrowing banks is slightly more complicated because it involves solving for the portfolio of the lending bank and the borrowing banks, and ensuring that the borrowing banks have the highest probability of default possible. This generates four special cases, as illustrated in the proposition below:

Proposition 3 *Under assumption 2, the worst case beliefs \widehat{PD}_i that solve 21 are given by the following:*

If $PD(\bar{\omega}) \geq PD(\underline{\omega})$, then for m_u and $\tilde{\omega}$ that satisfy the conditions,

$$\begin{aligned} m_u &= \max m, \quad m \in \{1, \dots, 2N\}, \quad \text{such that} \\ Y_M/A_k &= m\bar{\omega} + (2N - m - 1)\underline{\omega} + \tilde{\omega}, \\ \tilde{\omega} &\in \mathcal{C}(\underline{\omega}, \bar{\omega}), \end{aligned}$$

if $PD(\tilde{\omega}) \leq PD(\underline{\omega})$, then

$$\widehat{PD}_i = \frac{m_u PD(\bar{\omega}) + (2N - m_u - 1)PD(\underline{\omega})}{2N - 1};$$

if $PD(\tilde{\omega}) \geq PD(\underline{\omega})$, then

$$\widehat{PD}_i = \frac{m_u PD(\bar{\omega}) + (2N - m_u - 2)PD(\underline{\omega}) + PD(\tilde{\omega})}{2N - 1}.$$

If $PD(\bar{\omega}) \leq PD(\underline{\omega})$ then for m_u and $\tilde{\omega}$ that satisfy the conditions,

$$\begin{aligned} m_u &= \max m, \quad m \in \{1, \dots, 2N\}, \quad \text{such that} \\ Y_M/A_k &= m\underline{\omega} + (2N - m - 1)\bar{\omega} + \tilde{\omega}, \\ \tilde{\omega} &\in \mathcal{C}(\underline{\omega}, \bar{\omega}), \end{aligned}$$

if $PD(\tilde{\omega}) \leq PD(\bar{\omega})$, then

$$\widehat{PD}_i = \frac{m_u PD(\underline{\omega}) + (2N - m_u - 1)PD(\bar{\omega})}{2N - 1};$$

if $PD(\tilde{\omega}) \geq PD(\bar{\omega})$, then

$$\widehat{PD}_i = \frac{m_u PD(\underline{\omega}) + (2N - m_u - 2)PD(\bar{\omega}) + PD(\tilde{\omega})}{2N - 1}.$$

Proof: For each borrower bank, the portfolio that maximizes its probability of default maximizes the opposite of a Sharpe ratio given by

$$\frac{\frac{L_i}{1+L_i}R^D - \omega'_i\mu(1)}{\omega'_i\Sigma(1)\omega_i},$$

when treating $(R^D L_i / (1 + L_i))$ as the risk-free rate. Since the Sharpe ratio is a concave function of a portfolio weights, minus the Sharpe ratio is a convex function of the weights. The probability of default depends on $\Phi(\cdot)$ of the opposite of the Sharpe ratio. Under assumption 2, the operational part of the $\Phi(\cdot)$ function is its lower half, which is a convex function. Since convex functions of convex functions are convex, the probability that any bank defaults is a convex function of its portfolio weights. Since the objective function being maximized is a positive weighted sum of convex functions, it is also a convex function. Inspection will also quickly reveal that the feasible set of portfolios is a convex set. When a convex function is maximized over a convex set, the solution is at the extremes. This implies that no more than one of the $2N$ banks can have a portfolio weights that are not equal to either $\bar{\omega}$ or $\underline{\omega}$ since if two banks had portfolios in the interior of the set, they would be able to alter their portfolios until one of them hit a boundary. Finally, after the optimal portfolio weights are found, the worst case beliefs that a lender could have would involve a lender having the lowest risk possible, and the borrowing banks having the highest risk possible. Applying this criterion generates the four conditions in the final result for \widehat{PD}_i . \square .

Although the banks portfolios are multidimensional, because of convexity, and because the banks portfolios are mean-variance efficient, the solution for \widehat{PD}_i is simple to characterize. If instead each borrower bank's portfolio weights were known be an element of some more general convex set, then under assumption 2, the problem will generally remain convex, and the solution will lie on the boundary of the set, but it will be more difficult to solve the model. To illustrate ideas, I have opted to keep the model simple.

4.3 An example with two risky assets

With this solution for worst-case beliefs in hand, we now focus on illustrating how the anonymous brokered tier of the interbank market can break down, and why private efforts to restart it may fail while government efforts to restart it may succeed and improve welfare. To illustrate these ideas, suppose there are only two sectors. Loans to a pool of borrowers in sectors 1 and 2 will be referred to as assets 1 and 2. Core banks hold $Y_M[1]$ of asset 1, and each bank i has the capacity to hold the entire stock of that asset ($\bar{\omega}_i A_i[1] > Y_M[1]$).

If the news that arrives at date 1 is sufficiently bad for asset 1, then because of convexity, and our assumptions on capacity, the worst-case beliefs that maximize \widehat{PD}_i will involve one borrower bank holding all of asset 1, and other $2N - 1$ banks holding no exposure. This is equivalent to having the belief that there are $2N - 1$ banks with a low probability of default,

P_L and one bank with a high probability of default P_H , which implies,

$$\widehat{PD}_i = P_L + \frac{1}{2N-1}(P_H - P_L) \quad (24)$$

From equation 16, we know that if the worst-case beliefs \widehat{PD}_i exceeds some maximal level, denoted \overline{PD}_i , that financing costs for short-term loans will be too high to justify extending these loans, and hence the interbank market may break down. Assume that the news $f(1)$ is sufficiently bad that it does break down.

One possible solution would be for good banks to reveal information about themselves. Suppose the cost of signaling in this way is c , given by

$$c = (\bar{R}_i - R^D - P_L LGD_i) \times \frac{M}{2} + \epsilon, \quad (25)$$

for some $\epsilon > 0$. This cost is then equal to the expected surplus a good bank could hope to earn by signaling plus a little bit, which means for this c , no bank would be willing to signal on its own because it could not pass the costs of signalling on to its borrowers. In this case, the interbank market will fail because of uncertainty, and private incentives will be insufficient to restart the market.¹⁷

It turns out that if government pays to acquire the information on banks risk exposures, it can improve the function of the interbank market and create social surplus.¹⁸ To illustrate the mechanism for improving surplus, suppose one bank has default probability P_H and the other $2N - 1$ have default probability P_L , i.e. suppose that the worst case beliefs associated with uncertainty are correct. If the government searched for the one bad bank sequentially, publicized for each bank searched whether it is H or L and announced its risk exposures, and then stopped after the bad bank was found, its expected search costs are $(2N - 1)c/2$.¹⁹ After the bad bank is located and its presence is announced, then the other $N - 1$ low risk borrower-banks can borrow in the interbank market and then extend short-term loans. Across all M subperiods, assuming the high risk bank is shut, leaving one bank without an ability to fund its short-term loans, manipulation of equation 25 shows this will generate loans

¹⁷If information is gathered, P_L is the lowest possible risk that bank i could possibly have. Since its true PD will be greater, with this information cost the loan will not be extended.

¹⁸Note that in the present treatment government actions only create surplus by generating information and changing agents beliefs. In Caballero and Krishnamurthy (2008), government actions create surplus through an additional channel: welfare is measured relative to beliefs of a benevolent social planner, rather than relative to the beliefs of the agents in the model. We rule out this additional channel in our analysis.

¹⁹Because the banks are identical based on observable characteristics, this search strategy is optimal.

with surplus $2(N - 1)(c - \epsilon)$ before information costs. Which means that after information costs, the social surplus generated by the government information provision is on the order of $Nc > 0$, showing government sponsored sequential search and release of information can improve social welfare and the functioning of the interbank market. The same logic applies if the government's costs of revealing information are higher than that of the private sector; and as we show below there are special circumstances in which the surplus generated by government action can be arbitrarily large.

These results are formalized in the following proposition.

Proposition 4 *There exist $P_H, P_L, N, M, c, LGD_{i,j}$ and \bar{R}_i such that:*

1. *Because of uncertainty about banks risk exposures the interbank market may break down.*
2. *Private provision of information on the exposures may be too costly to restart the market.*
3. *Sequential government-supported inspections in which the government inspects banks, and announces their health and risk exposures may restore market functioning and improve social surplus. This may be possible even if the government faces higher costs of information gathering than the banking sector faces.*
4. *There are parameter values for which results 1 - 4 hold, and for which the expected social surplus from sequential government-supported inspections can be made arbitrarily large.*

Proof: For simplicity assume all banks with a surplus of funds are ex-ante symmetric, and that all banks with a shortage of funds are ex-ante identical, and that all short-term borrowers have the same reservation rate for borrowing given by \bar{R}_i . Furthermore, banks assets holdings correspond with those associated with worst case beliefs so that there are $2N - 1$ banks with low default probability P_L and one with high default probability P_H .

To facilitate the welfare calculations, assume that short-term loans are priced competitively so that short-term borrowers capture the social surplus from lower rates in the interbank market.²⁰

²⁰Short-term borrowers demand for lending is assumed to be inelastic when the interest rates they face are below their reservation value. Therefore, assuming they receive the surplus is just a redistribution of surplus without changing the total amount of surplus available.

If bank i makes a short-term loan and does not reveal information about itself first, then the social surplus created will be negative if P_H and P_L satisfy:

$$\text{Surplus}_i = \bar{R}_i - (R^D + [P_L + (1/(2N - 1))(P_H - P_L)] LGD_i) = \eta, \quad (26)$$

for some small $\eta < 0$.

If the cost of signalling the information to bank i is c given by

$$c = (\bar{R}_i - R^D - P_L LGD_i)(1 + \frac{M - 1}{2}) + \epsilon, \quad (27)$$

for some small $\epsilon > 0$, then the cost of signalling exceeds the expected benefits that the bank could hope to earn from signalling immediately and making a loan now plus the expected future benefits from having signalled.

To prove results 1 and 2, choose P_H and P_L and LGD_i such that $1 > P_H > P_L > 0$, and $1 > LGD_i > 0$. Given these choices, choose \bar{R}_i so that $\bar{R}_i = R^D + [P_L + (1/(2N - 1))(P_H - P_L)] LGD_i + \eta_i$ with $-(1/(2N - 1))(P_H - P_L)LGD_i < \eta < 0$. These conditions guarantee that without signaling the spread charged to bank i will be too high, and will cause the interbank market to breakdown, proving result 1. Choosing c as in equation 27 guarantees that bank i 's costs from signalling exceed its expected benefits, so it will not signal. The same will be true for all banks i , proving result 2.

