Estimating the Intertemporal Substitution Elasticity

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Abstract

To estimate the elasticity of intertemporal substitution (EIS), one traditionally assumed time-additive utility and constant relative risk aversion, but that model is often regarded as a failure as little or no link is detected between expected real consumption growth and the real interest rate. We test whether the inverse of the EIS is really all that different from the RRA estimates we get from *eg* equity markets, *ie*, whether time-additive utility is really such a bad assumption. We find that by simple modifications of the basic model—namely, accounting for seasonals and allowing for effects stemming from changes in wealth—produces elasticities that are quite compatible with other studies, including values implied by the more reasonable range of risk-aversion estimates obtained from CAPM tests. Our EIS estimates are obtained from GMM, after unsuccessfully trying the standard proxy for expected inflation. We also pool data from 24 countries, as single-country estimates are very imprecise and erratic. Our attempt to reduce the time-aggregation problem in consumption data, lastly, is successful in the sense that the resulting EIS estimates are even higher.

Keywords: seasonality, inflation rate, instrumental variable, time aggregation, varying relative risk aversion.
JEL-codes: G12, F43.

Estimating the Intertemporal Substitution Elasticity

Introduction

The Elasticity of Intertemporal Substitution (EIS), a key parameter in macro-economics and finance, measures the sensitivity of expected consumption growth to the real interest rate (*ie* the expected real return on a nominally risk-free asset). In the case of time-additive utility the EIS coincides with relative risk tolerance, the inverse of the Arrow-Pratt measure of relative risk aversion (RRA); thus, under that assumption the EIS can also provide an alternative estimate of RRA that does not rely on stock returns.

Assuming additive utility, EIS or RRA is a crucial parameter to determine the net effect of higher expected future aggregate pay-offs on its present value. The watershed case, where the opposing income and substitution effects[1] cancel out exactly, corresponds to an RRA of unity; for values above unity higher expected growth sends the stock market down and *vice versa*. Many economists would intuitively think that, given the level of current consumption, a prospect of higher future consumption should increase its discounted value, implying that the net outcome of income and substitution effects should be positive and RRA, accordingly, below unity. However, the RRA estimated from financial markets is normally at least 3 in unconditional tests of the CAPM, and much higher in conditional studies. One way to resolve the conflict is to blame the stock market: expected returns on equity are too high (the equity premium puzzle), possibly because of irrationality or perhaps because the observed returns ignore various deadweight costs associated with investments or because the mean-variance model that underlies many tests is otherwise flawed. Other ways out of the puzzle include blaming the utility function: if utility is not additive over time, the close link between the EIS and the RRA is severed. Attanasio and Weber (1989), Epstein and Zin (1989, 1991), Svensson (1989) and many others since, propose approaches to separate the EIS from the RRA, but at the cost of higher complexity. In this paper we want to test whether the inverse of the EIS is

[1] The most obvious force is the income effect: there is, by assumption, a higher pay-off to discount, so that at constant discount rates the present value rises. But there is an opposite force as well, a substitution effect: to make agents accept a higher consumption growth, the discount rate must rise. In (log)normality models, also an increased variance has similarly counteracting effects on the risk-adjusted expectation and on the discount rate.

really all that different from the RRA estimates we get from the traditional equity literature, *ie*, whether time-additive utility is really such a bad assumption. Thus, in the paper we adopt time additivity as a working hypothesis, and we study EIS-implied RRA estimates. To get these, we explore four avenues that might explain the often disappointingly low EIS estimates hitherto: (*i*) the treatment of expected inflation, an unobservable term in the real interest rate; (*ii*) the assumption of Constant RRA (CRRA), the workhorse model in this literature; (*iii*) the data, which has predominantly been U.S.-based; and (*iv*) time aggregation: theoretically, the 90-day interest rate of *eg* April 1 should be linked to the growth between the consumption levels on April 1 and July 1, not the aggregate consumption levels for the second and the third quarter. In the remainder of this introduction we provide more details on these four aspects of the paper, starting with the unobservability of expected inflation.[2]

In the past the unobservability of expected inflation has been handled via either surveys or Instrumental Variable (IV) regressions. We first document that the standard proxy method does not work, and then proceed with the Generalized Method of Moments (GMM). Specifically, in the regression estimating the EIS, a popular IV-like proxy for the expected real interest rate can be obtained by assuming inflation to be a martingale, and accordingly substituting actual inflation from the preceding period for the inflation expectation. Technically, this popular proxy boils down to a pared-down two-stage least square (TSLS) approach, using lagged inflation as the sole instrument and by a priori fixing the coefficient of the first-stage regression at unity. The method still introduces an errors-in-the-regressor bias if inflation is not a martingale, but the method does avoid the inefficiency stemming from estimation error at the first-stage regression of TSLS. A perhaps better justification for the proxy method is that it represents common practice in dailies and weeklies, where inflation over the past twelve months is almost invariably substituted for expected inflation. In this paper, however, inflation is the three-month percentage change of the price levels, for consistency with the quarterly consumption data; and non-overlapping three-month inflation turns out to behave very differently from the overlapping twelve-month figures we see published every month. For the developed countries, three-month inflation even turns out to be negatively autocorrelated and strongly seasonal rather than a unit-root process. For the emerging countries the martingale model is less grossly violated, but the amount of momentum remains mild, at best. In most cases, including

[2]Another big issue, the presence of durable goods in the spending data, is not discussed. Empirical results on the impact of durable goods are mixed; more practically, approximate adjustments for spending on durable goods are available for few countries only.

especially the emerging markets, the proxy method performs badly, generating negative EIS estimates. Much better inflation forecasts can be obtained using past inflation rates (four lags) and interest rates. Using these instruments, GMM provides EIS estimates that are all significantly positive.

The second potential problem with conventional tests is the CRRA assumption. One reason to doubt the validity of the model is that we observe strongly seasonal consumption growth, rising sharply around year-end and slowing down afterwards; yet there is no similar pattern in interest rates. One diagnosis, then, is that utility exhibits a seasonal that autonomously modifies the consumption pattern without all of the changes in marginal utility that would have followed if the utility function had been time invariant. Thus, in this view we spend so much more around Christmas not because interest rates are sky high in quarter 3 and abysmal in quarter 4, but because spending is exogenously more fun at that time. Stated differently, interest rates do not need to move strongly because marginal utility does not fall despite the higher spending. If this diagnosis is correct, the regular pattern of intertemporal substitution may be obscured by the seasonality in marginal utility, *ie* the CRRA model is mis-specified. Simple generalisations of the power utility model that can explain a seasonal in consumption lead to a seasonally changing RRA, which then dictates the addition of seasonal dummies in the intercept and slope of our main regression. We find that the dummy intercepts are significantly positive for growth between the third and fourth quarters, and negative in the fourth-to-first quarter, reflecting a year-end effect. The model's goodness-of-fit rises dramatically relative to the standard GMM. The pattern persists if we also let the slopes share the same seasonal with the intercepts, as the model suggests it should. Crucially, there also is a distinctly positive effect on EIS estimates.

Before moving on, we compare the above solution to the common way of dealing with the seasonal: deseasonalize the data first, and hope that that's all there is to it. But that solution implies that our consumption variable becomes a constructed one. In addition, pre-deseasonalisation would have largely destroyed the link between the seasonals in interest rates and (especially) inflation. Thus, one advantage of our procedure is that the deseasonalisation and the estimation of the EIS are done in one step, with the significance tests and confidence intervals fully taking into account the margin of error in the deseasonalisation, and allowing the estimation to pick up any correlated parts in the two seasonals.

This finishes our discussion of seasonal variation in marginal utility. Another, more standard, generalization of the CRRA model is to allow RRA to fall in wealth, that is, EIS to rise with wealth. Under a generalized power function, where only wealth in excess of a threshold produces utility,[3] RRA and EIS will vary as wealth is changing; accordingly, in our tests we allow risk tolerance to vary as a linear function of recent fluctuations in wealth, measured by the local stock index' deviation from its moving average (ΔSI). Our findings support the theory: the slope of the ΔSI is positive, suggesting that EIS and risk tolerance increase with wealth. The pattern can be combined with the seasonal.

This brings us to the third aspect of our paper. On the empirical front, the data scope is extended from the U.S., the standard sample in earlier work, to an international panel of 24 countries, including both developed and emerging economies. For efficiency, we estimate the equations via panels with a country-specific fixed effect and common values for the preference parameters. One question that arises in this connection is the heterogeneity among the estimates of the individual series: pooled regressions are consistent with equation-by-equation estimates only if the coefficients are identical across equations. Our purpose, however, is to get some average estimate. According to Pesaran and Smith (1995), there are four procedures that can be used to estimate an average effect: the mean group estimator (estimating separate regressions for each group and averaging the coefficients over groups, as in LS), pooled regression, aggregate time-series regressions and cross-section regressions on group means. In the static case, where the regressors are strictly exogenous and the coefficients differ randomly and are distributed independently of the regressors across groups, all four procedures provide a consistent and unbiased estimate of the coefficient means (Zellner, 1969).

Fourth, we explore the impact of the time-aggregation problem. An interest rate refers to a specific starting day and a specific end date, for instance February 15 and May 15, so ideally we should have daily consumption data for exactly these dates. Quarterly consumption growth, in contrast, compares one 90-day aggregate to another 90-day aggregate. This mismatch between the regressee and regressors may weaken the relation between consumption growth and the real rate, and then the EIS is underestimated. To reduce the impact of the aggregation problem, we propose to extract monthly data from the quarterly ones. Specifically, we assume that consumption is changing smoothly, so that we can make informed guesses about the distribution of consumption within the quarter by looking at consumption in the preceding and next quarters. We then extract, for each quarter, the middle-month figure and use this

[3] In equilibrium, patterns in marginal indirect utility of wealth must mirror those in marginal indirect utility of consumption.

series instead of the three-month aggregates. We find that aggregation seems to systematically reduce the estimated EIS of the developed countries: the EIS estimates are higher when the data have been corrected. This is observed under all methods, equations and samples except the emerging markets, but whether this is externally valid under all circumstances is of course far from obvious.

Given the tentative nature of the correction of aggregation, the conclusions that follow are based on regular data. Regarding the orders of magnitude of the RRA implied by the EIS, under the time-additivity assumption, we find coefficients of about 5 for emerging markets, and 10 for developed economies if we adopt a GMM/CRRA model. Allowing for either the seasonal or the changes in wealth always makes the implied RRA estimate drop substantially. The final estimate is around 2 for the full sample and for the emerging markets. For the continental European group the RRA estimates are even lower. These estimates are similar in magnitude to those obtained from exchange rates and real consumptions by Apte *et al* (2005) or from portfolio holdings data by Van Pée *et al* (2008). Thus, it seems that the equity puzzle may indeed have something to do with the stock market, and not necessarily with the additivity or non-additivity of utility over time.

1 The Models

The EIS is defined as the sensitivity of expected real consumption growth relative to the real interest rate, which in turn is defined as the expected real return on a nominally risk-free deposit or bill. In the case of time-additive utility the EIS coincides to the degree of risk tolerance, the reciprocal of RRA: after all, expected utility is additive over states already, so when we make lifetime expected utility also additive across periods, consumption variability across states affects lifetime expected utility in the same way as variability over time. In this section we discuss the modeling issues that motivate our regressions. We first review the standard test equation, then present the arguments for including equity data and sesonal dummies into the regression, and conclude with a proposed partial solution for the aggregation issue in the consumption data.

