Business Cycles and Regime-Shift Risk

Wei Yang[1]

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[1]William E. Simon Graduate School of Business Administration, University of Rochester, Rochester, NY 14627. wei.yang@simon.rochester.edu.

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Abstract

The consumption growth data strongly favor a two-regime specification. The high volatility, low growth regime is associated with deep recessions: the Great Depression, the recession of 1937–1938, the post-war recession of 1945, and the most recent financial crisis. I develop parsimonious models in which (i) consumption and dividend growth follow regime-switching dynamics, (ii) the regime characteristics are consistent with the empirical evidence from the consumption growth data, and (iii) the risks associated with regime shifts are priced in asset markets. The models explain major regime-dependent asset market phenomena. Regime-shift risk exhibits the dominant influence on asset prices: It generates a high equity premium, and also induces time-varying risk premiums and explains the return predictability.

1 Introduction

Business cycles are recurrent phases of expansions and contractions occurring at about the same time in many economic activities [Burns and Mitchell (1946)]. In this paper I report empirical evidence of business cycle related regimes in consumption growth, and present parsimonious models to demonstrate the strong influence of regime shifts on asset prices. The models explain major regime-dependent asset market phenomena, and show that regime-shift risk induces both a high and time-varying equity premium.

It is now well established that the conditional distributions of macroeconomic and asset-price series change with the stage of the business cycle, and many have argued that this state dependence is well described by parsimonious regime-switching models.[1] Yet most of the preference-based asset pricing literature has focused on single-regime models, and studies that do incorporate regimes[2] have relied almost exclusively on asset market data to identify the regimes.[3] In contrast, in this paper I develop preference-based asset pricing models in which (i) the conditional distribution of consumption growth has shifting regimes, (ii) the characteristics of these regimes are consistent with the empirical evidence from a long history of data on U.S. consumption growth, and (iii) the risks associated with changes in consumption-growth regimes are priced in asset markets. I document that major features

[1] For descriptive regime-switching models on aggregate economic series, see Hamilton (1989), Lam (1990), Goodwin (1993), and Filardo (1994), among others. For stock returns, see Turner, Startz, and Nelson (1989), Cai (1994), Hamilton and Susmel (1994), Hamilton and Lin (1996), Maheu and McCurdy (2000), and Perez-Quiros and Timmermann (2001), among others. For interest rates, see Garcia and Perron (1996), Gray (1996), Ang and Bekaert (2002a), and Ang and Bekaert (2002b), among others.

[2] For regime-switching dynamic term structure models, see Bansal and Zhou (2002), Bansal, Tauchen, and Zhou (2004), Dai, Singleton, and Yang (2007), and Ang, Bekaert, and Wei (2008), among others. For regime-switching asset pricing studies on stock returns, see Cecchetti, Lam, and Mark (1990), Kandel and Stambaugh (1990), Kandel and Stambaugh (1991), Cecchetti, Lam, and Mark (1993), Abel (1994), Whitelaw (2000), Lettau, Ludvigson, and Wachter (2008), Ozoguz (2009), and Ghosh and Constantinides (2010), among others.

[3] For example, regime-switching dynamic term structure models identify regimes from bond yields. As another example, in a recent study on return predictability, Ghosh and Constantinides (2010) identify regimes largely from asset pricing data. Both the empirical properties of the regimes and the regime classification in their study are very different from those in my paper.

of the conditional distributions of asset prices differ across the consumption-growth regimes that emerge from my historical analysis, and show that the models in the paper are able to explain these regime-dependent asset market phenomena.

I present empirical evidence that the annual consumption growth data of 1929–2010 (the top panel of Fig. 1) strongly reject a single-regime specification in favor of two regimes: a persistent regime with low volatility and high growth, and a less persistent regime with high volatility and low growth. Further, the key empirical properties of dividend growth and asset market data differ substantially across the two consumption-growth regimes. The high volatility regime (the bottom panel of Fig. 1) is associated with deep recessions: It spans over the Great Depression, the recession of 1937–1938, the post-war recession of 1945, and the most recent financial crisis. Measured by the peak-to-trough decline in the aggregate output, these are the four deepest recessions during the sample period.