To prove 3, assume the government performs sequential information collection (i.e. sequential search) in which it looks at each bank, certifies whether it is good, or bad, discloses information about the banks risk exposures, and stops searching after it has found the single bad bank, which is shut down. The expected search costs are equal to $c(2N - 1)/2$.²¹ Abstracting from the integer problem, of the $2N - 1$ good banks, each is expected to be a borrower in the interbank market in $1/2$ of the periods $1, \dots, M$; in each period the borrowing bank captures surplus $\bar{R}_i - R^D - P_L LGD_i$, for a total surplus net of borrowing costs of:

$$\begin{aligned} \text{Surplus} &= [(2N - 1)/2](\bar{R}_i - R^D - P_L LGD_i)M - [(2N - 1)/2]c \\ &= [(2N - 1)/2](\bar{R}_i - R^D - P_L LGD_i)M - [(2N - 1)/2][(\bar{R}_i - R^D - P_L LGD_i)(1 + \frac{M - 1}{2}) + \epsilon] \\ &= [(2N - 1)/2](\bar{R}_i - R^D - P_L LGD_i)(\frac{M - 1}{2}) - [(2N - 1)/2]\epsilon. \end{aligned}$$

²¹At most $2N - 1$ searches are needed, and the chance of finding the high risk bank on each search attempt without replacement is $\frac{1}{2N}$. Elementary calculations then show the expected search cost is as given.

The final line of the expression for surplus can be guaranteed to be positive by choosing ϵ sufficiently small. This establishes result 3 when the government and private sector have the same cost of monitoring. If c for the government is equal to ψ times the private cost of monitoring, for $\psi < 2$, it is straightforward to reevaluate expected surplus and show that it will still be positive, completing the proof of result 3.

Finally, holding N fixed, but allowing M to approach infinity, with the resulting consequences for c , shows from the last equation that there are economies in which the interbank market will break down, private efforts to restart it will fail, and for which the expected social surplus from the governments efforts to restart the market are arbitrarily large. This establishes result 4. \square .

The novel aspect of the example is that public provision of information on exposures is welfare improving when private information on exposures will not be provided. Private provision of information fails because the private costs of signalling that a bank is high quality are too high to be passed on to the banks short-term borrowers. Therefore, high quality banks will not signal they are high quality — and of course bad banks will not pay the costs to signal that they are low quality.

Public provision of information is welfare enhancing because when public search for bad banks succeeds, the knowledge that a weak bank has been located and closed, and the knowledge of its positions combined with knowledge of the outstanding assets Y_M allows banks to update their assessment of the worst-case risks of other banks, reducing economy-wide uncertainty, and creating an external benefit. The way the example is parameterized, for the expected external benefit to be positive, the government's search activity has to stop before every bank is examined because if every bank is examined the costs of searching would just be equal to the total private search costs, and the total surplus would be negative, as it is in the private case. Because on average the government only has to search in half of the banks, government search is welfare enhancing.²²

The proposition is proved in the special case where worst case beliefs on asset exposures and the exposures coincide. In a more general setting, the worst case beliefs will be the same, but the actual asset exposures will differ. As a consequence, the surplus that is generated by the government's search strategy will depend on how the assets are allocated, and the government's stopping rule for deciding when its bank inspections have located enough bad

²²It would of course be trivial to create examples where the government faces lower costs of signalling bank quality than the banks themselves. This could occur, for example, if the banks' attempts to signal their own quality are not credible, but that government signals are credible.

assets. To analyze how the distribution of risky assets across banks affects the surplus calculation I assume that government searches for risky asset exposures one bank at a time, and stops searching when the market restarts and surplus is at least 40% of what it would be if the risky assets were evenly dispersed among all banks. This search rule is not optimal, but was chosen so that search would continue beyond the point when interbank markets just restart, since more search is typically beneficial in those cases. However, search is not allowed to continue too long when the risky asset holdings appear to be very dispersed since in those circumstances more search is costly.

To analyze the more general setting requires simulation. In the simulations we assumed that c , $2N$, R_l , and the aggregate exposure vector of core banks Y_M , are such that the worst case beliefs about P_H , and P_L satisfy the properties in proposition 4. This means \widehat{PD}_i is such that the interbank market breaks down initially, and c is such that the surplus would be as in the proposition if the most pessimistic beliefs were actually consistent with the true allocation of risky assets among the banks. For simplicity, ϵ in the proof of the proposition is set to 0. In the simulations, there are $2N = 20$ banks that are ex-ante identical, but their portfolios are tilted by different amounts toward the high risk asset (asset 2), which can be interpreted as the real-estate sector. In particular, each bank's portfolio weight in risky asset 2, w_2 , can range from 0.4 to 0.6, and its weight in asset 1 is $1 - w_2$. Given the asset supplies, there is a $2N \times 2$ matrix of portfolio weights W_d (with the first and second columns corresponding to weights in assets 1 and 2) in which the holdings of asset 2 are maximally dispersed (all banks hold the same amount of asset 2), and a second matrix W_c in which the holdings of asset 2 are maximally concentrated (one bank has portfolio weight $w_2 = 0.6$, and the others have $w_2 = 0.4$).²³ The simulations are conducted for matrices of portfolio weights $W(\lambda)$ that are convex combinations of the maximally dispersed and maximally concentrated holdings:

$$W(\lambda) = \lambda W_c + (1 - \lambda)W_d, \quad \lambda \in [0, 1],$$

where λ can be interpreted as a concentration index that ranges from 0 (maximally dispersed) to 1 (maximally concentrated). For each matrix, the government's inspections are random searches that stop when the surplus created from the government's actions is high enough, as discussed above. To estimate the expected surplus generated for each matrix, 10,000 random searches are conducted, and the average surplus generated net of search costs is reported.

²³Note: in these simulations I have relaxed the assumption that one bank can hold all of the high risk asset in its portfolio since that assumption is not essential. For comparability purposes with proposition 4, the condition I did maintain is that Y_M is such that worst case beliefs involve one high risk bank and $2N - 1$ low risk banks.

The results from our baseline simulation are reported in Figure 3. The figure shows that the expected surplus generated by the government’s search policy depends on how the holdings of asset 2 are concentrated. The case of maximal concentration corresponds to the situation in the proposition, and yields the maximal expected surplus. When the exposure is more dispersed, the surplus is lower, and in the extreme when it is very dispersed, the surplus drops below 0 (Figure 3). The reason that concentration matters is that when exposures are dispersed, a large number of inspections are required to restart the interbank market—and each inspection is costly. Conversely, when exposures are concentrated, once the large exposures are identified, the interbank market quickly restarts and less surplus is exhausted on costly search and inspection. It should be added that there is also a dependence on the exact criterion for restarting search. In simulations that are not shown, I considered the possibility of stopping the search process after the interbank market restarted. With the exception of very concentrated asset holdings this often generated very little surplus because after a sufficient amount of risky asset is located the interbank market restarts, but with interbank spreads that make the banks nearly indifferent between making loans or not, thus generating little surplus.²⁴ Continuing to search beyond the point when markets restarted generated more surplus in most cases. Overall, the analysis shows that the government search strategy generates positive surplus for many, but not all asset concentrations.

An important assumption in the analysis of this example is that the government’s ability to assess a banks’ risk is just as good as that of banks themselves. With this assumption, in this stylized example, the government’s search policy always generates surplus. A more realistic assumption is that government has to pay higher costs to assess risk in the banking sector than banks’ themselves. To analyze the role of different costs for the government, we assumed that the government’s costs of evaluating banks range from being the same as the banks cost to 3x that amount. The analysis based on this range of costs shows that government intervention raises surplus when the asset holdings are concentrated enough and costs are low enough, but that there is a very broad range of concentration and costs of which the surplus genated by the government policy is negative (Figure 4).

One of the reasons the surplus is negative is that government search is completely random in the example which is equivalent to assuming that the government has no information about banks relative exposures to the distressed asset class. If we instead assume that government has prior information about the relative risk exposures, then using that information has a dramatic effect on the surplus calculations. In particular, under the strong, but not

²⁴Recall the banks are monopolists in lending to final borrowers that have inelastic demands for funds, so the total surplus is just the banks surplus.

unrealistic assumption that government knows the ranking of banks exposures to the distressed asset class, inspecting banks by ranking has a major effect on the surplus calculations, generating positive surplus for a very wide range of concentrations, and even for high government search costs (Figure 5).

In addition to the simulations with dispersed asset holdings, I also generalized proposition 4 to allow for more than bank to have a high probability of default, and the others have a low default probability. When there are $2N = 20$ banks and up to five banks have a high probability of default, the results in the proposition generalize. It generalizes for some situations when N is larger, but they have to be checked by simulation or very tedious combinatorics so I have not done a complete analysis.²⁵ When $2N = 20$ if for worst case beliefs the number of high risk banks exceeds 5, the expected number of random searches required becomes too large to generate positive surplus.

In sum, the findings on the government’s search and information provision policies show:

1. The external benefits of government search may justify a government search and information provision policy in order to restart markets.
2. Government search and information provision policies are more effective and less costly if government has prior information on the relative size of core banks exposures.
3. If the holdings of a distressed asset class are likely to be dispersed across core banks, and if information search costs are high, the surplus generated by government efforts may become negative, suggesting the government’s search and information revelation policy may need to be justified on broader grounds than are present in the current model.

Regarding point 3, there are two additional potential justifications for government intervention. The first is the government’s costs of learning information about banks and then revealing it to reduce uncertainty may be lower than the costs to the banks themselves. This is because banks may have a credibility problem in signaling that they are low risk during a financial crisis—while government audits of banks may be viewed as more credible. For this reasoning to apply, government’s should take steps to ensure that their information releases on banks’ condition is viewed as credible. A second basis for government intervention is

²⁵To prove the result for $2N = 20$, I assumed the government searched until all bad banks were found, and computed the expected number of searches required using Monte Carlo analysis. The other parts of the proof are similar to the proposition.

that the surplus calculations in the simulations conducted in this paper are incomplete. For example, if banks cannot raise lending rates on some borrowers because of adverse selection, then the present analysis underestimates the potential surplus from the government's policy.

In closing this subsection, it is useful to recall the past historical experience with information provision on bank's health. Although the details on the information provision differ from the interventions above, Park (1991) found that the Bank Holiday of (1993), which temporarily closed banks, and only reopened those deemed solvent, and earlier experiences with bank clearinghouse's that issued clearing loan certificates during a crisis both improved market functioning because it provided the public with information about the quality of bank assets, which allowed them to distinguish between solvent and insolvent financial institutions. More recently, the SCAP stress test that was conducted in the US during the global financial crisis of 2007-2009 has been perceived to have led to an easing of credit conditions because it also provided a clearer picture of bank's financial health.