1.1 The standard test equation

We indicate the timing of growth rates, inflation rates, asset returns, *etc* between times t and $t+1$ by a subscript $t+1$. With utility U generated by nominal consumption C, the pricing kernel m for an asset's nominal pay-off is the ratio of marginal utilities of nominal spending for dates $t+1$ and t, or unity plus the percentage change in marginal utility,

$$m_{t+1} = \frac{\Lambda_{t+1}}{\Lambda_t} = 1 + \frac{\Delta\Lambda_{t+1}}{\Lambda_t}, \tag{1}$$

where Λ denotes the marginal utility and $\Delta\Lambda$ is its change. For a risk-free bond with nominal interest rate R_{t+1} expiring next period, the future nominal payoff $1+R_{t+1}$ is discounted by the pricing kernel; under the no-arbitrage rule the result should equal the bond's initial value, unity. Familiarly, the expected kernel $E_t\left[m_{t+1}\right]$ then equals the inverse of future pay-off, $\frac{1}{(1+R_{t+1})}$:

$$\begin{aligned} \mathrm{E}_t\left[m_{t+1}(1+R_{t+1})\right] &= 1, \\ \mathrm{E}_t\left[m_{t+1}\right] &= \frac{1}{(1+R_{t+1})}, \\ ie\ \ \mathrm{E}_t\left[1+\frac{\Delta\Lambda_{t+1}}{\Lambda_t}\right] &= \frac{1}{(1+R_{t+1})}. \end{aligned} \tag{2}$$

If marginal utility were observable, the above would be easily tested by regressing its realised growth rates, $\Delta\Lambda/\Lambda$, on the discount factor, a regressor that is observed without error. To make utility growth observable, one generally restricts the utility function to depend on consumption only. Two such avenues are reviewed in the Appendix: imposing CRRA and lognormality, or adopting a more general linear expansion of $\Delta\Lambda/\Lambda$, an approach which imposes no extra restrictions on [2] but ignores higher-order terms. The latter avenue leads to

$$g^{'}_{t+1} = -\sigma_t\theta_t + \sigma_t\left[R^{'}_{t+1} - \mathrm{E}_t(i^{'}_{t+1})\right] + \tilde{\epsilon}_{g',t+1}. \tag{3}$$

where g' denotes the log change in consumption, σ the inverse of relative risk aversion, $R' := \ln(1+R)$ the continuously compounded rate of return, θ the time preference parameter, $\mathrm{E}_t(i^{'}_{t+1})$ conditional expected inflation, and $\tilde{\epsilon}_{g',t+1}$ conditionally unexpected consumption growth. Obviously, σ also doubles as the EIS, as it relates expected consumption growth to the real interest rate; that is, in the standard model one has EIS $= 1/$RRA even without CRRA, at least approximatively.

What the combination of CRRA and lognormality does add is an exact solution. The only difference with Equation (3) turns out to be the addition of a Jensen's Inequality term,

$$\frac{1}{2}\left(\frac{s^2_{t,g}}{\sigma_t} + 2\mathrm{cov}_t(g', i') + \sigma_t s^2_{t,i}\right), \tag{4}$$

where s^2_g and s^2_i denote the variances of the log changes in consumption growth and inflation, respectively, and $\mathrm{cov}_t(g', i')$ denotes their covariance. This extra term does not involve the real interest rate; and assuming constant moments and constant RRA, it can simply be stuffed into the intercept. All this suggests that the EIS estimated from the linear approximation model is probably robust to the details of the basic model but that the intercept implies no good estimate of time preference. A second implication is that the second moments in the Jensen's Inequality term are country-specific, so in the empirical work we add a country fixed effect to the basic equation.

1.2 State-dependent RRA and EIS

Most empirical studies adopt the assumption of constant relative risk aversion (CRRA).[4] The results from the CRRA model have been questioned for decades, however, because the EIS estimated under the standard approach is too small relative to what many economists expect. As noted in the introduction, one can object that decreasing RRA is far more plausible and that constant power utility is incompatible with the observed seasonal in consumption growth. Accordingly, we now propose simple models of preferences that are state-dependent with respect to seasonal circumstances and changing wealth levels.

As shown in Figures 1 to 3, quarterly consumption growth exhibits a strong seasonal pattern and much wider fluctuations than the ex post real interest rate, let alone the (unobservable) expected real rate. Most importantly, consumption is typically rising in the Christmas/year-end period and shrinking afterwards. If there is no strong link with the interest rate nor with prices, an economist's interpretation would be that there is an autonomous or exogenous change in the parameters of utility that prompts the investor to consume more even at constant prices and interest rates. If the marginal utility function changes, then the RRA and the EIS must almost surely change too; after all, the RRA is just the elasticity of marginal utility.

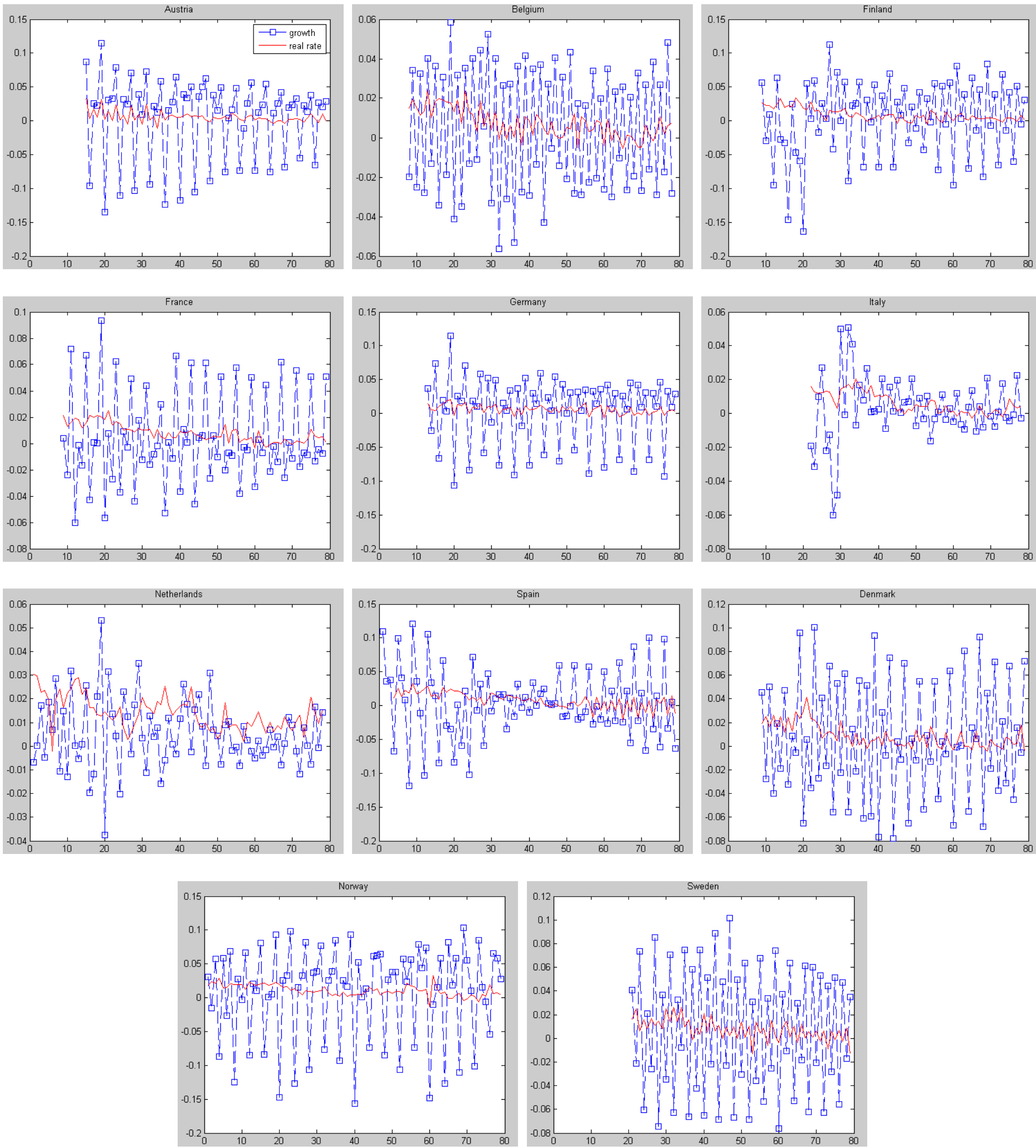
Let $q(t)$ denote the quarter corresponding to the t-th observation, *ie* $q = \{1, 2, 3, 4, 1, 2, ...\}$. One example of a season-dependent utility function that makes marginal utilities and consumption fluctuate could be a generalized power rule like

$$U(C_t/\Pi_t, q(t)) = \frac{1}{1-\eta}\left[C_t/\Pi_t - L_{q(t)}\right]^{1-\eta}, \qquad (5)$$

where $L_{q(t)}$ is the consumption standard for the q-th quarter. For instance, during the year-

[4] See Hall (1988) and Hansen and Singleton (1982), for an early analysis, and Attanasio et al. (2002), Vissing Jorgensen (2002) and Attanasio and Vissing Jorgensen (2003) for recent contributions.

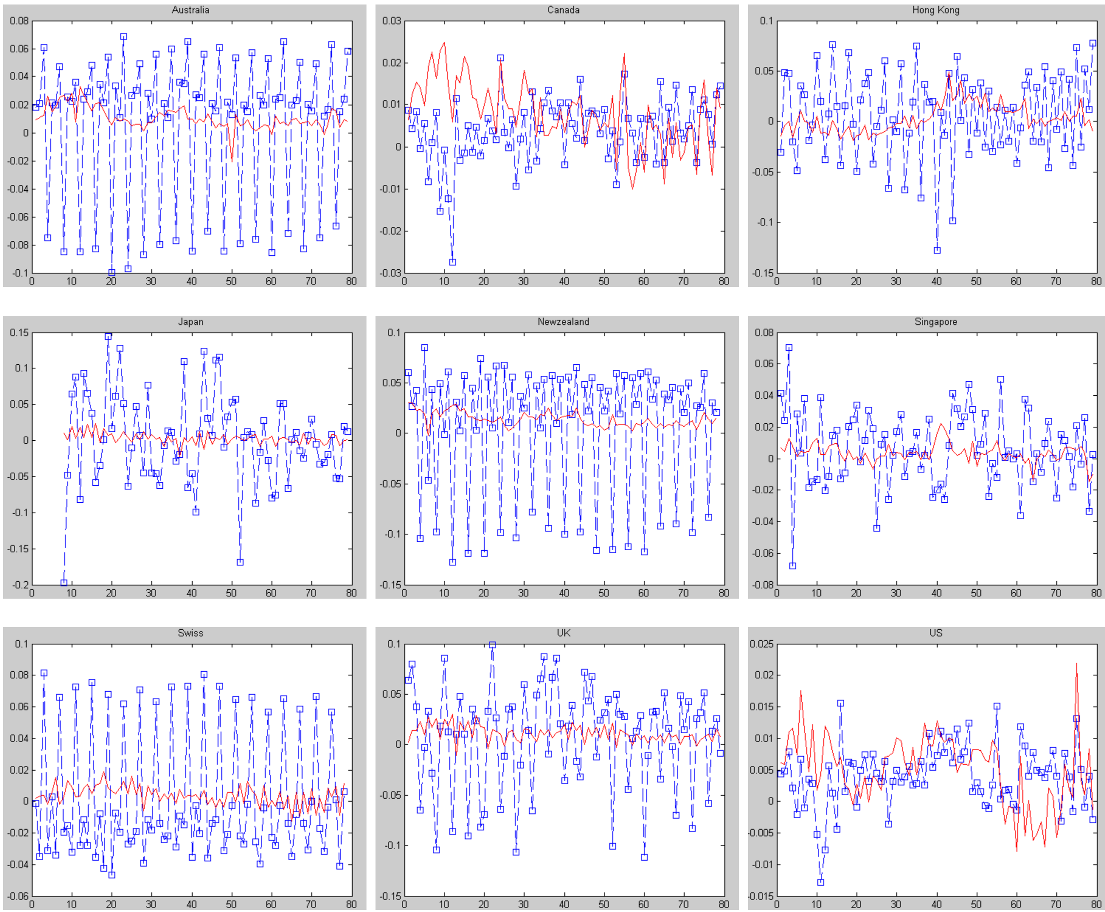
Figure 1: **Consumption growth and real rate (Continental European)**