My model results suggest that regime-shift risk exerts the dominant influence on asset prices. Regime-shift risk accounts for more than 80% of the variance of the pricing kernel in the low volatility regime, and more than 60% in the high volatility regime. The effect of regime-shift risk is particularly strong during good times. The equity premium in the low volatility regime is almost entirely attributed to regime-shift risk: The regime-switching models generate average equity premiums above 6%; without regime shifts, single-regime models that use the same parameters generate equity premiums much lower than 1%.

My models allow for regime-dependent risk aversion, and incorporate the economic insight that investors are more risk averse during recessions. The model results show that the equity premium in the low volatility regime is more sensitive to risk aversion in the high volatility regime than that in the low volatility regime. This asymmetric effect necessarily arises through the regime-shift risk channel.

Taken together, consumption-growth regimes provide a useful setting to investigate the asset pricing implications of business cycles. Regimes suggest persistent variations in the mean and the volatility, and thus corroborate the emphasis of the long-run risk study by Bansal and Yaron (2004). Regime-dependent risk aversion is also related to the habit formation model of Campbell and Cochrane (1999), in which risk aversion varies with surplus consumption. Different from existing studies, key modeling features in my paper arise in consumption-growth regimes, for which there is strong empirical support in the data. Further, the model results suggest that regime shifts, rather than within-regime variations, are the dominant driver of asset prices.[4] Regime shifts in my paper also present substantial differences from the low-probability, extreme left-tail events proposed in the studies of rare disasters.[5] In my paper, regimes are identified and the characteristics are also estimated from the observed consumption growth data. The high volatility regime occurs with a higher probability than that typically assumed for rare events, and is less destructive than disasters are usually calibrated to be.[6]

I investigate two models to highlight the contributions of regime-shift risk to the equity premium and return predictability, respectively. In the first model, consumption and dividend growth are correlated i.i.d. shocks within each regime, while the second model incorporates common expected components in the growth rates within each regime.[7] The first model highlights that regime-shift risk generates a high equity premium, and the second model further demonstrates that regime-shift risk induces time-varying risk premiums. Both models are parsimonious: the regime transition probabilities are constant; the growth rate volatilities are constant within each regime; and the agent knows the true regime. These features preclude additional time-varying mechanisms, and sharpen the focus on the asset

[4]Regime shifts in my paper are also related to jumps in Bansal, Kiku, and Yaron (2010), Bansal and Shaliastovich (2011), and Drechsler and Yaron (2011), and the bad environment-good environment framework of Bekaert and Engstrom (2010).

[5]For example, Rietz (1988), Veronesi (2004), Barro (2006), and Wachter (2011), among others.

[6]Over the entire sample period of 81 years, there are 16 years of the high volatility regime. The mean of 0% and the standard deviation of 4% are in line with the worst realized consumption growth of -8% (in 1932) in the high volatility regime (which is also the worst in the entire sample).

[7]In a single-regime setting, Bansal and Yaron (2004) study the asset pricing effects of the predictable components in consumption and dividend growth.

pricing implications of regime shifts in economic fundamentals.

Regime-shift risk is priced because with Epstein-Zin preferences, the agent cares about consumption wealth, which is high in the low volatility regime, and low in the high volatility regime. Stocks are risky because the stock price is also high in the low volatility regime and low in the high volatility regime. The positive co-variation between the stock price and consumption wealth across the two regimes, in conjunction with the agent's preference for early resolution of uncertainty, generate a high equity premium.