My interpretation of past information provision policies is they succeeded because they eliminated much of the ex-post uncertainty about banks during a crisis. The analysis above suggests we may want to follow policies that facilitate better ex-post information provision in case they are needed. There is also a case for encouraging the development of market institutions that reduce the effects of uncertainty ex ante. In the next subsection I discuss how the structure of the Fed Funds market serves that role, and then in the following subsection I discuss the role for government to further reduce uncertainty in that market.

4.4 Uncertainty, Information, and the Structure of the Interbank Market

In this section we explore how the tiered structure of the interbank markets helps to reduce the effect of Knightian uncertainty on loan spreads. The main ideas are best explained in the context of the example with two risky assets in section 4.3. To build on this example, suppose that in addition to the N large banks that receive lending shocks and attempt to borrow in the anonymous brokered tier of the interbank market there are also S small banks, $s = 1, \dots, S$, that could attempt to borrow in the top tier of the market. In this case the worst case probability of default is given by:

$$\widehat{PD}_i = \max_{\substack{\omega_k, k = 1, \dots, 2N - 1 \\ \omega_s, s = 1, \dots, S}} \frac{1}{2N - 1 + S} \left(\sum_{k=1}^N PD_k(\omega_k) + \sum_{s=1}^S PD_s(\omega_s) \right) \quad (28)$$

subject to the constraint that:

$$\sum_{k=1}^{2N} \omega_k A_k + \sum_{s=1}^S \omega_s A_s = Y_M, \quad (29)$$

where $\sum_{s=1}^S A_s \ll Y_M[1]$.

This is a case where the size of small banks collectively is much smaller than the amount of loans that have been made to sector 1, which is a sector of the economy whose loans are expected to performing poorly conditional on time 1 information. Since each bank's probability of default is increasing in the amount of assets it holds in sector 1, because of the convexity of the maximization problem, from the proof of proposition 3, it follows that the portfolio weights which maximize the robust probability of default set $\omega_s[1] = 1$ for small banks $s = 1, \dots, S$, and then allocates all remaining holdings of asset 1, to a single large bank. As in the two asset example, the other large banks concentrate their asset holdings in asset 2. To see how including small banks in the anonymous brokered top-tier of the interbank market affects robust default probabilities versus the case when they are not included, note that each small bank has a default probability P_S where $P_S > P_H$ in the example with two risky assets. On the other hand, the large bank that has an exposure to asset 1, now has default probability P'_H where $P'_H < P_H$. Provided that N is greater than 2 (which is required for an anonymous brokered market to make any sense) and $S > 2$ (which is realistic) allowing the small banks to participate in the center of the interbank market increases the worst-case probability of default. This result is summarized in the following proposition

Proposition 5 *If there are 2 or more small banks and two or more large banks that want to borrow in the anonymous brokered tier of the interbank market, then if $\sum_{s=1}^S A_s \ll Y_M[1]$, then the worst-case probability of default for the two asset example in equation 28 is increasing in the percentage of small banks that participate in the anonymous brokered tier of the market.*

Proof: See the appendix.

The intuition for the result is that small banks by virtue of their small size can hold portfolios that are less diversified than a large bank could hold. Therefore, small banks have greater potential to increase the worst-case probability of default than large banks do when trading in the interbank market. As a result small banks cannot participate as borrowers in the center of the interbank market unless they are monitored, but since monitoring is costly and time consuming, small banks essentially cannot participate as borrowers in the core of the interbank market. Small banks could of course be lenders in the anonymous brokered part of the market. They could also participate in a relationship lending arrangement in which they are monitored by a very small number of borrowers. This type of arrangement for small banks is consistent with the structure of the market.

One possible objection to this analysis is that the portfolios that small banks are assumed to potentially hold in the two asset example could not have been optimal ex-ante. This reasoning is not necessarily correct because loans to small sectors may have high expected returns and high risk ex-ante, and so lie on the efficient frontier — it is precisely because of the possibility that small banks may take undiversified positions in these loans that large banks will not usually make unsecured loans to small banks in the interbank market.²⁶ Another interpretation of this result is that banks pay an uncertainty premium when there is uncertainty over their positions. Therefore, if they are constrained in their possible actions, it may reduce the uncertainty premia that they pay. In the case of large banks, they are constrained from taking large positions in small sectors of the economy—and this constraint reduces the uncertainty premia that they pay and thus allows them, but not small banks, to trade in the top tier of the interbank market.

When large banks trade with large banks, they may need to trade with many large banks to fulfill their liquidity needs. In particular, Bech and Atalay note that in 2006 banks in the center of the Fed Funds market received funds from 19.1 banks on average and lent funds to 9.3 banks on average. These average numbers can be misleading because the distribution is very skewed. For example, the most active bank in the center received funds from 127.6 banks in 2006, and lent funds to 48.8 banks. Because the banks in the center lend to a much larger number of counterparties in a brokered market, monitoring the banks individually to reduce uncertainty can be costly. This raises the question of what type of uncertainty

²⁶Although small banks do not borrow in the center of the Fed Funds market, they do participate in the market, but primarily by making relationship loans to large banks. The basis for trade is that small banks have access to a larger deposit base than they need, while large banks have outstipped their ability to fund themselves solely through customer deposits. Because an individual large bank's funding needs typically swamp any single small banks available deposits, small banks typically establish a relationship with a few large banks to which it repeatedly lends through time [Furfine, (2001)].

premium should the banks in the center pay when lending to each other. In this respect, it is useful to examine the role of N . Recall that in the two asset example when all banks are large, $\widehat{PD}_i = P_L + \frac{1}{2N-1}(P_H - P_L)$. As N grows large, with Y_M fixed, this expression approaches P_L , which means that a large number of large banks interacting anonymously in the brokered market reduces the spread that large banks require for uncertainty when lending in the interbank market. In other words, a large number of large banks interacting in an anonymous brokered market reduces the spreads that must be paid for uncertainty about counterparties risk exposures.

Overall, the analysis shows that large N and eliminating small banks from the center of the interbank market are both features of the interbank market that reduce premia for uncertainty. This allows large banks to transact with each other on very short notice without having to know very much about the composition of each others balance sheet. In other words, the structure of the interbank market reduces costs due to uncertainty and reduces information costs. The effectiveness of this structure, and in particular on whether a large N provides substantial insurance depends on which assets are most affected by the news at date 1.

The earlier analysis assumed the news primarily affected asset 1, and that large banks holdings of asset 1 were small enough that one large bank had enough assets on its balance sheet, that it could absorb all of the core banks total exposure to asset 1. In this circumstance, because there are many core banks, large N provides dampened the amount of spread widening. Suppose instead asset 2 was severely affected and core banks total exposures is much greater than any banks ability to absorb it on its balance sheet $Y_M[2] \gg A_k$. Then worst case beliefs imply that many banks ($J > 1$) may have a large exposure to asset 2, which implies a much larger value for the worst case probability of default: $\widehat{PD}_i = P_L + \frac{J}{2N-1}(P_H - P_L)$, and a market that is more prone to collapse due to uncertainty. In practical terms, this analysis means the interbank market may be resilient to problems by a major firm (such as WorldCom, GM, or Chrysler), but it may be less resilient when a severe shock hits a sector that many banks are exposed to, such as housing.

In conclusion, the structure of trading in the Fed Funds market reduces the effect that uncertainty about banks' position have on interbank spreads. Government actions to further reduce these spreads ex-ante are proposed below.

4.5 Role for government in shaping prior information

The model suggests two informational roles for government policy. The first is to provide information that helps to resuscitate markets when they collapse. The second is to shape the information environment to improve market function and reduce the likelihood of future collapses. To illustrate the role for shaping the information environment, consider again the problem of solving for worst case beliefs in equation 21, but that banks information sets are coarser. They don't know Y_M , the aggregate asset holdings of the banks in the center of the Fed Funds market, but instead only know \tilde{Y}_M which is equal to core banks holdings Y_M plus ϵ_{BO} , the vector of asset holdings of banks and others that are not core banks. The information may be coarser if information on core banks is not broken out in published banking statistics, or if available information on core banks is not sufficiently detailed. In either case, the coarsened information changes the constraint 22 to

$$\sum_{k=1}^{2N} \omega_k A_k + \epsilon_{BO} = \tilde{Y}_M, \quad (30)$$

I will assume ϵ_{BO} is bounded below by 0, and has no upper bound. For simplicity, I also assume all asset holdings are positive, implying $\tilde{Y}_M > Y_M$.

When the information about core banks aggregate holdings in 30 is coarsened, it relaxes a constraint in solving for worst case beliefs. Uncertainty averse banks and others can form more pessimistic beliefs as a result, which can generate higher uncertainty premia. For this reason, more precise information on core-banks asset holdings Y_M reduces uncertainty premia, and more coarse information makes it worse. This is stated formally below:

Proposition 6 *When the information on core banks total asset holdings Y_M is coarsened as in equation 30, then worst case beliefs about core banks probability of default increase, and uncertainty premia increase as well. Less coarse information about Y_M has the opposite effect.*

Proof: Obvious because coarser information about Y_M relaxes a constraint in the optimization problem 21 that solves for worst case beliefs regarding default probabilities. \square .

The proposition suggests that more precise information about Y_M should be provided. This raises the question of why it is not provided either publicly or privately. There is a

two part answer. The first is that the information is less valuable when times are good, and this weakens the incentives for producing it. This is illustrated in Figure 6, which plots spreads over the riskfree rate as a function of $Y_M[2]$ the supply of asset 2 held by core banks, when information at date 1 shows its expected returns are high. In a departure from earlier analysis in the paper, spreads are presented not just for the worst case beliefs about banks asset holdings, but for all beliefs about asset holdings that are possible conditional on $Y_M[2]$. The resulting relationship is the correspondence Q which maps $Y_M[2]$ into the set of spreads that are possible given $Y_M[2]$. The upper blue-dashed and lower solid green boundaries of the correspondence are the spreads that correspond to the most pessimistic and optimistic beliefs about asset exposures, with points in between corresponding to all other beliefs on asset allocations.²⁷ For all values of $Y_M[2]$ spreads never exceed 1/2 basis point, which confirms that information on $Y_M[2]$ is less valuable during good times. Because the figure is based on all possible beliefs, it also shows that during good times that the spreads that result when there is uncertainty and uncertainty aversion will not be very different from the spreads that would occur without uncertainty. This can make it appear as if uncertainty is not present or not important.