Key: The figure shows quarterly consumption growth rates (broken lines linking the squares) and realised real interest rates (the full lines) for a number of countries.

end season the standard may simply be higher. The argument of U being excess consumption rather than consumption itself, this model induces a seasonal into the RRA that mimicks the fluctuations in consumption, a familiar result re-derived below. From the first and second derivatives, the RRA fluctuates proportionally to the ratio of consumption relative to excess

Figure 2: **Consumption growth and real rate (non-European developed)**



Key: The figure shows quarterly consumption growth rates (broken lines linking the squares) and realised real interest rates (the full lines) for a number of countries.

consumption:

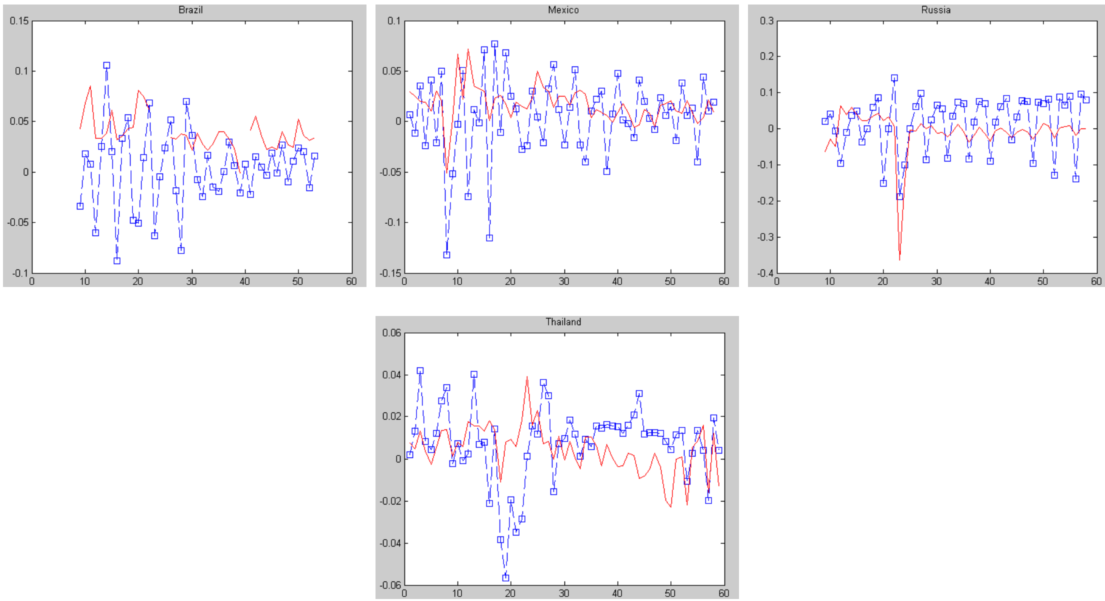
$$\frac{\partial U}{\partial C_t/\Pi_t} = (C_t/\Pi_t - L_{q(t)})^{-\eta} \quad , \quad \frac{\partial^2 U}{\partial C_t/\Pi_t^2} = -\eta(C_t/\Pi_t - L_{q(t)})^{-\eta-1}, \tag{6}$$

$$\begin{aligned} \Rightarrow -C_t/\Pi_t \frac{\frac{\partial^2 U(C_t/\Pi_t)}{\partial C_t/\Pi_t^2}}{\frac{\partial U(C_t/\Pi_t)}{\partial C_t/\Pi_t}} &= -C_t/\Pi_t \frac{-\eta(C_t/\Pi_t - L_{q(t)})^{-\eta-1}}{(C_t/\Pi_t - L_{q(t)})^{-\eta}}, \\ &= \eta \frac{C_t/\Pi_t}{C_t/\Pi_t - L_{q(t)}}. \end{aligned} \tag{7}$$

At constant real interest rates, the investors will tend to stabilize lifetime excess consumption rather than gross consumption as under the standard model. This means that in periods of high standards, consumption rises while excess consumption does not, which means that risk aversion should endogenously rise, and EIS fall, towards the peak quarter.

But that is not the only possible prediction. Another simple way to make marginal utility rise in particular seasons is to let the exponent itself follow a seasonal. If U is specified as

Figure 3: **Consumption growth and real rate (emerging)**



Key: The figure shows quarterly consumption growth rates (broken lines linking the squares) and realised real interest rates (the full lines) for a number of countries.

$(C/\Pi)^{1-\eta_q}/(1-\eta_q)$, marginal utility equals $1/(C/\Pi)^{\eta_q}$. Therefore, rising marginal utility for given consumption now requires a fall in η rather than the rise that was predicted by the first seasonal-utility model. We conclude that this theory is agnostic about the seasonal pattern in the risk aversions; which model does best is an empirical matter.

A far more familiar notion of state dependence is that RRA may also change with the level of wealth. In this statement, utility is the indirect utility of (real) wealth instead of the direct utility of consumption, but the two are related in the sense that in the optimum both marginal utilities are equalized. If there is a minimum-consumption standard, a minimum-wealth standard is implied, *viz* the risk-free investment needed to lock-in all future minimum consumption budgets. Again in a generalized power-utility function, for example, only wealth W above this threshold level $\underline{W}$ then produces (indirect) utility. The familiar result is that RRA will fluctuate with W:

$$-W\,\frac{\frac{\partial^2 J(W)}{\partial W^2}}{\frac{\partial J(W)}{\partial W}} = \eta\frac{W}{W-\underline{W}}$$

So, RRA falls when wealth increases and vice verse, or, equivalently, the EIS rises with wealth according to this utility function. This pattern can, of course, arise over and above the seasonal.

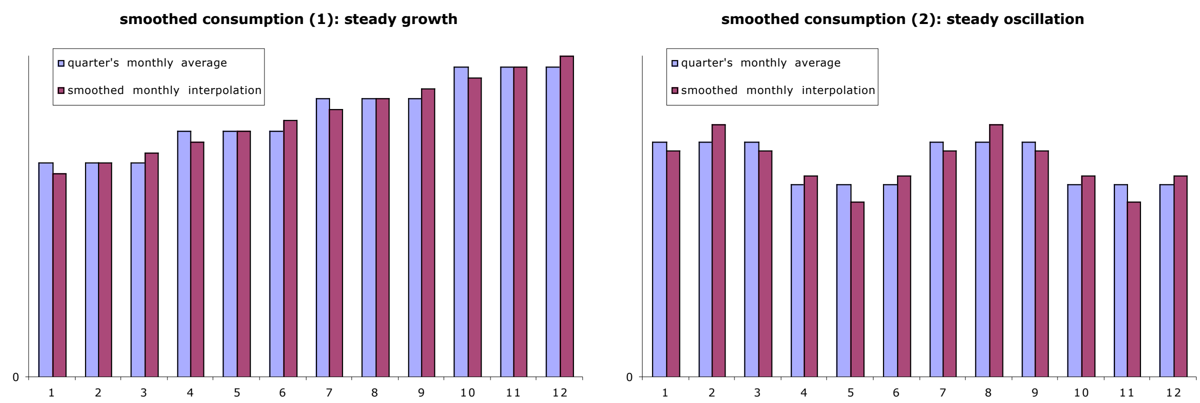
1.3 The Time Aggregation Problem

The time aggregation problem is another possible reason for a low EIS. If we use *eg* the February 15 observation of the 90-day interest rate, this should ideally be related to consumption growth of May 15 relative to February 15, or maybe the week of May 15 relative to that of February 15. But in practice, consumption data bears on a much more extensive period, for instance one quarter. Hall (1988) assesses the time aggregation issue by comparing annual to quarterly and monthly data. However, for most countries monthly consumption data are not available, so this test is difficult to replicate in an international context.

Of course, there would be no issue if consumption were constant within each quarter, but that is hard to believe. In this paper, we come up with a partial solution by calculating a monthly consumption figure for the middle month from the aggregate quarterly data. This interpolator works a bit like a Hodrick-Prescott smoother. It starts from the quarter's monthly average consumption, $C_t/3$, as the first-pass estimate of the month's level, and then administers changes subject to the constraint that the sum of the corrections within the quarter is to be zero. For the first month in the quarter, this first-pass value is adjusted in light of what happened in the preceding quarter: if consumption was on the rise, the quarter's first month is lowered (and the preceding quarter's third month is raised), thus smoothing out the observed growth. Similarly, for the third month in the quarter, the first-pass value is adjusted in light of what happened in the subsequent quarter. If consumption was still rising, the quarter's third month is again increased (and the next quarter's first month is lowered) so as to smooth out the growth. In contrast, if the next quarter consumption was lower, the original quarter's third month is decreased relative to its first-pass value and the next quarter's first month is increased to smooth out the drop. What happens to the middle month of the quarter, lastly, depends on both the preceding and the following quarter. If we saw quarterly consumption rise twice in a row, the figure for the middle month of the central quarter should be between those for the first and third months. If, in contrast, the central quarter represented a peak, then the revised peak will show up most clearly in that quarter's middle month, with a value above the quarter's average while the first and last month end up with figures that are below the quarter's average.

Figure 4 shows two theoretical examples. The first subfigure shows what would happen if we had an infinite chain of steadily rising quarterly data: the result would be a series of steadily rising monthly data. The second subfigure indicates what would happen in an infinite chain of oscillating quarterly data, alternating between a high and a low level. The resulting

Figure 4: Extracting monthly data from quarterly use of the smoother



monthly series shows peaks and troughs in each quarter's central month.