The size of the equity premium is determined by the differences in both consumption wealth and the stock price between the two regimes.[8] Time-varying differences thus lead the equity premium to vary over time. In the model with expected growth components, consumption wealth and the price-dividend ratio increase with the expected component, and are much more sensitive in the high volatility regime due to the higher loadings of the growth rates on the expected component. Thus, an increase in the expected component not only raises the stock price, but also shrinks the gaps in consumption wealth and the stock price between the two regimes, which, in turn, lowers the equity premium.[9] This mechanism gives rise to the negative relation between the price-dividend ratio and the equity premium, even though the growth rate volatilities are constant within each regime. In addition, the model also generates the volatility feedback effect.

The calibrated models are capable of replicating the key features of the regime-dependent asset market phenomena in the empirical data. In the high volatility regime, both models generate lower risk-free rates, higher stock return volatilities, and lower price-dividend ratios; the model with expected components further generates a higher volatility for the risk-free rate, lower autocorrelations for the price-dividend ratio, and more negative slope coefficients

[8]Specifically, the difference in consumption wealth between the two regimes determines the market price of regime-shift risk. The difference in the stock price between the two regimes measures the stock return exposure to regime-shift risk.

[9]Therefore, both the market price of and the return exposure to regime-shift risk are time-varying.

and larger R^2 in the excess stock return predictions by the price-dividend ratio. All these model implications are consistent with the regime-dependent empirical results.

The rest of the paper proceeds as follows. In Section 2, I report the empirical results. Section 3 specifies the regime process and the agent preferences. Sections 4 and 5 present the details of the asset pricing models and compare the model implications with the empirical data. Specifically, Section 4 studies the regime-switching model with i.i.d. consumption and dividend growth within each regime, while Section 5 incorporates expected growth components into the dynamics. Section 6 concludes, and the appendices collect additional details.

2 Regimes in empirical data

The annual consumption data are from the Bureau of Economic Analysis (BEA). Following the convention in the literature, consumption is the sum of real personal consumption expenditures on nondurable goods and services, and divided by the population to obtain per capita values. The stock market returns are value-weighted annual returns for NYSE, and the risk-free rates are 3-month T-bill rates, both adjusted for inflation. All asset data series are obtained from the CRSP. Year-end price-dividend ratios and annual real dividend growth rates are computed from the value-weighted annual returns for NYSE with and without distributions, and adjusted for inflation. The sample period is 1929–2010, and thus 1930–2010, or 81 years, for the growth rates.

2.1 Two-regime descriptive model of consumption growth

I estimate a simple, two-regime descriptive model for annual consumption growth rates, in which the mean and the standard deviation are different across the two regimes, and the regime transition probabilities are constant.[10] The model is estimated using maximum

[10] I also estimate two-regime descriptive models in which the regime transition probabilities vary with the price-dividend ratio or the risk-free rate, and find that the results on time-varying regime transition

likelihood. Hamilton (1994) provides the standard reference on the construction of the likelihood function and the estimation. Table 1 presents the estimation results and compares with the single-regime specification.

I test the single-regime null hypothesis using a simulation approach.[11] I simulate 10,000 samples using the single-regime parameter estimates. For each sample, I estimate both the two-regime and the single-regime models and compute the likelihood ratio. All the likelihood ratios obtained from the simulated samples are much lower than that obtained from the empirical data. I thus conclude that the single-regime null hypothesis is strongly rejected with a p-value less than 10^{-4}.[12]

The two regimes differ considerably in the model parameter estimates. In the single-regime model, the volatility of consumption growth is about 2%. In the two-regime model, the volatilities are about 1% in one regime, and almost 4% in the other. I thus label them low and high consumption volatility regimes. In the high volatility regime, consumption growth also has a lower mean of almost 0%; in the low volatility regime, the mean is more than 2%. Consistent with existing studies on regime-switching models, the within-regime transition probability is lower for the high volatility regime; the low volatility regime is more persistent.