In very bad economic conditions, knowledge of $Y_M[2]$ can be very important because the average risk of core banks varies with $Y_M[2]$, while the precise amount of risk depends on how the assets are allocated for a given level of $Y_M[2]$. This is illustrated in Figure 7, which illustrates that when bad news arrives about asset 2, then depending on $Y_M[2]$, interbank loan spreads in the brokered tier of the market can range from 20 basis points when the supply of assets held by the banks is as low as possible, to 380 basis points when the supply of risky asset 2 held by the banks is as high as possible (Figure 7).²⁸ Although knowledge of $Y_M[2]$ is valuable, the extent to which information provision is favored by banks depends on whether banks are uncertain and uncertainty averse, KU hereafter, or whether they understand their environment and behave like Bayesian Expected Utility maximizers, BEU hereafter. I discuss the implications of both specifications for policy below.

Policy When Banks and their Financiers are Uncertainty Averse

To illustrate the role of uncertainty aversion in formulating information release policy, suppose all banks are uncertainty averse and bad news arrives about asset 2 at date 1.

²⁷The best beliefs are found by choosing allocations to minimize \widehat{PD}_i . The minimum is attained when one bank has a high risk portfolio and is lending in the market, and the other banks all hold the same diversified portfolio.

²⁸In the example there are 20 banks who each have assets normalized to 1. Because the minimal portfolio weight in asset 2 is 0.4 and the maximal weight is 0.6, the smallest amount of asset 2 that the 20 banks can collectively hold is 8, while the maximum amount is 12.

If $Y_M[2] = 10$, but that the only public information available is so coarse that the belief $Y_M[2] = 14$ is possible, then each core bank could reason that the other core banks could have maximal exposure to asset 2. On that basis, when interbank spreads are set based on worst case beliefs, the resulting interbank spread would be $380bps$. Denote this maximal spread by $Spread_{KU}$, where KU stands for Knightian Uncertainty. Because of Knightian Uncertainty about Y_M and banks holdings, each bank in the example would feel that the spread it was charged in the interbank market is too high given its credit risk. Therefore, all would favor public provision of information that breaks the pooling equilibrium, and brings spreads down by revealing more information about individual banks risk exposures. In addition, all banks would favor information that made knowledge about $Y_M[2]$ more precise since this would bring down the spreads that all of them faced. In particular, if it was known that $Y_M[2] = 10$, then if all banks are uncertainty averse, the spread would come down to 200 basis points, and all banks would favor the release of this information.²⁹

If banks have noisy information, but the information is less coarse so that worst case beliefs are that $Y_M[2] = 12$, then the spreads they pay in the interbank market will be lower than that associated with each bank having maximal exposure to asset 2. Thus, those banks that have maximal exposure would not want their information revealed through bank inspections, but all banks would still prefer $Y_M[2]$ to be revealed, and many banks would still favor inspections because it would reduce their spreads relative to the worst case.

From the government's perspective, if all banks are uncertainty averse, then provision of better information on $Y_M[2]$ is an unambiguously good policy because it would lower spreads in the interbank market and make it function better. Sequential information inspections would also reduce average spreads since with uncertainty aversion spreads are set based on worst case beliefs. So, abstracting from costs of information collection, the government would also favor the release of information on the bank's health and risk exposures.

The policy prescriptions are more ambiguous when banks are not uncertainty averse, as shown below.

Policy When Banks and their Financiers are Bayesian Expected Utility Maximizers

Suppose banks and their financiers are not uncertainty averse, and have well formed

²⁹All banks would favor this intervention whether the interbank market was functioning or not. If it was not functioning, all banks are made better off because the intervention restores market function. If it is functioning, all banks are made better off because all receive higher surplus when they pay lower spreads.

beliefs that take the form of a unique common posterior distribution over \tilde{Y}_M and the portfolio allocations among the core banks. This implies they have a well formed posterior over all points in the correspondence Q , where Q is the set of possible interbank spreads that would result if $Y_M[2]$ was known.³⁰ Given the posterior, the equilibrium spread, $Spread_{BEU}$ is just the conditional expected spread over the set Q given $\tilde{Y}_m[2]$. Because $Spread_{KU}$ is based on worst case beliefs, $Spread_{BEU} \leq Spread_{KU}$. Therefore, there may be banks that have maximal risk, but benefit from being pooled with lower risk banks. Banks with maximal risk will not favor public provision of information via bank inspections because it will increase their own spreads. This is in contrast to the Knightian uncertainty case in which all banks favored inspections when worst case beliefs about $Y_M[2] = 14$.

Suppose better information about $Y_M[2]$ is revealed by government actions. From an ex-post perspective, this action could raise spreads since the information revealed by a better signal could be worse than what market participants expect ex-ante. For that reason, ex-post information revelation may be opposed by some banks. This differs from the Knightian Uncertainty case in which all banks are made better off by revealing the information on $Y_M[2]$.

Finally, from the government's perspective, when banks do not behave in an uncertainty averse fashion then inspections and release of information can reveal unfavorable information and raise spreads; and release of better information on $Y_M[2]$ can also reveal unfavorable information and raise spreads. Therefore, the case for revealing information is less clear cut when banks are not uncertainty averse — and when they do not face structural uncertainty.

Interpretation

The analysis above highlights one of the dilemmas faced by policy makers in setting optimal transparency and disclosure policies. In particular, during a crisis if the economic environment is one in which banks (or their financiers) behave in an uncertainty averse fashion when setting spreads and extending credit, then steps that improve transparency are more likely to improve the functioning of markets and calm the crisis. Conversely, if bank's behavior is not characterized by uncertainty aversion, then weak banks may benefit from the lack of transparency while strong banks are hurt by it. If policy makers are especially concerned about the weak banks during a crisis, they may be hesitant to release public information during a crisis because it may make the crisis worse for banks that are weak.

³⁰In Figure 7, Q is the set of points between the solid and dashed lines since the solid and dashed lines measure the highest and lowest possible spreads for a given level of $Y_M[2]$.

One way to judge whether the KU or BEU views of bank and financier behavior during the crisis is more descriptive is to examine banks responses to transparency type policy interventions since they are more likely to be favored in a KU world. The recent experience with Bank Stress tests is suggestive of the KU view. Those banks that have been stress-tested have had an easier time raising capital, and several banks that were not initially stress tested have requested that they be stress-tested. This experience suggests that serious consideration should be given to transparency type initiatives that improve transparency about core banks before a crisis, as well as policies that inspects banks and reveal information during a crisis.

The uncertainty view of policy has two additional implications for policy. The first is that it suggests an optimal sequencing of different types of steps to address a crisis. More specifically, capital injections, loan guarantees, and information provision via the SCAP stress test have all been used as policy actions. How should these policy actions be sequenced? If banks and other financiers are uncertainty averse, and information provision takes time, the optimal sequencing should first use temporary interbank loan guarantees to prevent markets from total collapse during a crisis. If equity injections to reduce spreads follow next, then if spreads are based on worst-case beliefs, and banks are perceived to be in terrible shape, massive injections of capital would be required to have the desired effect on spreads. For example, without transparency initiatives, if $Y_M[2]$ is perceived to be 14 when it is 10, to reduce interbank spread to 40 basis points requires 82 percent capital injections for all core banks. If instead before the capital injections, information was provided that $Y_M[2] = 10$, this action alone would reduce the required injection to 60 percent. If in addition, the government inspected all banks before capital was injected, then the injections could be better tailored to individual bank needs, which in some cases could reduce the total amount of equity capital that needed to be injected by 50 percent.³¹ Therefore, the best sequence of policy actions during a crisis should be loan guarantees, information collection and dissemination, and then capital injections.

An additional implication is that making information on $Y_M[2]$ more precise can help restore the functioning of multiple interbank markets. To see how, suppose that banks that participate in the interbank market for Euro-reserves and banks that participate in the Fed Funds market both hold some of the same long-term assets and that one class of

³¹For example, when $Y_M = 10$ worst case asset holdings for interbank spreads would imply that 10 banks need 82 percent capital injections, and 10 don't need any injections. Thus the information would reduce the required equity injections by 50 percent relative to the case when $\tilde{Y}_M[2] = 14$ and the government did not engage in information gathering. However, the asset holdings associated with worst case interbank spreads in a pooling equilibrium are not necessarily those that require the largest capital injections. For example, when $Y_M = 10$, the asset holdings that generate the best case interbank spreads require a 62 percent capital injection per bank to bring interbank spreads down to 40 basis points.

assets is impaired. Suppose, for simplicity, that both sets of interbank markets have similar structures, and that the only information available is on the aggregate holding of banks in the core of the Fed funds market. If the information is imprecise, then in the US worries that US banks exposures may be high to the class of impaired assets can cause the US interbank market to collapse. For the same reason, in Europe, worries that European exposures are high (which implies US are low) could cause the European interbank market to collapse. In fact, both the US and European markets could collapse from uncertainty even though the worst case asset holdings that are envisioned in each market may be jointly impossible.³² In such circumstances, it is straightforward to construct examples where the provision of more precise information on aggregate asset holdings in one market could help restart one or both markets.

In all of the above analysis, banks are only linked to each other by holding similar assets, but not through any linkages. Yet, how the banks are linked to each other, which I refer to as the financial architecture, is important. This is briefly discussed in the next section.

5 Financial Architecture

Proposition 4 showed that there are circumstances when government provision of information on bank's health can restart the interbank market and improve welfare. A critical assumption in the analysis is that when the government inspects banks, it can easily infer their solvency, and when it releases information about other banks, the release of that information allows others to improve their inferences concerning the solvency of the remaining banks in the system. The extent to which the government's policy is successful depends upon the architecture of the banks in the financial system, and how they may be linked. In particular, there are settings where knowledge of a banks own risk exposures are not sufficient to infer its solvency, which means that there are situations where release of information on individual banks health may not be sufficient to restart the markets. A stark example of this possibility, based on Allen and Gale (2000), is the following: Consider four banks, A,B,C,D. The balance sheets of bank A is as follows:

Table 1: Balance Sheet of Bank A

³²Note: these worst case beliefs in each market are not internally consistent when held by market participants who participate in the European interbank market, or the US market, but not both. For participants in both markets, internal consistency would require solving for beliefs that satisfy adding up constraints for counterparty credit risk in both markets simultaneously.

| | |
|-----------|-------------|
| Assets | Liabilities |
| Loans 100 | Deposits 80 |
| | Equity 20 |

Off balance sheet: Sold “bullet” protection on bank D that pays 25 if bank D defaults.

Banks B,C, and D, have identical balance sheets to bank A, except that bank B has sold protection on A that pays 25 if A defaults. Similarly C has sold protection on B, and D has sold protection on C.

Examination of Bank A in isolation shows that A is solvent if D is, but not if D is not. Therefore, examination of bank A alone does not establish whether A is solvent. Examination of D shows D is solvent if C is solvent, but not otherwise. This means that one cannot know if A is solvent unless one knows whether C is solvent. Therefore knowledge of the exposures of bank A, and of the exposures of the bank that A is directly exposed to does not provide information that is sufficient to know whether A is solvent. Because the example is symmetric, what is true for bank A is true for the other banks. No bank in this example can know its own solvency based solely on its own direct risk exposures.