The objective of smoothness is operationalized by minimizing the sum of the squared percentage growths in the resulting monthly figures.[5] Formally, the procedure first trisects the quarterly consumption. Time subscripts t still refer to quarters, as before. We denote the first-, second- and third-month corrections of the consumptions in quarter t by u_t, v_t, w_t, and we impose $w_t = -u_t - v_t$ so that the total adjustments within each quarter are zero. So, the consumption in the first month of each quarter is $C_{1,t} := (u_t + \frac{C_t}{3})$, where the subscript $\{1, t\}$ means the first month of the t-th quarter. Similarly, the second-month consumption is $C_{2,t} := (v_t + \frac{C_t}{3})$ and the third-month consumption is $C_{3,t} := (w_t + \frac{C_t}{3}) = (\frac{C_t}{3} - u_t - v_t)$. Secondly, our objective function is to minimize the total variance of month-by-month consumption growth rates, ie the sum of the squares of the monthly consumption growths:

$$\begin{aligned} \{u_t, v_t\}_{t=1,\ldots T} &= \operatorname{argmin} \left(\frac{C_{1,1}}{C_{3,0}}\right)^2 + \sum_{t=1}^{T} \left\{ \left(\frac{C_{2,t}}{C_{1,t}}\right)^2 + \left(\frac{C_{3,t}}{C_{2,t}}\right)^2 + \left(\frac{C_{1,t+1}}{C_{3,t}}\right)^2 \right\}, \\ &= \operatorname{argmin} \left(\frac{u_1 + \frac{C_1}{3}}{C_{3,0}}\right)^2 \\ &\quad + \sum_{t=1}^{T} \left\{ \left(\frac{v_t + \frac{C_t}{3}}{u_t + \frac{C_t}{3}}\right)^2 + \left(\frac{\frac{C_t}{3} - u_t - v_t}{v_t + \frac{C_t}{3}}\right)^2 + \left(\frac{u_{t+1} + \frac{C_{t+1}}{3}}{\frac{C_t}{3} - u_t - v_t}\right)^2 \right\}. \qquad (8) \end{aligned}$$

So, we can numerically find the values of (u_t, v_t) that minimize the total variance of monthly

[5] In the solution shown in the graphs, the objective actually was to minimize the sum of the squared changes not the percentage changes, because that allows for an analytical solution, at least in an infinite chain of linearly rising data or perfectly oscillating data: equal absolute month-to-month changes.

growth, or, stated differently, maximize the smoothness of the resulting monthly data. Note a pre-added third month from quarter 0 and, less visibly, an extra data point for month 1 of quarter T+1. Without these, the procedure has no "anchor" for the first and last observations, so the first and last consumption adjustments, u_1 and $-u_T - w_T$, respectively, can be (and will be) totally out of line. The notional initial consumption $C_{3,0}$ can start from the first revised figure and correct for one month of growth, calculated as the average growth observed between the first and very last revised figure:

$$C_{3,0} = \frac{C_{1,1}}{(C_{3,T}/C_{1,1})^{1/(3T-1)}}. \tag{9}$$

The notional extra final monthly number, $C_{1,T+1}$, can be created in the same style.

The revised consumption growth, denoted as g''_{t+1}, is the change between the second-month real consumption in the next quarter and the second-month real consumption in the present quarter,

$$g''_{t+1} := \ln(\frac{C_{2,t+1}}{C_{2,t}}) - \ln(\frac{CPI_{2,t+1}}{CPI_{2,t}}) = \ln(\frac{\frac{C_{t+1}}{3} + v_{t+1}}{\frac{C_t}{3} + v_t}) - i'_{t+1}, \tag{10}$$

where $CPI_{2,t}$ is the consumer price level of the second month in the t-th quarter. To jointly assess the effects of aggregation and the adequacy of the proposed procedure, we re-test all the models in terms of $g''_{q2,t2+h}$, and compare the new estimated EIS with the ones based on the quarterly aggregation data.

After this review of some modeling issues we now proceed to the statistical issues in estimating the models.

2 Estimating the EIS

We first discuss the estimation methods, starting from the problem of an unobservable regressor, expected inflation. For simplicity, much of that discussion is in a constant-σ framework, but the results easily carry over to our more general model. We then present our operational equations.

2.1 Estimation Methods

The empirical literature usually assumes the EIS and the RRA to be constant, and finds that the EIS is small and insignificant, suggesting that consumption growth is insensitive to the expected real interest rate and vice versa. One practical difficulty is that one ingredient of the expected real interest rate, the inflation expectation, is unobserved. A direct solution is to use survey data on expectations. For example, Hall (1988) employs the Livingston surveys on inflation expectations, and Choi (2005) similarly works with surveys of expected stock returns for the U.S. However, in most countries survey data are unavailable, while in those countries where surveys do exist, the comparability across the samples depends on the survey's scope, the quality of questionnaires, etc. So in this paper we consider the proxy method and one instrumental variable (IV) method, GMM.

The first method proxies the expected inflation $\mathrm{E}(i'_{t+1})$ by the realized inflation rate from the preceding period i_t:

$$\mathrm{E}_t(i'_{t+1}) = i'_t. \tag{11}$$

That is, when inflation is regarded as a martingale process, the expectation has become observable, and thus, the EIS links consumption growth to the difference between the interest rate and the actual inflation from last period $[R'_{t+1} - i'_t]$,

$$g'_{t+1} = -\sigma_p \delta + \sigma_p \left[R'_{t+1} - i'_t\right] + \epsilon_{g',t+1}, \tag{12}$$

where σ_p denotes the EIS estimated under the proxy method. The proxy method may give us the real rate as perceived by investors, because that's how the real rate is commonly implemented. Otherwise, however, it looks like a dubious simplification of TSLS, as stated in the intro. The instrumental variable (IV) methods relax the assumption of martingale inflation. In this paper, we consider the Generalized Method of Moments (GMM) as it combines the merits of TSLS with corrections for autocorrelation and heteroscedasticity. Frequently used IVs, in this literature, are the lag of consumption growth, the stock index return, nominal interest rate and inflation. Because consumption growth and stock return are found to have low predictability at the quarterly frequency,[6] in this paper we start from the nominal interest rate on 3-month interbank contracts and four lags of inflation. The reason why we include four lags of inflation is that the AR model shows a seasonality in the quarterly inflation: the autoregression coefficients are usually negative at the first lag but almost always positive at the fourth lag. Thus, we put all four lags into the IV group to capture the seasonality and have a good fit. In fact, the five instruments, taken together, do quite a good job in tracking the inflation rate, which is what we would need in TSLS. We need more instruments, though: in a more complete model with a stock-market variable and seasonals, each equation has six parameters to be estimated (see section 2.2), so the required number of instruments (and, therefore, orthogonality conditions)

[6] See Hall (1988) and Yogo (2004).

has to be at least six. We add three more lagged observations of the risk-free rate to the set. In all IV estimations, we use the same set of eight instruments.

2.2 Test Equations

To test for wealth-related variation in the EIS (RRA), we invoke the familiar home-bias phenomenon and accordingly proxy for changing wealth via the country's stock index. Specifically, we use the 'stock index deviation' ΔSI, a variable picked up from technical trading:

$$\Delta SI := \frac{S_t - \overline{S_{t-260,t}}}{S_t}, \tag{13}$$

where S_t is the spot price of the stock index, and $\overline{S_{t-260,t}}$ is the moving average over the past year (260 trading days, actually). In technical trading, the moving average is regarded as a reference point: when S_t is above $\overline{S_{t-260,t}}$, the deviation index ΔSI is positive, indicating a buy signal or profit signal for the market and *vice versa*. Here we simply use this the deviation index to see whether wealth has been rising or not, recently; there is no presumption that $\overline{S_{t-260,t}}$ equals $\underline{W}$. A higher ΔSI should be related to a higher risk tolerance according to the theory.

In a general model including both seasonal consumption and wealth effect, we allow σ_t to vary across quarters with a seasonal fixed effect d_q and to change with stock index deviation ΔSI,

$$\sigma_{j,t} = \beta_0 + \sum_{q=\{1,3,4\}} d_q \mathbf{1}_{q=q(t)} + \beta_w \Delta SI_{j,t} \;\; (q = 1, 3, 4). \tag{14}$$

The dummies are for observations based on quarters 1, 3 and 4. By a growth rate being 'based on quarter q' we mean that it compares quarters $q+1$ and q. Thus, the year-end peak should be reflected in the seasonal for quarter 3, d_3, and the first-quarter decline in d_4. As there is no dummy for quarter 2, β_0 estimates the base-rate RRA for a second quarter of consumption seasonality when the stock market has not moved. When we replace the σ_t from the general first-approximation model—now including a country fixed effect γ_j reflecting the nonlinearities—by this expression, we have our general state-dependent model for country j:

$$\begin{aligned} g'_{j,t+1} &= \gamma_j - \sigma_t \delta + \sigma_t [R'_{j,t+1} - E_t(i'_{j,t+1})] + \epsilon_{g'_j,t+1}, \\ &= \gamma_j + \left[\beta_0 + \sum_{q=1,3,4} d_q \mathbf{1}_{q=q(t)} + \beta_w \Delta SI_{j,t} \right] \{\delta + [R'_{j,t+1} - E_t(i'_{j,t+1})]\} + \epsilon_{g'_j,t+1}. \end{aligned} \tag{15}$$

Thus, the constant δ and expected real rate are multiplied by a time-varying slope, σ_t, which depends on consumption seasonals and on wealth levels.

The full model is implemented in steps. We start with the CRRA model, estimated via the proxy method and GMM. We then move on to the state-dependent-RRA models, again moving gradually. The first non-CRRA model we estimate is one where we only have a seasonal in the intercept. Then, we also let the slope follow the same pattern. Lastly, we look at the complete model. Below, we show the equations — labeled CRRA, s1, s2 (for seasonal models), w (for wealth-based model), and c1, c2 (for combined) — and the way the substitution elasticities are extracted:

$$(\text{CRRA}:) \quad g'_{j,t+1} = \gamma_j + \sigma[R'_{j,t+1} - E_t(i'_{j,t+1})] + \epsilon_{g'_j,t+1}; \tag{16}$$

$$(s1:) \quad g'_{j,t+1} = \gamma_j + \left[\beta_0 + \sum_{q=1,3,4} d_q \mathbf{1}_{q=q(t)}\right]\delta + \sigma_{s1}[R'_{j,t+1} - E_t(i'_{j,t+1})] + \epsilon_{g'_j,t+1}; \tag{17}$$

$$(s2:) \quad g'_{j,t+1} = \gamma_j + \left[\beta_0 + \sum_{q=1,3,4} d_q \mathbf{1}_{q=q(t)}\right]\{\delta + [R'_{j,t+1} - E_t(i'_{j,t+1})]\} + \epsilon_{g'_j,t+1}, \tag{18}$$

$$\sigma_{s2,q} = \begin{cases} \beta_0 & , \quad q = 2, \\ \beta_0 + d_q & , \quad q = 1,3,4; \end{cases} \tag{19}$$

$$(w:) \quad g'_{j,t+1} = \gamma_j + [\beta_0 + \beta_w \Delta SI_{j,t}]\{\delta + \sigma_w[R'_{j,t+1} - E_t(i'_{j,t+1})]\} + \epsilon_{g'_j,t+1}, \tag{20}$$

$$\sigma_{w,j,t} = \beta_0 + \beta_w \Delta SI_{j,t};$$

$$\overline{\sigma_w} = \text{AVERAGE}_{\forall j,t}(\sigma_{w,j,t}); \tag{21}$$

$$(c1:) \quad g'_{j,t+1} = \gamma_j + \left[\beta_0 + \sum_{q=1,3,4} d_q \mathbf{1}_{q=q(t)} + \beta_w \Delta SI_{j,t}\right]\delta + \sigma_{c1}[R'_{j,t+1} - E_t(i'_{j,t+1})] + \epsilon_{g'_j,t+1}, \tag{22}$$

$$(c2:) \quad g'_{j,t+1} = \gamma_j + \left[\beta_0 + \sum_{q=1,3,4} d_q \mathbf{1}_{q=q(t)} + \beta_w \Delta SI_{j,t}\right]\{\delta + [R'_{j,t+1} - E_t(i'_{j,t+1})]\} + \epsilon_{g'_j,t+1}, \tag{23}$$

$$\sigma_{c2,q,j,t} = \begin{cases} \beta_0 + \beta_w \Delta SI_{j,t} & , \quad q = 2, \\ \beta_0 + d_q + \beta_w \Delta SI_{j,t} & , \quad q = 1,3,4, \end{cases} ,$$

$$\overline{\sigma_{c2,q}} = \text{AVERAGE}_{\forall j,t}(\sigma_{c,j,q,t}). \tag{24}$$

2.3 Priors and Hypotheses about EIS and RRA

With additive utility the EIS also provides us with a way to measure RRA, which is one of the key numbers in asset pricing theory. To judge whether EIS-implied numbers are reasonable we need some priors.