To examine whether A is solvent, pretend that all of the risk exposures among the banks are known. With this knowledge is A solvent? One definition of solvency is if A defaults, then can A pay off all of its obligations. By this definition, A is certainly insolvent. To see how, note that if A defaults, then B will become insolvent, which will cause C and then D to become insolvent. Finally, when D is insolvent, A will not be able to pay its obligations, so A is definitely insolvent by this definition.

Now, suppose instead that bank C has 30 in equity, instead of 20, and has as an additional 10 in US treasuries on the other side of its balance sheet. With these asset holdings, if A defaults, then so will B, but C and D will not. Hence, A will be able to pay off its liabilities and hence will not be insolvent. Put differently, A will not have a reason to default in the first place.

Now, suppose instead we start the chain with C, then if C defaults, D defaults, which causes A and then B to default. However, when B defaults, if we tally up C’s losses, then C has more than enough assets to pay its obligations. So, it is not consistent to start a chain with C defaulting.

This reasoning suggests that depending on the financial architecture and firms positions:

1. There may be circumstances where no bank knows its own solvency based on its knowledge of its direct risk exposures to other banks.
2. Private parties may not have sufficient information to evaluate the risk of their individual counterparties even if they know that counterparties exposures.
3. The government's approach to restarting markets in the previous section may fail because the government's provision of information on asset positions does not provide sufficient information on the health of individual banks to establish whether they are solvent. In particular, in the network above, if the government made public bank A's balance sheet, and its connections to banks B and D, that information would not be sufficient to establish whether bank A is solvent.

These results point towards the importance of the architecture of the financial system and the need for a comprehensive approach in crafting remedies for market breakdowns since the likelihood of a programs success depends on the financial environment in which it is implemented.

6 Conclusion

When one bank makes a loan to another, the credit risk of the bank that lends funds is related to the risk of the assets on the borrowing banks balance sheet, but because of opaqueness, the creditor bank will be uncertain about the borrowing banks portfolio composition, and thus uncertain about the risks that it faces. Nevertheless, despite the uncertainty, unsecured overnight interbank borrowing and lending is common — and spreads for interbank loans at the top tier of the Fed Funds market is typically low. In this paper, I show that interbank spreads should contain an uncertainty premium related to credit risk, but because of institutional features of the Federal Funds interbank market, the spread can usually be kept low. Nevertheless, market collapses due to uncertainty are possible. In those circumstances, private efforts to restart markets may not be successful — but sometimes government provision of information as advocated in this paper, or the Bank Holiday of 1933 or the recent Stress Capital Tests that were conducted on US banks can help alleviate uncertainty, restart markets, and improve welfare. Our analysis shows that there is scope for governments to step in ex-post to reduce uncertainty, and furthermore that policies which release better aggregate information on core banks that are at the center of the financial system can help reduce uncertainty ex-ante. An advantage of policies that release aggregate information is

that it keeps banks individual exposures private, while still providing information that may improve transparency and financial stability. The success of transparency efforts also depends on how banks are linked to each other in the financial system. How to account for these linkages while formulating a transparency policy remains a topic for future research.

Appendix

A Propositions and Proofs

Proposition 1: Under the regularity condition that for all $F(2)$

$$0 < X_i - \theta_i - \gamma_i F(2) < \bar{u}_i, \quad (1)$$

the return on a well diversified portfolio of regional loans between periods 0 and 2 is given by

$$R_i(2) \sim \text{approx}\mathcal{N}(\alpha_i, \sigma_i^2), \quad (\text{A1})$$

where,

$$\begin{aligned} \alpha_i &= \left[X_i + (RGD_i - X_i) \left(\frac{X_i - \theta_i}{\bar{u}_i} \right) \right], \\ \beta_i &= \left[\frac{(X_i - RGD_i)\gamma_i}{\bar{u}_i} \right] \\ \sigma_i^2 &= \beta_i'(\Sigma_f(1) + \Sigma_f(2))\beta_i. \end{aligned}$$

In addition, conditional on the information bank i has at time 1 ($I_i(1)$),

$$R_i(2)|I_i(1)\text{approx.} \sim \mathcal{N}(\alpha_i + \beta_i f(1), \beta_i' \Sigma_f(2) \beta_i). \quad (\text{A2})$$

Proof: After screening out bad loans ($\eta_{s,i} < 0$), the probability that good loan s defaults, conditional on $F(2)$ is given by

$$\begin{aligned} PD[(s, i)|F(2)] &= \text{Prob}(r_{s,i} < X_i) \\ &= \text{Prob}(\theta_i + \gamma_i F(2) + u_i < X_i) \\ &= \text{Prob}\left(\frac{u_i}{\bar{u}_i} < \frac{X_i - \theta_i - \gamma_i F(2)}{\bar{u}_i}\right) \end{aligned}$$

When F satisfies the regularity condition in equation 1, this probability is given by:

$$PD[(s, i)|F(2)] = \frac{X_i - \theta_i - \gamma_i F(2)}{\bar{u}_i}, \quad (\text{A3})$$

By the law of large numbers when lending a total of W to a continuum of regional investors, the amount wealth returned is $W\{D^i(1 - PD[(s, i)|F(2)]) + PD[(s, i)|F(2)]RGD_i\}$ generating a return of

$$\begin{aligned} [R_i(2)|F(2)] &= X_i(1 - PD[(s, i)|F(2)]) + RGD_i \times PD[(s, i)|F(2)] \\ &= \left[X_i + (RGD_i - X_i) \left(\frac{X_i - \theta_i}{\bar{u}_i} \right) \right] + \left[\frac{(X_i - RGD_i)\gamma_i}{\bar{u}_i} \right] F(2) \\ &= \alpha_i + \beta_i F(2) \end{aligned}$$

Because $F(2)$ is normally, distributed, the return will be normally distributed. From here computing the distribution of $R_i(2)|I_i(1)$ is straightforward. \square .

Note: when F is Gaussian, the regularity condition will not be exactly satisfied, which implies the probability of default is not precisely linear in F , but it will be very close provided that \bar{u}_i is large, and the variance of $F(2)$ is small.

Proposition 2: Although there is Knightian uncertainty about bank i 's positions, under some economic conditions (such as case 1), favorable economic news can reduce the loan spread that bank i pays for borrowing in the interbank market even though there is Knightian uncertainty over bank i 's positions. Conversely, under other economic conditions, such as those in case 2, sufficiently unfavorable news about some sectors of the market can destroy bank i 's ability to finance its new lending opportunities in the interbank market, effectively causing the interbank market to break down.

Proof: Assume $\omega_i \in C(\omega_i^*)$ maximizes $PD_i(\omega, 0)$ the probability that i defaults conditional on information at time 0, and $\tilde{\omega}_i$ maximizes $PD_i(\omega, 1)$, the probability that i defaults conditional on the information at time 1. $PD_i(\omega_i, 0) > PD_i(\tilde{\omega}_i, 1)$ if:

$$\frac{\frac{L_i}{1+L_i}R^D - \omega_i'\alpha}{\sqrt{\omega_i'\Sigma\omega_i}} > \frac{\frac{L_i}{1+L_i}R^D - \tilde{\omega}_i'\alpha}{\sqrt{\tilde{\omega}_i'\rho\Sigma\tilde{\omega}_i}}.$$

Adding and subtracting $\frac{\frac{L_i}{1+L_i}R^D - \omega_i'\alpha}{\sqrt{\tilde{\omega}_i'\rho\Sigma\tilde{\omega}_i}}$ to the right hand side and rearranging shows the

inequality holds if:

$$\begin{aligned} \frac{\frac{L_i}{1+L_i}R^D - \omega'_i\alpha}{\sqrt{\omega'_i\Sigma\omega_i}} &> \frac{\frac{L_i}{1+L_i}R^D - \tilde{\omega}'_i\alpha}{\sqrt{\tilde{\omega}'_i\Sigma\tilde{\omega}_i}} \\ &+ \left(\frac{L_i}{1+L_i}R^D - \tilde{\omega}'_i\alpha \right) \times \left(\frac{1}{\sqrt{\tilde{\omega}'_i\rho\Sigma\tilde{\omega}_i}} - \frac{1}{\sqrt{\tilde{\omega}'_i\Sigma\tilde{\omega}_i}} \right) \\ &- \frac{\tilde{\omega}'_i\beta f(1)}{\sqrt{\tilde{\omega}'_i\rho\Sigma\tilde{\omega}_i}} \end{aligned}$$

The left hand side of the inequality is positive because ω_i maximizes the default probability at time 0. By the assumptions of the proposition, $\left(\frac{L_i}{1+L_i}R^D - \tilde{\omega}'_i\alpha \right)$ is negative, and $\rho < 1$. This guarantees that the first expression on the right hand side is negative. Since $\omega_i > 0$ and $\beta f(1) > 0$, the second term on the right hand side is also negative. This establishes the inequality is true, and shows that under some conditions the news at time 1 is unambiguously good.

To show that in the second case the news can be unambiguously bad, note that the risk exposure to asset 1 can be positive and the news about the mean return of asset 1 is negative. It then follows that if the mean on asset 1 is sufficiently low, and the possible exposure is sufficiently high, then the interbank market will freeze up, as claimed in the proposition. \square .

Proposition 5: If there are 2 or more small banks and two or more large banks that want to borrow in the anonymous brokered tier of the interbank market, then if $\sum_{s=1}^S A_s \ll Y_M[1]$, the robust probability of default for the two asset example in equation 28 is increasing in the percentage of small banks that participate in the anonymous brokered tier of the market.

Proof: The robust probability of default is increasing if and only if

$$\frac{P'_H + (2N - 1)P_L + SP_S}{2N + S} > P_L + (1/2N)(P_H - P_L).$$

Since $P_H > P_L$, the inequality will be satisfied if

$$\frac{P_L + (2N - 1)P_L + SP_S}{2N + S} > P_L + (1/2N)(P_H - P_L),$$

which reduces to the condition

$$\frac{P_S - P_L}{P_H - P_L} > \frac{1}{2N} + 1S.$$

Because $P_S > P_H$ the left hand side is greater than 1, while since $2N$ and S are greater than two, the right hand side is less than 1. \square .