One familiar watershed value is unity, and many economists would have an implicit prior that RRA is likely to be below unity. The reason is that only for RRA below unity (or EIS above unity, more precisely) a higher expected total future income increases its present value; and the same holds for a lower variance for total future income, in a normality or lognormality model. In the real world, the market is unlikely to be insensitive to the growth prospects, and it would be deemed even more counterintuitive if good news (more output or lower total risk of output) sent the markets down. Instead, once people perceive a higher future aggregate pay-off, given current consumption, they would probably be willing to pay more for this claim in the current period. If that is one's prior, the implied belief is that the RRA is smaller than unity.

A second benchmark to judge our estimates by is the numbers obtained from stock markets. There, one usually adopts a mean-variance approach, under which the RRA can be measured either as the market risk premium divided by the market's variance, or as the ratio of individual assets' risk premia over their covariance risk. However, these studies typically find that the value of the RRA exceeds unity, and more like 3 or more. Cooper and Kaplanis (1996), for instance, advance the range 3-to-10 as reasonable. Conditional studies, especially covariance-based ones, come up with multiples of 10, occasionally even exceeding 100. Many would think that even 7 is already anomalously high: with a moderate long-run market volatility of, say, 15%, the implied long-run risk premium would be an implausible 16% ($=0.15^2*7$). For risk premia between 6 and 11% we would need the RRAs of 3 to 5. Preferring to err on the safe side we take 10 as the maximum plausible number.

The discrepancy between the estimated price of assets' covariance risk (or even the estimated price of market variance) and what economists would deem reasonable is called the equity premium puzzle. But a stock-market-based RRA may be overestimated, for instance because it ignores dividend taxes, price pressure and other transaction costs, information issues and other deadweight costs (Cooper and Kaplanis, 1986), non-equity assets, and non-traded wealth. Thus, if we study consumption data and calculate the RRA as a reciprocal of the EIS, we can see whether the resulting the RRA is smaller than the ones from the stock markets and perhaps even lower than the threshold value of unity.

Apart from compatibility of the EIS estimates with priors, we can also wonder about international differences. We estimate a general-average EIS (see below), but also parameters for subsamples, namely a continental-European and an emerging-market one. A marked trait of the continental EU group is an extensive social safety net. How this affects RRA is unclear, *a priori*: the safety net may be a symptom of high risk aversion, but its availability may also stimulate more risk taking in one's private investments. The prior about the emerging group is also unclear. In view of their lower per capita wealth, these countries would have a higher RRA and thus, tentatively, a lower EIS. However, this view implicitly adopts a representative-consumer view, which may become less appropriate the higher the internal income inequalities are. The typical emerging-market investor may, in fact, have an income comparable to western countries, in which case little difference in RRA is to be expected.[7]

3 Empirical Results

3.1 Data

Our data cover twenty-four countries. Four are emerging countries: Brazil (BR), Mexico (MX), Russia (RU) and Thailand (TH), for which we have data from the first quarter (Q1) of 1993 to the last quarter (Q4) of 2007. The twenty developed countries contain eleven continental EU countries—Austria (AT), Belgium (BE), Denmark (DK), Finland (FI), France (FR), Germany (DE), Italy (IT), Netherlands (NL), Norway (NO), Spain (ES) and Sweden (SE)— and nine other economies, often quite heterogeneous in terms of geography, cultures, policies or economic weight: Australia (AU), Canada (CA), Hong Kong (HK), Japan (JP), New Zealand (NZ), Singapore (SG), Switzerland (CH), United Kingdom (UK) and United States (US). The sample of the developed countries starts earlier, from Q1 of 1988 to Q4 of 2007. The consumption data are quarterly household expenditures per capita, the interest rates are the three-month interbank rates on the middle date of each quarter, and the price level is the CPI of the quarter's middle month. Consistent with quarterly consumption, the inflation rate is calculated as a quarterly price change, rather than the twelve-month one conventionally reported in the media. All the data are from Datastream.

[7]If the class of investors is very different from the general population, also aggregate consumption data become less appropriate, though.

Table 1: Testing the martingale assumption on inflation

$$i'_{t,t+\Delta} = \rho_0 + \rho_1 i'_{t-\Delta,t} + \xi.$$

Europe		Emerging		The rest	
	ρ_1		ρ_1		ρ_1
AT	***-0.417	BR	***0.502	AU	*0.173
BE	***-0.383	MX	***0.622	CA	-0.014
FI	0.057	RU	0.475	HK	***0.582
FR	-0.073	TH	*0.210	JP	***-0.347
DE	-0.172			NZ	***-0.267
IT	***0.501			SG	***-0.267
NL	***-0.411			CH	**-0.215
ES	***-0.408			UK	**-0.207
DK	***-0.477			US	-0.121
NO	-0.049				
SE	***-0.435				
$\sharp(-\rho_1)$	10/11		0/4		7/9

Here, "$\sharp(-\rho_1)$" is the number of negative ρs out of total in each subsample.

Panel regressions offer the advantage of increasing observation numbers and being able to model time and space simultaneously. Therefore, we estimate the EIS with a fixed effect model, where different intercepts reflect a country's heterogeneity and the EIS (σ) is estimated via a common slope across the panel. The empirical tests are based on three pooled data sets: the whole sample (including all countries, developed and developing), and two meaningful subsets, the continental European and the emerging countries. We have discussed possible priors about the EIS for the European and emerging subsamples. In addition, the EU group is characterized by a high degree of integration. As to the emerging-market group, their consumption patterns are more noisy, being more vulnerable to the macro risks (*eg* inflation risk, policy risk, etc.) relative to developed economies. Most notably, each of the emerging countries in our sample experienced a financial crisis during the sample period: the Mexican Crisis from 1994 to 1995, Thailand's crisis in 1997 and later the crises in the Russian Federation (1998) and in Brazil (1999).

We now move on to the empirics. Most of the analysis is based on uncorrected quarterly data. The results for the constructed middle-month data are, in fact, slightly but systematically better—for anyone who believes in low risk aversions or high EIS, that is—but the method has not earned any credibility elsewhere. To set a standard for the state-dependent models we start with the traditional workhorse, the CRRA model.

Table 2: Pooled estimates of EIS under different models

Panel A: Proxy method	Whole sample		Europe		Emerging countries	
	σ_p	η_p	σ_p	η_p	σ_p	η_p
	***-0.174	-5.747	**-0.261	-3.831	**-0.284	-3.521
$F[\eta = 3,\ 10]$		*** < 3		*** < 3		*** < 3
$F[\eta = 1]$		*** < 1		*** < 1		*** < 1
Panel B: IV methods: GMM and TSLS						
	σ_{GMM}	η_{GMM}	σ_{GMM}	η_{GMM}	σ_{GMM}	η_{GMM}
	***0.107	9.346	0.120	8.333	0.223	4.484
$F[\eta = 3,\ 10]$		[$\approx< 10$]		[$\approx< 10$]		[$\approx< 10$]
$F[\eta = 1]$		*** > 1		*** > 1		*** > 1
	σ_{TSLS}	η_{TSLS}	σ_{TSLS}	η_{TSLS}	σ_{TSLS}	η_{TSLS}
	**-0.144	-6.944	0.180	5.556	-0.320	-3.125
$F[\eta = 3,\ 10]$		*** < 3		[$\approx< 10$]		*** < 3
$F[\eta = 1]$		*** < 1		*** < 1		*** < 1

"$F[\eta = 3,\ 10]$" refers to two F-tests, viz "$F[\eta = 3]$" and "$F[\eta = 10]$". Asterixes before the 10 [or after the 3] mean that 10 [or 3] is statistically too far from the estimate to be a possible true answer. Similarly, "*** < 3" means the η is clearly smaller than 3 at the 1% significant level; while an expression like [$\approx< 10$] means the η is insignificantly smaller than 10. "***", "**" and "*" are the significance level at 1%, 5% and 10%.

3.2 EIS Estimated Under CRRA Assumptions

We start from the proxy method where the actual inflation rate from last period substitutes for expected inflation $E_t(i'_{t+1})$. In Table 1 we first examine the martingale assumption of the proxy method via an AR(1) model of inflation for each country. Under a martingale, where inflation is assumed to be wandering randomly, the first-order autocorrelation coefficients, ρ_1, should be unity. However, strongly negative ρ_1s dominate: ten of eleven continental European countries have negative ρ_1s, and so have seven of nine remaining developed countries. Although there is a pattern of mild momentum in the emerging group, these ρ_1s are still much smaller than unity. So, the martingale is rejected, and the estimates from the proxy method may be unreliable. The potentially redeeming factor of this approach is that the estimation errors at the first stage of TSLS are avoided; but whether this actually means a lot remains to be seen.

Table 2 summarizes the estimates of the parameters of interest obtained from CRRA models considered in this paper. From top to bottom, the panels refer to the proxy and GMM methods, with TSLS added for comparison's sake, while from left to right we show the columns based on, respectively, the whole sample and two sub-sets, the continental European and the emerging countries. In Panel A, the EIS (σ) is estimated via the proxy method.[8] All the values (σ_p) are significantly negative, which is incompatible with risk aversion for a concave utility function. In short, the proxy method performs badly. It is not obvious whether the flawed martingale assumption is the main cause: for emerging markets, where the ρ_1s were clearly positive rather than usually negative (Table 1), we actually see the most negative estimate for the EIS among the three.

In Panel B, the EIS is first estimated under GMM. We have strikingly higher σs than under the proxy and TSLS method; in fact, all the σs become positive and two out of three are significant. The emerging countries now come up with the highest σ_{GMM} estimate, 0.535, unconvincingly suggesting that they would be the least risk-averse group.

It is tempting to blame the failure of the proxy method on the inappropriateness of the martingale assumption. Still, we have already noted that for emerging markets, where first-order autocorrelation was at least positive rather than negative, the proxy method did worse rather than better. The same insight is obtained when we consider the TSLS estimates added below the GMM results in the table. TSLS works with an augmented AR (4) model rather than assuming a martingale, but is otherwise similar to the proxy method in that the final estimates come from an OLS regression. We see that correcting the martingale assumption has a strong beneficial effect for the European group only; it hardly improves the all-country estimate and actually worsens the emerging-market one somewhat. Thus, most of the improvement in GMM must have come from the extra information from the orthogonality constraints and the more judicious weighting of the information.

To close the discussion of the CRRA model, we calculate the implied RRA as the reciprocal of the estimated EIS and test whether the RRA estimate is significantly below 3, above 3 but below 10, or above 10, conventional intervals from financial markets. The other critical value that is tested is RRA < 1, a key benchmark in the equity puzzle. From Table 2, the RRA estimates under GMM are very close to the upper bound and insignificantly different from 10, so at this stage we do not find that consumption data generate lower RRA than standard financial ones. From the equity-puzzle perspective, the estimated EISs are still too low, though, *ie* the implied RRAs are still too high. Let's consider to what extent the CRRA assumption is responsible. We start with seasonally varying utility. The CRRA/GMM estimates from the current section serve

[8] In appendix, as a complement of the pooled regressions, we report the estimates for each country, equation by equation.

as the benchmark model.

3.2.1 The Pattern of EIS with Seasonal Consumption

We take into account the consumption seasonality in two steps, first by just adding dummy variables in the intercepts (which is similar in spirit to just deseasonalising the data), and later by also allowing the slope coefficient of the real rate to follow the same pattern. Thus, the first seasonal model is

$$g'_{j,t+1} = \gamma_j + \left[\beta_{s1,0} + \sum_{q=1,3,4} d_q \mathbf{1}_{q=q(t)}\right]\delta + \sigma_{s1}[R'_{j,t+1} - E_t(i'_{j,t+1})] + \epsilon_{g'_j,t+1}, \qquad (25)$$

where γ_j is the fixed effect, and $\beta_{s1,0}$ is the intercept for quarter 2, and where the d_qs for quarter 1, 3, and 4 indicate to what extent these quarters' intercepts deviate from that of quarter 2.

We report the estimates of the dummy intercepts and common slope coefficients in Panel A of Table 3. For the whole sample and the continental-Europe subset, the dummy intercepts are significantly different from zero, while they are insignificant for the emerging countries. This result is consistent with Figures 1 to 3, where the emerging countries exhibit a weaker seasonal pattern than the developed countries. The less significant seasonality may be because all the observed emerging countries experienced a financial crisis during the sample period, which may have had a pervasive influence on the economies and weakened the seasonal pattern in consumption. Equally probable, the year-end seasonal may be a symptom of affluence. Still, regardless of significance, d_3 is in fact positive (and the most positive among the seasonal effects) in all cases, pointing towards the Christmas/New-Year holidays in the fourth quarter which boost the consumption growth based on the third quarter. In keeping with these observations, d_4 is negative everywhere, reflecting a falling consumption in the quarter following the gift season. Secondly, we find that for the developed countries, σ_{s1}s are higher than their counterparts σ_{GMM}s under the standard GMM. (In the table, an increase in an EIS estimate relative to the GMM/CRRA figure is denoted with '+', otherwise we show a '−'.) Accordingly, while the implied RRA η_{s1} estimates are still significantly above unit, they are now in the range 2-3. The estimate for the emerging-country subset, 1.869, is still the lowest, but is also very imprecise. Thirdly, the model's R^2s increase dramatically except for the emerging countries.[9]

[9] The R^2s are reported in the Appendix, equation by equation.

Table 3: Estimating state-dependent EIS

	Whole sample		Continental Europe		Emerging countries	
Panel A: Seasonal Intercepts:						
d_1		***0.012		***0.025		0.008
d_3		***0.028		***0.040		**0.018
d_4		***-0.056		***-0.050		***-0.018
σ_{s1}	***0.306	+	***0.282	+	0.535	+
η_{s1}	3.268	***[3,10]***	3.546	[3,10]***	1.869	***[1,3]***
Panel B: Full Seasonal Model:						
$\sigma_{s2,1}$	***0.132		***0.676		0.072	
$\sigma_{s2,2}$	0.012		0.002		0.030	
$\sigma_{s2,3}$	***0.691		***1.916		***0.492	
$\sigma_{s2,4}$	0.038		***-1.519		0.184	
$\overline{\sigma_{s2}}$	0.218		0.269		0.195	
$\overline{\eta_{s2}}$	4.582		3.721		5.141	
F_{eq}	***		***		—	
Panel C: wealth-related:						
$\beta_{w,0}$		***0.255		***0.237		***0.339
$\beta_{w,1}$		***0.068		***0.032		***0.076
$\overline{\sigma_w}$	0.257	+	0.238	+	0.345	+
$\overline{\eta_w}$	3.891		4.202		2.899	
Panel D: seasonal- and wealth-related intercepts						
d_1		***0.181		***0.253		*-0.009
d_3		***0.142		***0.208		0.004
d_4		***-0.018		***0.024		***-0.031
$\beta_{c1,1}$		***0.078		***0.207		***0.082
σ_{c1}	***0.671	+	***1.468	+	*0.241	+
η_{c1}	1.490	*** <[3,10]***	0.681	*** <1	4.149	***[3,10]***
Panel E: Combination of seasonal- and wealth-related model						
d_1		***0.005		0.000		0.032
d_3		***0.020		0.002		*0.036
d_4		***-0.059		***-0.057		-0.013
$\beta_{c2,0}$		***0.766		**10.130		***1.169
$\beta_{c2,1}$		***0.055		**0.052		***0.089
$\sigma_{c2,1}$	0.365		1.166		0.081	
$\sigma_{c2,2}$	0.769		10.133		1.176	
$\sigma_{c2,3}$	0.414		2.332		0.035	
$\sigma_{c2,4}$	0.032		-1.455		0.275	
$\overline{\sigma_{c2}}$	0.395	+	3.044	+	0.392	+
$\overline{\eta_{c2}}$	2.532		0.329		2.551	

"+" indicates that the estimated σ is higher than its counterpart under the standard GMM. "F_{eq}" is the F-test on the equality of the EISs across quarters in the second seasonality model. Asterixes before the 10 [or after the 3] mean that 10 [or 3] is statistically too far from the estimate to be a possible true answer. Similarly, "*** < 3" means the η is clearly smaller than 3 at the 1% significant level; while an expression like [$\approx<$ 10] means the η is insignificantly smaller than 10. "***", "**" and "*" are the significance level at 1%, 5% and 10%.

In short, for the developed countries, when we allow for the seasonality in consumption in a very simple way, both the explanatory power and the estimated EIS rise.

In the second seasonal model, we extend the seasonality to the EIS, as the formal model suggests:

$$g'_{j,t+1} = \gamma_j + \left[\beta_{s2,0} + \sum_{q=1,3,4} d_q \mathbf{1}_{q=q(t)}\right] \{\delta + [R'_{j,t+1} - E_t(i'_{j,t+1})]\} + \epsilon_{g'_j,t+1}. \tag{26}$$

In Panel B of Table 3, we report the $\sigma_{s2,q}$ and an F-test of equality of the EISs across quarters, F_{eq}. Both the t- and the F-tests confirm that the EIS has a seasonal pattern in the developed countries, but for the emerging group the evidence remains unclear. Again as before, the σs for base quarters 1 and (especially) 3 are still the highest, following the pattern of consumption growth itself. This is more consistent with a model where the exponent η fluctuates seasonally rather than the minimum consumption standard L. The average substitution elasticities are somewhat lower than those in the seasonal-intercept model (Panel A), therefore the corresponding risk-aversions are somewhat higher. This time, emerging countries do appear to have a marginally higher RRA than the other countries.

3.2.2 Wealth-related Fluctuation in EIS and RRA

The national stock-market index deviation ΔSI, our proxy for changing wealth, should positively correlate with the EIS if agents have decreasing RRA, *ie* increasing relative risk tolerance. Thus, we expect a positive $\beta_{w,1}$ in

$$\sigma_{j,t} = \beta_{w,0} + \beta_{w,1} \Delta SI_{j,t}, \tag{27}$$

with the betas estimated from

$$g'_{j,t+1} = \gamma_j + [\beta_{w,0} + \beta_{w,1} \Delta SI_{j,t}] \{\delta + \sigma_w [R'_{j,t+1} - E_t(i'_{j,t+1})]\} + \epsilon_{g'_j,t+1}. \tag{28}$$

From Panel C of Table 3, the base EIS rates, $\beta_{w,0}$, are somewhat higher than the estimates from the seasonal model, and all are significantly positive. $\beta_{w,1}$ is always positive and significant, indicating that risk tolerance seems to be higher after a rise in the stock market, as hypothesized, even though the effect is algebraically small (0.03 to 0.075).[10] When we compute average EIS numbers and their corresponding RRA estimates, we get numbers like 3 or 4.

3.2.3 Joint Impact of Seasonal Consumption and Wealth

Lastly we merge the impacts of seasonal consumption with the one of changing wealth. Again, the tests are implemented in two steps: first we give the coefficient of time preference δ a weight

[10] As the local stock market index must be an imperfect proxy for true wealth, the effect is probably underestimated.

that changes with seasonal consumption growth and wealth, while in step 2 the coefficients of both δ and the real rate are varying jointly. The first-stage model is,

$$g'_{j,t+1} = \gamma_j + \left[\beta_{c1,0} + \sum_{q=1,3,4} d_q \mathbf{1}_{q=q(t)} + \beta_{c1,1}\Delta SI_{j,t}\right] \delta + \sigma_{c1}[R'_{j,t+1} - E_t(i'_{j,t+1})] + \epsilon_{g'_j,t+1}, \quad (29)$$

where the intercepts capture both the seasonality and the direct effect, if any, from wealth variations on consumption. The relevant estimates are presented in Panel D of Table 3. The seasonal dummy variables are again significant except for the emerging subset, suggesting the existence of seasonality: d_1 and d_3 are still higher than d_4, indicating year-end holiday effects on consumption. $\beta_{c1,1}$ is significantly positive everywhere. Finally, we obtain EISs (σ_{c1}s) which, for the developed countries and the total sample, are even higher than the ones estimated from Equation [25], the numbers σ_{s1} from Panel A, while the estimate for the emerging markets is lower. The estimated RRA for continental Europe is even below unity, the first such number in this study. If one's prior is a low value for RRA in general but a higher one for less wealthy countries, these results make sense.

In the second-stage model, the time preference as well as the real rate share a state-dependent EIS coefficient:

$$g'_{j,t+1} = \gamma_j + \left[\beta_{c2,0} + \sum_{q=1,3,4} d_q \mathbf{1}_{q=q(t)} + \beta_{c2,1}\Delta SI_{j,t}\right] \{\delta + [R'_{j,t+1} - E_t(i'_{j,t+1})]\} + \epsilon_{g'_j,t+1}. \quad (30)$$

The estimates are presented in Panel E. The seasonality and stock-market coefficients are qualitatively unaltered. One unexplained effect is the hard-to-believe rise in the base rate of EIS for continental Europe following a rise in the stock market, $\beta_{c2,1} = 10$. However, note that the coefficient has a high standard error (at 10, the coefficient is significant at the 5% level only, while the other base rate estimates are about 10 times smaller but significant at the 1% level or better.). This should probably make us take the European estimated average RRA, .33, with a grain of salt; the other estimates are about 2.5.

In short, the EIS implies quite credible numbers for the RRA, at least if one takes the lower end of stock-index-based as the norm rather than unity. But in order to get these numbers one needs to take into account simple and plausible source of fluctuations in the RRA, and bear in mind that these fluctuations also affect the regression term that, in the CRRA model, would have been the constant. All these estimates are obtained from standard consumption data. In the next subsection we briefly show that our simple consumption-smoothing/intrapolation algorithm systematically increases the EIS estimates, *ie* further lowers the corresponding RRA estimate.