B Adequacy of the normality approximation

When the returns for infinitesimal investors in sector m satisfy equation 4:

$$r_{s,m} = \theta_m + \gamma_m F + \epsilon_m + u_{s,m}. \quad (4)$$

where F and ϵ_m are independent and gaussian, and $u_{s,m} \sim \text{Uniform}[0, \bar{u}_m]$.

then for F and ϵ_m that satisfy the regularity condition 1 :

$$X_m - \theta_m > \gamma_m F + \epsilon_m > X_m - \theta_m - \bar{u}_m, \quad (1)$$

the probability that a loan made at rate X_m to entrepreneur s in sector m , will default conditional on F, ϵ_m is

$$\text{Prob}(r_{s,m} < X_m | F, \epsilon_m) = \frac{X_m - \theta_m - \gamma_m F - \epsilon_m}{\bar{u}_m}. \quad (\text{pdfm})$$

This implies that in a well diversified portfolio, conditional on the information provided at time 0, the expected proportion of defaults, which is the unconditional probability of default is $\frac{X_m - \theta_m}{\bar{u}_m}$. If the rates on the loans are set competitively at time 0, then X_m should be set so that investors are indifferent between extending this loan or holding the riskfree asset. This implies that X_m solves the quadratic equation:

$$X_m \left[1 - \frac{X_m - \theta_m}{\bar{u}_m} \right] + \left[\frac{X_m - \theta_m}{\bar{u}_m} \right] RGD = R_f, \quad (\text{A4})$$

where R_f is the gross riskfree rate and X_m is the gross interest rate in the sector.

To illustrate whether our approach can be used to generate reasonable default probabilities for one sector alone we solved the model with one macro factor F and without a sector specific shock (which is unnecessary to include in the one sector case). The parameters are as follows:

| | |
|---------------------|------|
| θ_m | 0.55 |
| γ_m | 1 |
| $\sqrt{\sigma_F^2}$ | .1 |
| \bar{u}_m | 10 |
| RGD | 0.5 |
| R_f | 1.02 |

With these parameter choices, assuming regularity condition 1 is satisfied, and that the probability of default can have a Gaussian distribution, the quadratic equation implies X_m is 1.0472. Given this choice of X_m the unconditional probability of default, $\frac{X_m - \theta_m}{\bar{u}_m}$, is .04972. To check the approximate internal consistency of these values, I generated 10 million monte-carlo draws from the true return distribution and estimated the default probability to be the fraction of return draws below 1.0472. A plus or minus two standard deviation confidence interval for the default probability is [.0495, .0498], which contains the approximate probability of default.

To investigate the probability that condition 1 will be violated, note that the probability of a violation is

$$\begin{aligned}
\text{Prob(Violation)} &= 1 - \text{Prob}(X_m - \theta_m > \gamma_m F + \epsilon_m > X_m - \theta_m - \bar{u}_m) \\
&= 1 - \left[\Phi\left(\frac{X_m - \theta_m}{\sqrt{\gamma'_m \Sigma_F^2 \gamma_m + \sigma^2(\epsilon_m)}}\right) - \Phi\left(\frac{X_m - \theta_m - \bar{u}_m}{\sqrt{\gamma'_m \Sigma_F^2 \gamma_m + \sigma^2(\epsilon_m)}}\right) \right] \\
&= 1 - (\Phi(4.9721) - \Phi(-95.028)) \\
&= 3.31 \times 10^{-7}
\end{aligned}$$

The requirement that the fraction of defaults predicted by the model conditional on the factors is between 0 and 1, i.e. that the normality assumption is reasonable reduces to the condition in 1.

The results here illustrate that using a normal distribution as an approximation for default probabilities and the return on a portfolio of loans from different sectors of the economy can be reasonable method for approximating default behavior, and creating returns on sector portfolios that are normally distributed.

There is one drawback of this approach. For simplicity, I used a uniform distribution for the variable $u_{m,s}$ over a support on $[0, 10]$. If one uses a uniform random variable for the idiosyncratic return component, then its support has to be very large to generate plausible

default probabilities. For example the upper bound 10 corresponds to a gross return of 1000 percent, with a gross average return of around 500 percent. This is not a problem unless one is hoping to also match data on the returns that entrepreneurs actually earn.

To match the entrepreneurs returns, then it is better to model $u_{s,m}$ as a random variable that has constant density for low values and then has a different distribution (such as terminating at a mass point) for higher values. For example, if $\text{Prob}(u_{s,m} < k) = k/10$ for $0 \leq k < 1.5$, and $\text{Prob}(u_{s,m} = 1.5) = 0.85$, then the upper bound on gross returns is much smaller, and will better fit the data, and by the law of large numbers the return for a well diversified portfolio will still turn out to be approximately Gaussian.

BIBLIOGRAPHY

- Acharya, V., 2009, "A Theory of Systemic Risk and Design of Prudential Bank Regulation," *Financial Stability Review*, Forthcoming.
- Allen, F., and D. Gale, 2000, "Financial Contagion," *Journal of Political Economy* 108, 1-33.
- Allen, L.A., Peristani, S., and A. Saunders, 1989 "Bank Size, Collateral, and Net Purchase Behavior in the Federal Funds Market: Empirical Evidence," *The Journal of Business*, 62, no. 4, 501-515.
- Anderson, E. W., L.P. Hansen, and T.J. Sargent, 2003 "A Quartet of Semigroups for Model Specification, Robustness, Prices of Risk, and Model Detection," *Journal of the European Economic Association*, 1, no. 1, 68-123.
- Black, F., and J.C. Cox, 1975, "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," *The Journal of Finance* 31, no. 2, 351-367.
- Bartolini, L., S. Hilton, and J. McAndrews, 2008, "Settlement Delays in the Money Market," Federal Reserve Bank of New York Staff Report no 319.
- Bech, M.L., and E. Atalay, 2008, "The Topology of the Federal Funds Market," Federal Reserve Bank of New York Staff Report no. 354.
- Caballero, R., and A. Krishnamurthy, 2008, "Collective Risk Management in a Flight to Quality Episode," *Journal of Finance*, 63, no. 5, 2195-2236.
- Cocco, J.F., Gomes, F.J., and N.C. Martins, 2009, "Lending Relationships in the Interbank Market," *Journal of Financial Intermediation* 18, 24-48.
- Covitz, D., and C. Downing, 2007, "Liquidity or Credit Risk Risk? The Determinants of Very Short-Term Corporate Yield Spreads", *Journal of Finance* 62(5), 2303-2328.
- Duffie, D., and D. Lando, 2001, "Term Structures of Credit Spreads with Incomplete Accounting Information," *Econometrica* 2001, p 633-664.
- Duffie, D., and A. B. Ashcraft, 2007, "Systemic Illiquidity in the Federal Funds Market," *American Economic Review*, 97, no. 2., p. 221-225.
- Easley, D., and M. O'Hara, 2009, "Ambiguity and Nonparticipation: The Role of Regulation," *Review of Financial Studies* 22, no. 5, 1817-43.
- Furfine, C.H., 1999, "The Microstructure of the Federal Funds Market," *Financial Markets, Institutions & Instruments*, 24-44.
- Furfine, C.H., 2001, "Banks as Monitors of Other Banks: Evidence from the Overnight Federal Funds Market," *Journal of Business* 74, 33-57.
- Gilboa, I., and D. Schmeidler, 1989, "Maxim Expected Utility with Non-Unique Prior," *Journal of Mathematical Economics* 18, 141-153.
- Gorton, G., 2008, "The Panic of 2007," Maintaining Stability in a Changing Financial System, Federal Reserve Bank of Kansas City Jackson Hole Symposium, 131 - 262.

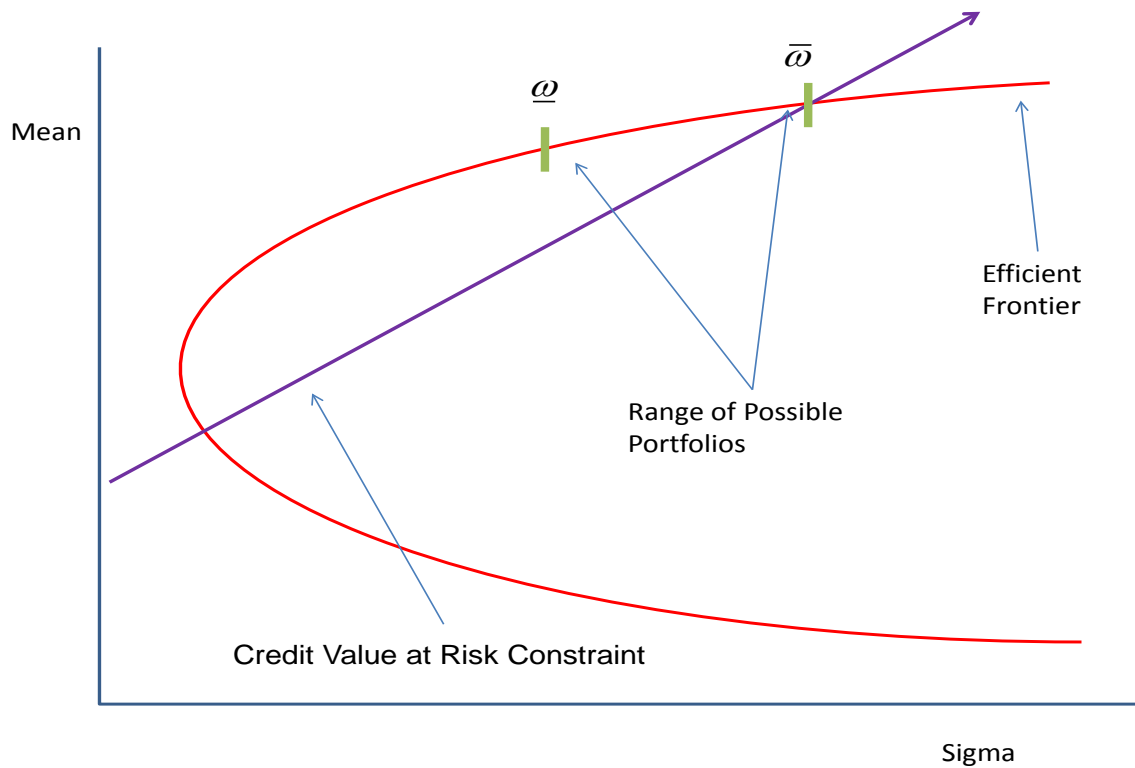
- Hansen, L.P., and T.J. Sargent, 2008, *Robustness*, Princeton University Press, Princeton, N.J.
- Heider, F., Hoerova, M., and C. Holthausen, 2009, "Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk," Working Paper, The European Central Bank.
- Huang, J.Z., and M. Huang, 2003, "How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?," Working Paper.
- Merton, R.C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *The Journal of Finance*, 29, 2, 449-70.
- Park, S., 1991, "Bank Failure Contagion in Historical Perspective," *Journal of Monetary Economics*, 28, 271-286.
- Taylor, J.B., and J.C. Williams, 2009, "A Black Swan in the Money Market," *American Economic Journal: Macroeconomics* 1, no 1, 58-83.
- Vasicek, O., 2002, "The Distribution of Loan Portfolio Value," *Risk*, December, p. 160-62.