Table 4: Results from revised consumption growth rates: system estimates of EIS under various models

	Whole sample		Continental Europe		Emerging countries	
			Panel A: RRA, Proxy method			
σ_p	***-0.147	+	**-0.319	+	**-0.269	+
			Panel B: RRA estimated with GMM			
σ_{GMM}	***0.120	+	**0.272	+	***0.443	+
η_{GMM}	8.333		3.676		2.257	
			Panel C: Seasonality intercepts:			
d_1		***0.024		***0.039		0.014
d_3		***0.041		***0.053		**0.023
d_4		***-0.063		***-0.057		**-0.016
σ_{s1}	***0.358	+	***0.528	+	0.552	+
η_{s1}	2.793		1.894		1.812	
			Panel D: Full seasonal model			
$\sigma_{s2,1}$	***0.395	+	***1.112	+	0.047	−
$\sigma_{s2,2}$	***0.007	−	-0.002	−	0.028	−
$\sigma_{s2,3}$	***0.750	+	***2.688	+	0.484	−
$\sigma_{s2,4}$	-0.024	−	***-2.017	−	0.173	−
$\overline{\sigma_{s2}}$	0.282	+	0.445	+	0.183	−
$\overline{\eta_{s2}}$	3.546		2.246		5.464	
F_{eq}	***		**		—	
			Panel E: RRA decreasing in wealth			
$\beta_{w,0}$		***0.276		***0.403		***0.399
$\beta_{w,1}$		***0.063		***0.025		***0.080
$\overline{\sigma_w}$	0.278	+	0.404	+	0.405	+
$\overline{\eta_w}$	3.597		2.475		2.469	
			Panel F: Seasonal- and Wealth-related intercepts			
d_1		***0.281		***0.309		*-0.026
d_3		***0.217		***0.247		-0.012
d_4		***0.107		***0.164		-0.012
β_{c1}		***0.169		***0.467		***0.091
σ_{c1}	***1.294	+	**2.029	+	0.251	−
$\overline{\eta_{c1}}$	0.773		0.493		3.984	
			Panel G: Combination of Seasonal- and Wealth-related model			
d_1		***0.016		**0.008		0.022
d_3		***0.033		**0.007		*0.038
d_4		***-0.064		***-0.062		-0.014
$\beta_{c2,0}$		***0.859	+	***11.302		**1.140
$\beta_{c2,1}$		***0.053	+	***0.056		***0.092
$\sigma_{c2,1}$	0.499	+	1.677	+	0.094	+
$\sigma_{c2,2}$	0.859	+	11.306	+	1.149	−
$\sigma_{c2,3}$	0.458	+	3.025	+	0.058	+
$\sigma_{c2,4}$	0.012	−	-2.064	−	0.220	−
$\overline{\sigma_{c2}}$	0.457	+	3.486	+	0.380	−
$\overline{\eta_{s2}}$	2.188		0.287		2.632	

"+" indicate that the EIS in terms of middle-month growth is higher than the one of raw growth, otherwise we use "−". "F_{eq}" is the F-test on the equality of the EISs across quarters in the second seasonality model.

3.3 Estimates From Constructed Consumption Levels for the Mid-Quarter Month

Recall that by comparing each quarter with the preceding and the subsequent quarter, one can assess how total quarterly consumption is distributed across the three months, at least if one accepts the prior that changes tend to be smooth. Applying the algorithm, we retrieve the consumption data for each quarter's middle month, and recompute three-month growth rates. These are then subjected to the same estimation procedures as the original data.

Table 4 reports the new EISs estimates. In addition, we compare the new figures with the corresponding ones obtained from uncorrected data. If the new EIS is higher than the one obtained from the raw growth data, we denote this occurrence as "+", otherwise, we show a "–". Going from top to bottom, the test results from the revised growth data evolve in a very similar pattern as those from the raw data. Especially, the proxy method still generates negative σs in the whole sample and in the two subsets. GMM estimation, in contrast, produces positive σs, and significant ones at that. In the seasonality models we observe strong seasonality in consumption growth, except for the emerging countries; and after controlling for the seasonality, both the EIS estimate and the models' explanatory power become higher than the ones under the standard GMM for the developed countries. In the second-stage seasonality model, we find that the EIS significantly varies across quarters; in the third-to-fourth quarter growth is again the highest, even for the emerging group. In the decreasing-RRA models, stock-market sensitivities β_w are clearly positive all the time, supporting the hypothesis that the EIS is co-varying positively with wealth. Across the panels, the continental European subset consistently has the highest EISs, *ie* the lowest RRAs.

When we compare the EISs based on the middle-month growth with the ones on the quarterly growth, we find that the new estimate is typically higher than the raw one. Of the total 39 EIS estimates (not counting the means, for the models with seasonal σ), we observe 27 rises against 12 drops. If, in the models with seasonally varying EIS, we just consider the means instead of the (noisy) four quarterly estimates, we even note that 18 out of 21 σs are higher. The gains are most systematic for the all-country and continental European samples. Perhaps the emerging group has inherently less stable consumption patterns, in which a smoothing-based correction would achieve little.

This, of course, is just a first application of an otherwise untested method. While the results are encouraging, they are by no means conclusive. Thus, in the concluding section of the paper the main message of the paper is based on the raw data.

4 Conclusions

The standard model for estimating an EIS, with time-additive utility and constant relative risk aversion, is often regarded as a failure as little or no link is detected between expected real consumption growth and the real interest rate. We show that by simple modifications of the basic model—namely, accounting for seasonals and allowing a role for stock prices as proxies for wealth—produces elasticities that are quite compatible with other studies, including values implied by the more reasonable range of risk-aversion estimates obtained from CAPM tests. Our EIS estimates are obtained from GMM, after unsuccessfully trying the standard proxy for expected inflation. We also pool data from 24 countries, as single-country estimates are very imprecise and erratic. Our attempt to reduce the time-aggregation problem in consumption data, lastly, is successful in the sense that the resulting EIS estimates are even higher.

The proxy method uses the realized inflation from the preceding period as a substitute for the expected inflation. In reality, however, in the emerging countries, inflation exhibits only a weak first-order positive autocorrelation, while in the developed economies we even find significantly negative first-order autocorrelations. Moreover, for many countries we see a strong seasonal in their inflation data. Also in terms of results the proxy method performs badly, with negative σs everywhere. Unexpectedly, TSLS brings no improvement; but GMM does generate higher σs than the proxy method.

There is a clear seasonal pattern in consumption growth, with pronounced consumption growth in the last quarter relative to the third, and lower or even negative growth in the first quarter of the next year relative to the fourth in the current year. This strongly suggests a Christmas-NewYear effect. In the testing models, the seasonal dummy intercepts explain a substantial part of the variance of consumption growth, and the slopes σs turn out to be higher than before controlling for seasonality. The EIS also correlates positively with changes in wealth, as one expects under *eg* generalized power utility; this indeed is the outcome of tests where the stock index deviation ΔSI proxies for the changing wealth. Finally, when we merge the wealth impact with the seasonal impact on the intercepts, we find that the estimated EIS again rises, although some of that effect disappears when we also let the coefficient for the real rate follow the same pattern. The general conclusion is that the EIS estimates are not as small (or even negative) as is traditionally reported. By implication, the RRA numbers inferred by inverting the EIS estimates, as one can do under the assumption of time-additive utility, are by no means extravagantly high. Rather, they come up at the lower end of what one infers from market risk premia and variances. Occasionally, RRA estimates even fall below unity.

Finally, we re-run all models based on revised data, obtained by constructing a monthly figure for each quarter's middle month. The procedure assumes that monthly data are smooth, so that information about the distribution of consumption within the quarter can be gleaned from looking at the adjacent quarters. Based on a comparison of the σs in terms of revised and original consumptions, it looks as if the time aggregation problem biases the EIS downwards, notably for the developed countries.

Appendices

The CRRA/lognormal model

If we assume CRRA, the general pricing equation features the expectation of the marginal utility ratio $e^{-\theta}(1+g_{t+1})^{-\eta}$, multiplied by the future value of a bond deflated by $(1+i_{t+1})$. The resulting present value must be equal to unity, the initial investment:

$$\mathrm{E}_t\left[e^{-\theta}(1+g_{t+1})^{-\eta}\frac{1+R_{t+1}}{1+i_{t+1}}\right]=1.$$

We move the variables known at time t to the right-hand side of equation, and leave the expectation terms on the left-hand side. The result is

$$\mathrm{E}_t\left[\frac{(1+g_{t+1})^{-\eta}}{(1+i_{t+1})}\right]=e^{\theta-\ln(1+R_{t+1})}. \tag{31}$$

If consumption growth $1+g$ and inflation $1+i$ are lognormally distributed with conditional means $(\mu_{g,t},\ \mu_{i,t})$ and standard errors $(s_g,\ s_i)$, the expected values of their logs equal the lognormal means plus the Jensens inequality term, *i.c.*, $E_t(1+x)=\mu_{x,t}+\frac{1}{2}s_x^2$ with $1+x=(1+g)^{-\eta}/(1+i)$. So, $\mathrm{E}_t\left[\frac{(1+g_{t+1})^{-\eta}}{(1+i_{t+1})}\right]$ can be written as the exponential function,

$$\mathrm{E}_t\left[\frac{(1+g_{t+1})^{-\eta}}{(1+i_{t+1})}\right]=\mathrm{E}_t\left[e^{-\eta\ln(1+g_{t+1})-\ln(1+i_{t+1})}\right]=e^{(-\eta\mu_{g,t}-\mu_{i,t})+\frac{1}{2}\left(\eta^2s_g^2+2\eta\mathrm{cov}_t(g',i')+s_i^2\right)}.$$

Then we replace the expectations on the left-hand side of Equation [31] with the exponential function of the lognormal means and variance-covariances:

$$e^{\left[-\eta\mu_{g,t}-\mu_{i,t}+\frac{1}{2}\left(\eta^2s_g^2+2\eta\mathrm{cov}_t(g',i')+s_i^2\right)\right]}=e^{\theta-\ln\left(1+R_{t+1}\right)}.$$

Therefore,

$$\begin{aligned}-\eta\mu_{g,t}-\mu_{i,t}+\tfrac{1}{2}\left(\eta^2s_g^2+2\eta\mathrm{cov}_t(g',i')+s_i^2\right)&=\theta-R'_{t+1},\\ \mu_{g,t}&=\left[\tfrac{1}{2\eta}\left(\eta^2s_g^2+2\eta\mathrm{cov}_t(g',i')+s_i^2\right)-\tfrac{1}{\eta}\theta\right]+\tfrac{1}{\eta}\left(R'_{t+1}-\mu_{i,t}\right).\end{aligned} \tag{32}$$

We replace the dependent variable $\mu_{g,t}$ by its realized value g'_{t+1} by adding the error term $\epsilon_{g',t+1}$ to the right-hand side. The scaler $\frac{1}{\eta}$ is replaced with σ, which results in

$$g'_{t+1}=\left[\frac{\sigma}{2}\left(\eta^2s_g^2+2\eta\mathrm{cov}_t(g',i')+s_i^2\right)-\sigma\theta\right]+\sigma\left(R'_{t+1}-\mu_{i,t}\right)+\epsilon_{g',t+1}. \tag{33}$$