Table 1: Balance Sheet of Bank A

| Assets | Liabilities |
|-----------|-------------|
| Loans 100 | Deposits 80 |
| | Equity 20 |

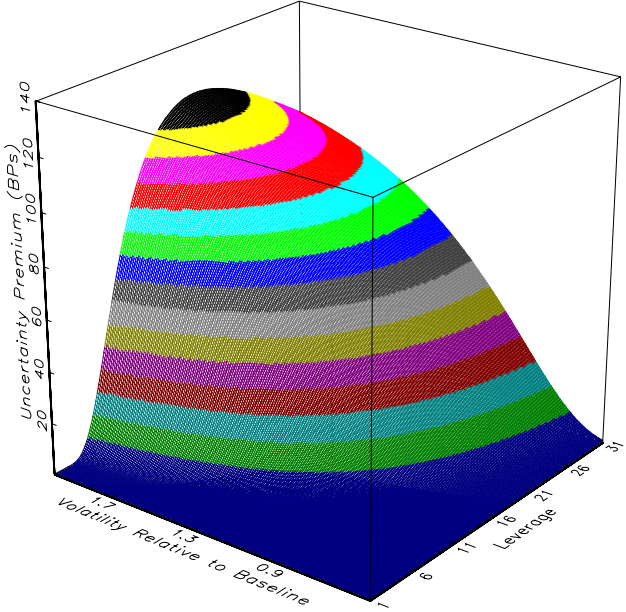
Off balance sheet: Sold “bullet” protection on bank D that pays 25 if bank D defaults.

Figure 1: Investment Opportunity Set for a Bank

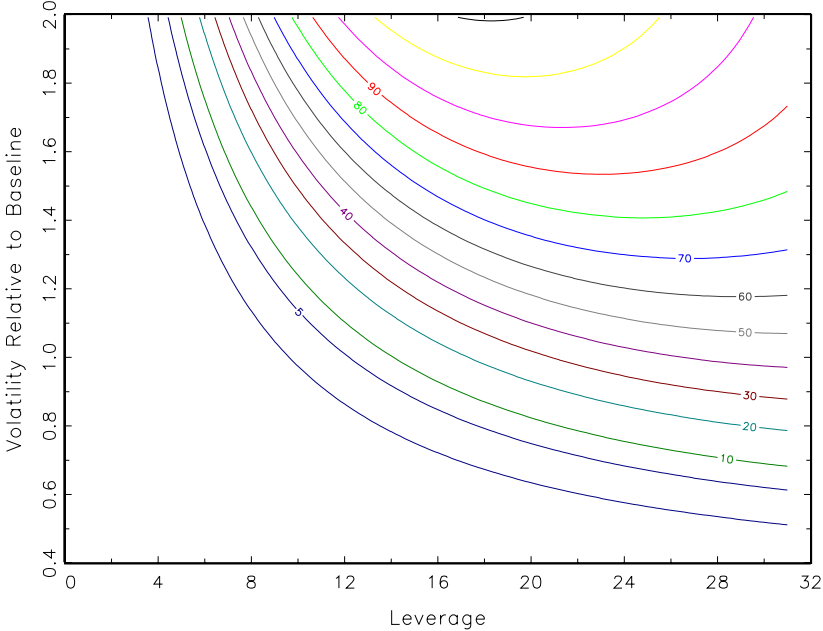


Notes: For the bank optimization problem in section 3.3, the Figure illustrates the banks investment opportunity set in terms of the mean and standard deviation of the portfolios it can choose. The bank chooses its portfolio subject to a credit value at risk constraint (points on and above and to the left of the purple line satisfy the constraint). Its optimal portfolio is $\bar{\omega}$, but because of internal constraints, it is assumed the bank instead chooses some mean-variance efficient portfolio between $\underline{\omega}$ and $\bar{\omega}$.

Figure 2: **Uncertainty Premium as a Function of Leverage and Loan Volatility**



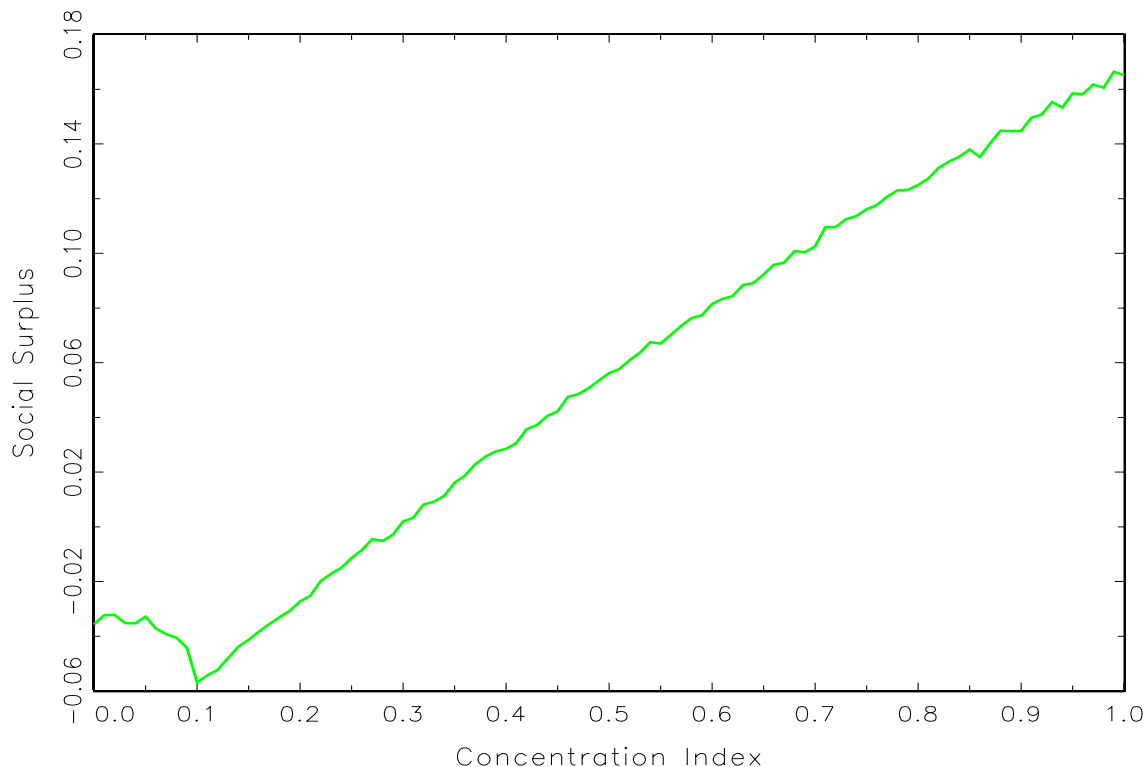
(a) Surface Plot



(b) Contour Plot

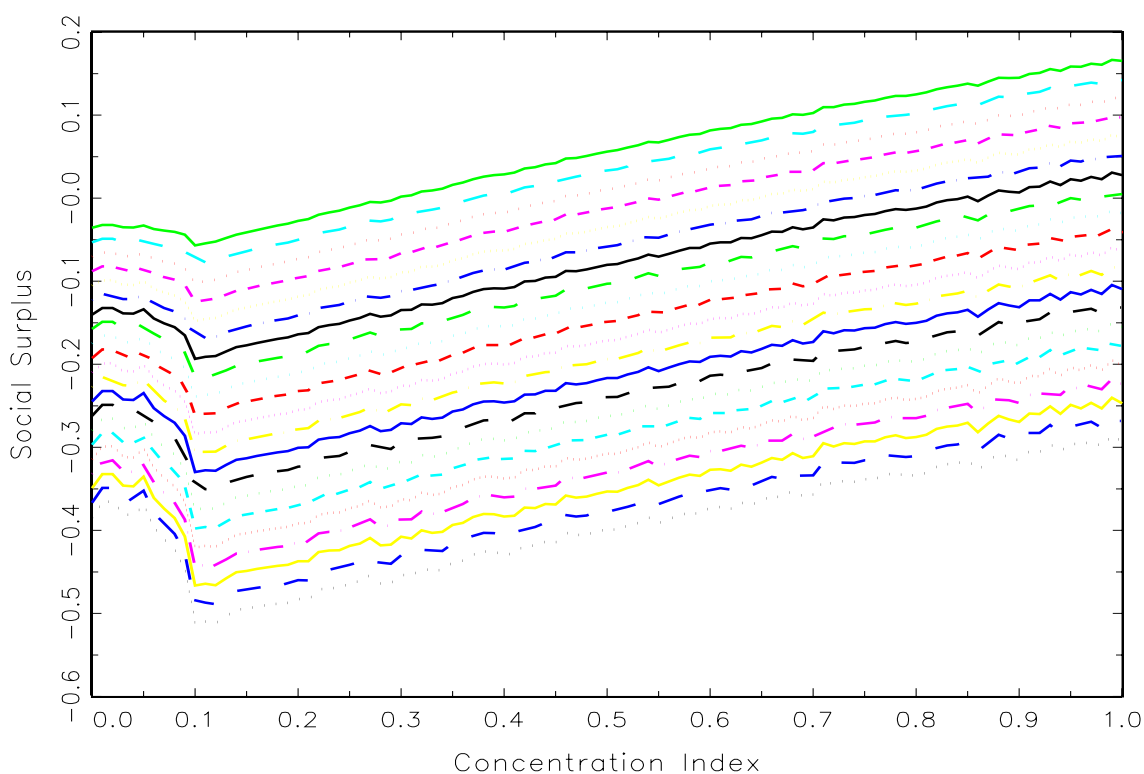
Notes: This Figure is reproduced from Pritsker (2009). For a stylized bank that holds two risky assets in its loan portfolio, the figure presents surface and contour plots of the uncertainty premium (in basis points) that the bank pays for its short-term unsecured interbank borrowing as a function of its leverage and as a function of the volatility (standard deviation) of it's assets relative to its baseline value.

Figure 3: **Economic Surplus from Government’s Sequential Inspection and Announcement of Bank’s Risk Exposures**



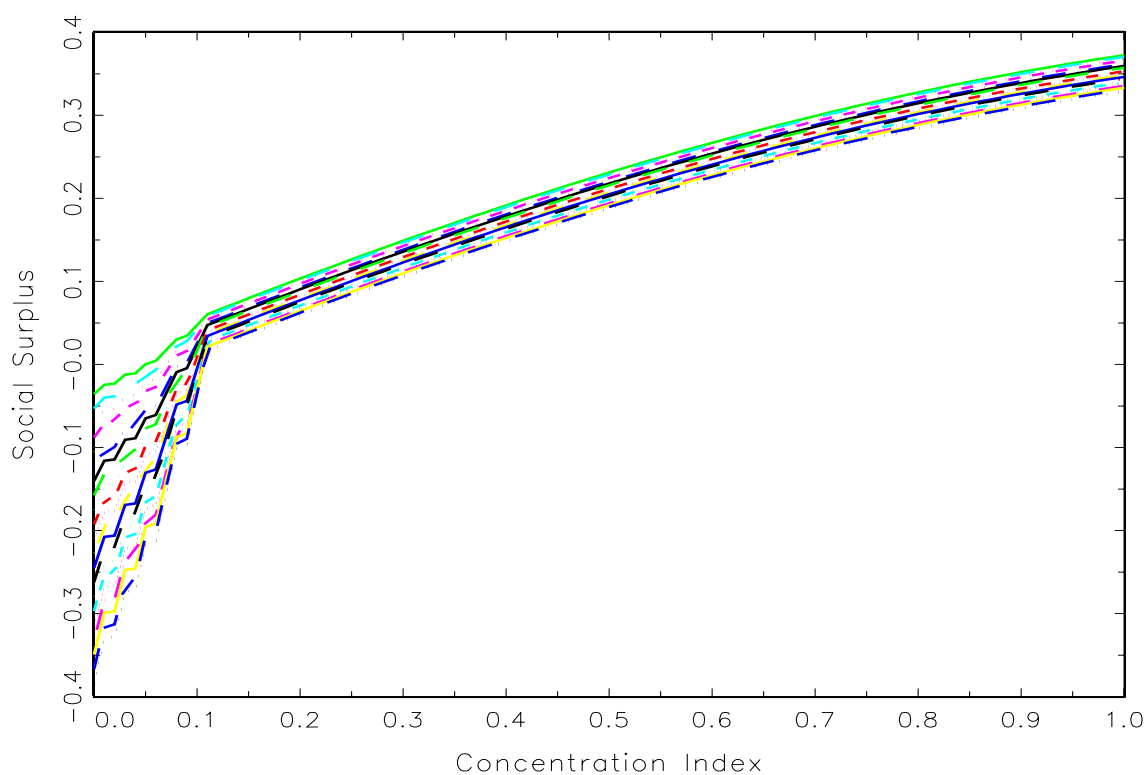
Notes: For a stylized example in which the interbank market breaks down due to uncertainty over banks risk exposures to a distressed class of assets, the figure presents a graph of the social surplus that is generated when the government follows a policy that involves sequentially inspecting and announcing the health and asset holdings of individual banks. The searches stop when liquidity is restored to the interbank market and when the surplus from interbank loans is judged to be high enough. Social surplus is a function of the true concentration of risk exposures within the banking sector to the distressed class of assets. Social surplus is measured in units of return per dollars of new entrepreneurial loans due to restoration of lending in the interbank market. Concentration is measured by an index that varies from 0 to 1, where 0 represents minimal concentration of risky holdings, and 1 denotes maximal concentration of risky holdings. Additional details are provided in the text.

Figure 4: **Economic Surplus from Government’s Sequential Inspection and Announcement Policy with Varying Government Inspection Costs and Random Search**



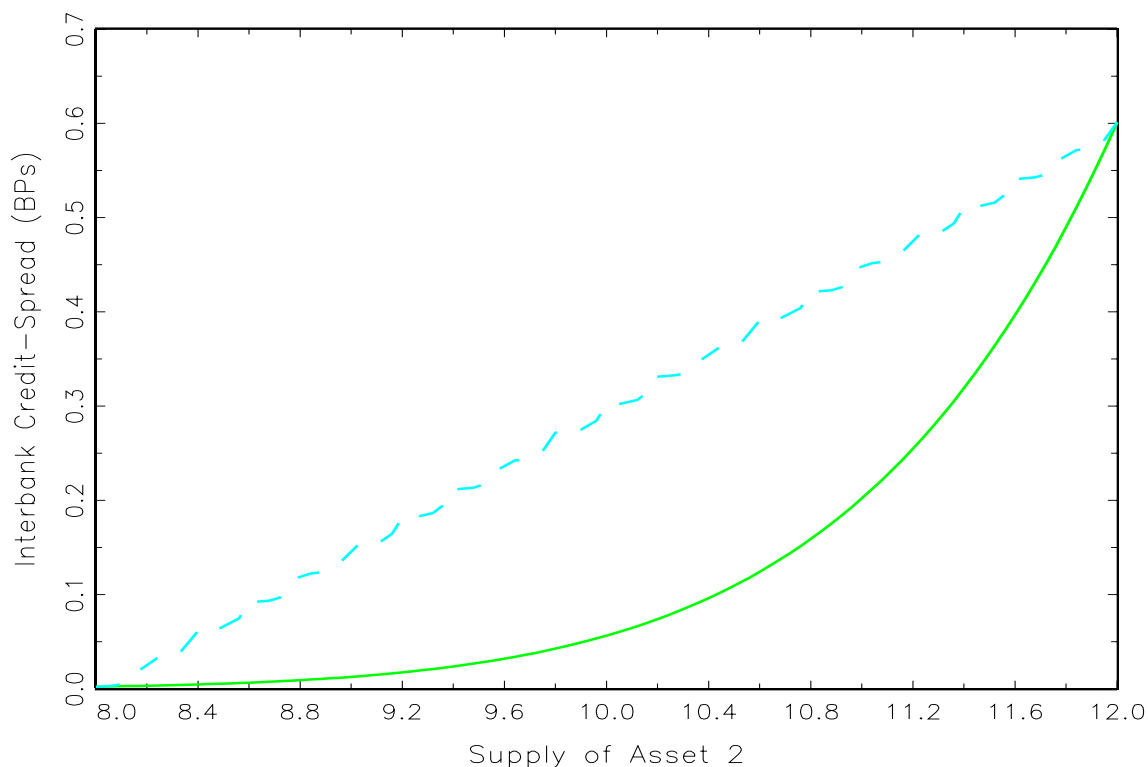
Notes: For the inspect and announce policy in Figure 3, this figure reports the surplus that is created if government’s costs of evaluating each bank are equal to the banking sectors own costs multiplied by factors of 1.0, 1.1, 1.2, 1.3, etc... Results are presented for differing levels of concentration of banking sector exposure to the distressed asset class. For further details see Figure 3.

Figure 5: **Economic Surplus from Government’s Sequential Inspection and Announcement Policy with Varying Government Inspection Costs and Search Based on Rank of Exposure**



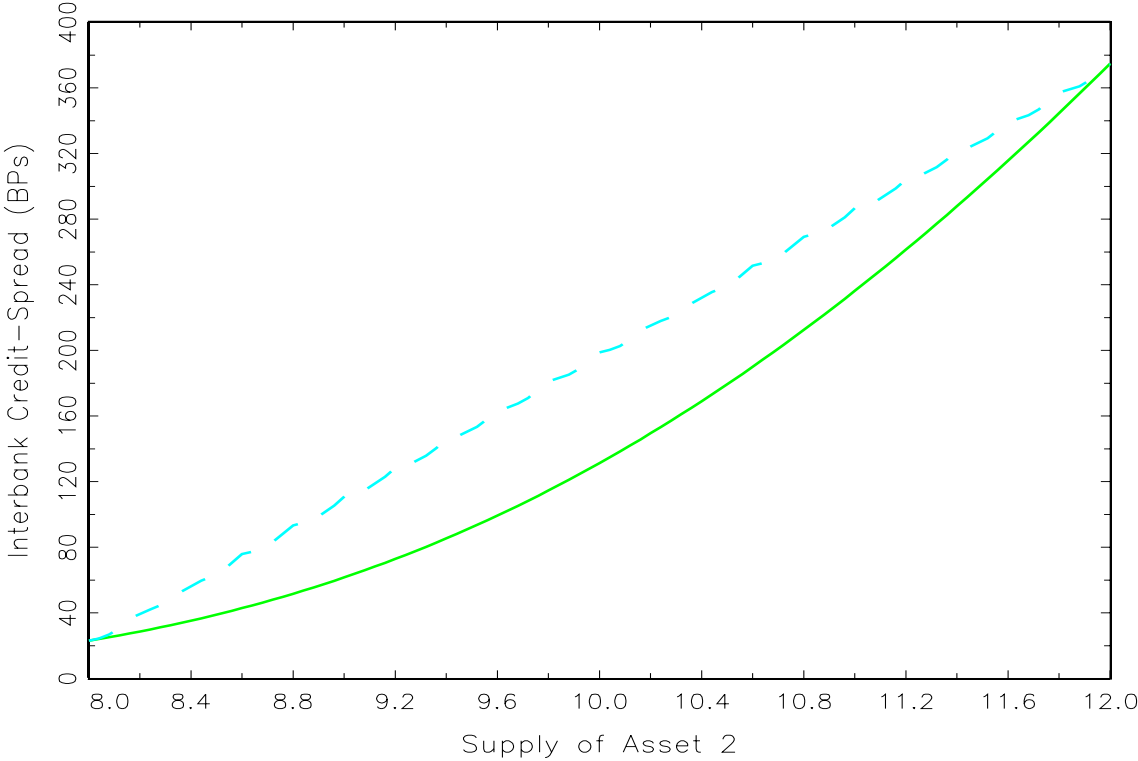
Notes: For the inspect and announce policy in Figure 3, and the cost structure in Figure 4, this figure reports the surplus that is created if government knows the relative magnitude of banks exposures to the distressed asset class and follows an inspect and announcement policy in which banks with higher risk exposures are inspected first, and in which inspections cease after the interbank market restarts.

Figure 6: **Interbank Spread as a Function of High Risk Assets Held by Banks: Good Economic Conditions**



Notes: For the two risky asset example in section 4.3, for various levels of outstanding supply of risky asset 2, the figure illustrates the spread in a pooling equilibrium that banks have to pay when borrowing in the interbank market when assets are distributed among banks to maximize the spread (blue dashed line) and when assets are distributed among banks to minimize the spread (solid green line). The units on the y-axis are basis points. The figure shows regardless of supply, the spread is very low, i.e. at most it is just over 1/2 basis point.

Figure 7: **Interbank Spread as a Function of High Risk Assets Held by Banks: Weak Economic Conditions**



Notes: For the two risky asset example in section 4.3, for various levels of outstanding supply of risky asset 2, the figure illustrates the spread in a pooling equilibrium that banks have to pay when borrowing in the interbank market when assets are distributed among banks to maximize the spread (blue dashed line) and when assets are distributed among banks to minimize the spread (solid green line). The units on the y-axis are basis points. The figure shows in weak economic conditions the spread is very sensitive to the known outstanding amount of the risky asset held by banks.