Generalisation up to a Linear Approximation

The marginal utility change $\Delta\Lambda$ can be expanded into a total differential involving its three determinants: nominal consumption level C, time t, and the price level Π, as we do in the first line of Equation [34].[11] Moreover, the marginal utility Λ is the derivative of utility with respect to consumption, so it can be rewritten as $\frac{\partial U}{\partial C}$ as we do in line two below. According to Roy's Identity, the derivative $\frac{\partial U}{\partial \Pi}$ equals $-\frac{C}{\Pi}\frac{\partial U}{\partial C}$. So, in the third line of the equation we make the substitution and expand the product, within the brackets in the fourth line, using the derivatives of $\frac{\partial U}{\partial C}$ and of $\frac{C}{\Pi}$. In the last line, we combine the second and third items, and extract $C\frac{\partial^2 U}{\partial C^2}$. The remaining expression between brackets is actually the real consumption growth:

$$\begin{aligned} \mathrm{d}\Lambda &= \frac{\partial \Lambda}{\partial t}\mathrm{d}t + \frac{\partial \Lambda}{\partial C}\mathrm{d}C + \frac{\partial \Lambda}{\partial \Pi}\mathrm{d}\Pi, \\ &= \frac{\partial^2 U}{\partial C \partial t}\mathrm{d}t + \frac{\partial^2 U}{\partial C^2}\mathrm{d}C + \frac{\partial^2 U}{\partial \Pi \partial C}\mathrm{d}\Pi, \\ &= \frac{\partial^2 U}{\partial C \partial t}\mathrm{d}t + \frac{\partial^2 U}{\partial C^2}\mathrm{d}C - \frac{\partial \frac{\partial U}{\partial C}\frac{C}{\Pi}}{\partial C}\mathrm{d}\Pi, \\ &= \frac{\partial^2 U}{\partial C \partial t}\mathrm{d}t + \frac{\partial^2 U}{\partial C^2}\mathrm{d}C - \left[\frac{\partial^2 U}{\partial C^2}\frac{C}{\Pi} + \frac{\partial U}{\partial C}\frac{\partial \frac{C}{\Pi}}{\partial C}\right]\mathrm{d}\Pi, \\ &= \frac{\partial^2 U}{\partial C \partial t}\mathrm{d}t + C\frac{\partial^2 U}{\partial C^2}\left[\frac{\mathrm{d}C}{C} - \frac{\mathrm{d}\Pi}{\Pi}\right] - \frac{\partial U}{\partial C}\frac{\mathrm{d}\Pi}{\Pi}. \end{aligned} \tag{34}$$

Lastly, we obtain the percentage change in marginal utility $\frac{\mathrm{d}\Lambda}{\Lambda}$ by dividing Equation [34] by $\frac{\partial U}{\partial C}$:

$$\begin{aligned} \frac{\mathrm{d}\Lambda}{\Lambda} &= \frac{\frac{\partial^2 U}{\partial C \partial t}\mathrm{d}t + C\frac{\partial^2 U}{\partial C^2}\left[\frac{\mathrm{d}C}{C} - \frac{\mathrm{d}\Pi}{\Pi}\right] - \frac{\partial U}{\partial C}\frac{\mathrm{d}\Pi}{\Pi}}{\frac{\partial U}{\partial C}}, \\ &= -\theta_t \mathrm{d}t - \eta_t\left[\frac{\mathrm{d}C}{C} - \frac{\mathrm{d}\Pi}{\Pi}\right] - \frac{\mathrm{d}\Pi}{\Pi}, \\ &= -\theta_t \mathrm{d}t - \eta_t\left[\mathrm{d}\ln\frac{C}{\Pi}\right] - \mathrm{d}\ln\Pi, \end{aligned} \tag{35}$$

where $\theta_t := -\frac{\partial^2 U/(\partial C \partial t)}{\partial U/\partial C}$ measures time preference, and $\eta_t := -C\frac{\partial^2 U/\partial C^2}{\partial U/\partial C}$ measures the RRA. As indicated by the time subscripts, neither of the two parameters is necessarily constant.

According to Equation [2] and Equation [35], if we can discretize the differentials, we obtain

$$\mathrm{E}_t\left[1 - \theta_t \Delta t - \eta_t\left(\Delta\ln\frac{C}{\Pi}\right) - \Delta\ln\Pi\right] \approx \frac{1}{1+R_{t+1}},$$

Now $\frac{1}{1+R_{t+1}}$ approximately equals $1 - \ln(1+R_{t+1})$, so we can simplify the above into

$$\mathrm{E}_t\left[\theta_t \Delta t + \eta_t\left(\Delta\ln\frac{C}{\Pi}\right) + \Delta\ln\Pi\right] \approx \ln(1+R_{t+1}).$$

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[11] The second-order terms in an Ito expansion are omitted, here, for simplicity; the second order terms are similar to the Jensen's Inequality term in the CRRA-lognormal model.

Next we move time preference $\theta_t \Delta t$ and expected inflation $\mathrm{E}_t(\Delta \ln \Pi)$ to the right-hand side, and divide the equation by $-\eta_t$. So, in Equation [36] the real consumption growth $\left(\Delta \ln \frac{C}{\Pi}\right)$ is the dependent variable, explained by time preference, RRA, the interest rate and expected inflation:

$$\begin{aligned} \mathrm{E}_t\left(\Delta \ln \frac{C}{\Pi}\right) &\approx -\frac{1}{\eta_t}\theta_t \Delta t + \frac{1}{\eta_t}\left[\ln(1+R_{t+1}) - \mathrm{E}_t(\Delta \ln \pi)\right], \\ \mathrm{E}_t(g'_{t+1}) &= -\sigma_t \theta_t + \sigma_t \left[R'_{t+1} - \mathrm{E}_t(i'_{t+1})\right]. \end{aligned} \tag{36}$$

In the second line, above, the scaler $\frac{1}{\eta_t}$ has been replaced by σ_t; Δt is normalized to unity; and real growth $\Delta \ln C/\Pi$ and inflation $\Delta \ln \pi$ have been denoted by g'_{t+1} and i'_{t+1}, respectively. The prime notation indicates a continuously compound rate, *eg* g'_{t+1} and i'_{t+1}, to distinguish them from the simple growth rates, *eg* $g' = \ln(1+g)$.

All of the above still refers to conditional expectations. The realized values equal the expectations plus shocks,

$$\begin{aligned} g'_{t+1} &= \mathrm{E}_t(g'_{t+1}) + \epsilon_{g',t+1}, \\ i'_{t+1} &= \mathrm{E}_t(i'_{t+1}) + \epsilon_{i',t+1}. \end{aligned}$$

Thus, if we add unexpected consumption growth on both sides, Equation [36] describes realized growth g'_{t+1} as follows:

$$g'_{t+1} = -\sigma_t \theta_t + \sigma_t \left[R'_{t+1} - \mathrm{E}_t(i'_{t+1})\right] + \tilde{\epsilon}_{g',t+1}. \tag{37}$$

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Appendix: Estimates of the Models: Equation by Equation

Table 5: Estimating the EIS in the proxy method

$$g'_{t+1} = -\sigma\delta + \sigma\,[R'_{t+1} - i'_t)] + \epsilon_{g',t+1}.$$
$$i'_{t+1} = \rho_0 + \rho_1 i'_t + \xi.$$

Panel A: Continental Europe			Panel C: the Rest Developed			Panel C: Emerging		
	σ	ρ_1		σ	ρ_1		σ	ρ_1
AT	***-2.271	***-0.417	AU	-0.234	*0.173	BR	-0.428	***0.502
BE	*-0.578	***-0.383	CA	***-0.275	-0.014	MX	***-0.763	***0.622
FI	***-3.046	0.057	HK	-0.089	***0.582	RU	0.093	0.475
FR	**-0.900	-0.073	JP	**-0.878	***-0.347	TH	***-0.528	*0.210
DE	***-2.141	-0.172	NZ	-0.138	***-0.267			
IT	0.296	***0.501	SG	-0.102	***-0.267			
NL	-0.028	***-0.411	CH	-0.021	**-0.215			
ES	**-1.152	***-0.408	UK	***-0.491	**-0.207			
DK	**0.769	***-0.477	US	-0.097	-0.121			
NO	0.194	-0.049						
SE	***2.300	***-0.435						
$\overline{\sigma}$	-0.596		$\overline{\sigma}$	-0.258		$\overline{\sigma}$	-0.407	
$\sharp(+/N)$	4/11	2/11	$\sharp(+/N)$	0/9	2/9	$\sharp(+/N)$	1/4	4/4

Here, "$\overline{\sigma}$ is the average σ and "$\sharp(+/N)$" is the number of positive σs over the total number of cross-section in each panel.

Table 6: instrumental variable methods

	First-stage regression R^2_{s1}			σ_{TSLS}		$Z' = R'$, $i_{-1 \text{ to } 4}$			
	$Z' = R', i_{-1 \text{ to } 4}$	$Z' = R'$	$Z' = i_{-1 \text{ to } 4}$	$Z' = R'$	$Z' = i_{-1 \text{ to } 4}$	σ_{TSLS}	$\sigma_{TSLS} > \sigma_{proxy}$	σ_{GMM}	$\sigma_{GMM} > \sigma_{TSLS}$
Panel A: Continental Europe									
AT	0.67	0.22	0.58	0.171	***2.386	***1.776	T	***4.598	T
BE	0.70	0.62	0.17	0.049	* 1.465	0.721	T	***1.273	T
FI	0.72	0.70	0.19	***-2.114	-0.292	**-1.540	T	-0.775	T
FR	0.79	0.80	0.06	-0.193	0.261	0.323	T	0.261	F
DE	0.51	0.19	0.45	0.751	***4.436	***3.120	T	***6.497	T
IT	0.77	0.77	0.31	0.136	0.542	0.350	T	-0.010	F
NL	0.75	0.47	0.25	0.342	-0.493	0.249	T	-0.043	F
ES	0.79	0.55	0.59	-0.304	***2.537	***1.416	T	***1.716	T
DK	0.86	0.71	0.03	0.041	*-7.552	-0.750	F	***-2.002	F
NO	0.52	0.56	0.33	-0.697	-0.835	-1.260	F	-2.522	F
SE	0.58	0.32	0.26	0.031	***-7.845	***-2.713	F	***-4.598	F
$\overline{\sigma}$				-0.169	-0.490	0.154		0.400	
$\sharp(+/N)$, $\sharp(T/N)$				5/11	6/11	7/11	8/11	5/11	5/11
Panel B: Developed Countries — The Rest Developed									
AU	0.35	0.39	0.08	-0.254	-0.069	0.223	T	-0.165	F
CA	0.39	0.41	0.20	***-0.497	-0.307	***-0.416	F	**-0.301	T
HK	0.66	0.02	0.36	-2.137	-0.128	-0.375	F	**-0.545	F
JP	0.56	0.21	0.51	1.827	1.586	**2.364	T	*1.536	F
NZ	0.38	0.47	0.13	-0.387	0.289	0.057	T	-0.135	F
SG	0.30	0.27	0.06	*-1.277	-1.055	-0.880	F	-0.615	T
CH	0.50	0.23	0.43	-1.078	-0.038	-0.483	F	-0.744	F
UK	0.66	0.18	0.58	-0.908	0.158	0.100	T	-0.608	F
US	0.48	0.47	0.11	**-0.287	**-0.541	**-0.250	F	**-0.212	T
$\overline{\sigma}$				-0.555	-0.012	0.038		-0.199	
$\sharp(+/N)$, $\sharp(T/N)$				1/9	3/9	4/9	4/9	1/9	3/9
Panel C: Emerging Countries									
BR	0.51	0.41	-0.05	-0.663	0.703	-0.186	T	-0.175	T
MX	0.34	0.26	0.35	*-0.911	-0.397	-0.258	T	0.238	T
RU	0.06	-0.02	0.08	-16.428	0.583	*0.619	T	***0.849	T
TH	0.12	0.17	0.07	***-1.940	***-1.841	***-1.854	F	**-1.961	F
$\overline{\sigma}$				-4.986	-0.238	-0.418		-0.263	
$\sharp(+/N)$, $\sharp(T/N)$				0/4	2/4	1/4	3/4	2/4	3/4
Total (+) or (*T*)				6/24	11/24	12/24	15/24	8/24	11/24

R^2_{s1} is the R^2 of the first-stage model. "$\sigma_{TSLS} > \sigma proxy$" means that value of the σ estimated from the TSLS is larger than the proxy method, and if that is true, denoted with "T", otherwise, "F". In the same rule, "$\sigma_{GMM} > \sigma_{TSLS}$" indicates the σs with the GMM larger than the TSLS estimate.