Following Hamilton (1994), I compute the smoothed regime probabilities implied by the estimation results. The bottom panel of Fig. 1 plots the smoothed probability of the high consumption volatility regime, and the shaded areas mark the NBER-dated recessions. The years for which the probability is greater than 0.5 are classified as the high volatility regime, and they span over three segments: 1930–1938, 1945–1948, and 2008–2010. These three

probabilities are insignificant.

[11] The standard likelihood ratio test does not satisfy usual regularity conditions, because under the null hypothesis there are unidentified nuisance parameters (e.g., the transition probabilities). See Hansen (1992) and Garcia (1998) for more details.

[12] I also estimate a three-regime descriptive model. The simulated likelihood ratio test indicates that the two-regime model cannot be rejected at conventional confidence levels in favor of the three-regime model.

segments correspond to the Great Depression and the recession of 1937–1938, the post-war recession of 1945, and the most recent financial crisis.

When measured by the peak-to-trough decline in the aggregate output, these recessions are the four deepest among all the recessions since the Great Depression. The real GNP data in Balke and Gordon (1986) indicate that the peak-to-trough declines are -36.2% for the Great Depression, -10.0% for the recession of 1937–1938, and -14.5% for the recession of 1945. The real GDP data from the BEA indicate a peak-to-trough decline of -5.1% for the most recent financial crisis. In short, the high consumption volatility regime is associated with severe economic contractions.

In between the three episodes of the high volatility regime, there are two stretches of the low-volatility regime: the World War II years of 1939–1944 and the post-war 1949–2007. The former coincides with the wartime expansion. The latter is more than 50 years that cover the post-war economic boom and the long expansions of 1980s, 1990s, and 2000s. During this period, there are three small bumps in the high volatility regime probability around the recessions of 1969–1970, 1980, and to 1990–1991.[13]

Lastly, the stationary distribution of the low and high regimes implied by the transition probabilities is 81.5%:18.5%, consistent with 65 years of the low volatility regime and 16 years of the high volatility regime.

2.2 Empirical data properties by regime

Table 2 reports the key summary statistics of the annual consumption and dividend growth rates within the two consumption volatility regimes as classified by the smoothed regime probabilities. The means and the volatilities of consumption growth are almost identical to, and thus confirm, the maximum likelihood estimates. Consumption growth exhibits a

[13]See Section 6 for the estimation of a two-regime descriptive model on the post-war quarterly consumption growth data.

significant first-order autocorrelation of 0.32 in the low volatility regime. In the high volatility regime, the first-order autocorrelation of 0.47 is even higher.[14]

Dividend growth also exhibits a higher standard deviation and a lower mean in the high consumption volatility regime. Specifically, the means and the standard deviations are about -2% and 20% in the high regime, and about 2% and 10% in the low regime. The first-order autocorrelations are similar across the two regimes. Finally, the correlation between dividend and consumption growth rates is much higher in the high volatility regime.[15]

Taken together, there are substantive differences in the empirical properties of economic fundamentals across the two consumption-growth regimes. As corroborating evidence, the growth rates of GDP and industrial production also exhibit higher volatilities and lower means in the high consumption volatility regime. For the GDP growth, the means and the standard deviations are -1.47% and 7.63% in the high regime, and 2.92% and 3.58% in the low regime. For industrial production growth, the means and the standard deviations are -2.81% and 15.17% in the high regime, and 4.50% and 6.13% in the low regime.

Panel A of Table 3 reports the key properties of asset market data within the two consumption-growth regimes. The equity premium is high in the low volatility regime. In the high volatility regime, the realized average excess stock return is negative but insignificant. The large standard error is due to the large return volatility in this regime. Indeed, the stock return volatility is about twice as high in the high volatility regime as that in the low volatility regime. The risk-free rate is low and its volatility is high in the high volatility regime. Finally, the price-dividend ratio is higher, more volatile, and exhibits higher autocorrelations in the low volatility regime. Panel B of Table 3 reports the excess stock return

[14]Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007) document that consumption growth is persistent. Their results are based on the implicit assumption that the data are of a single regime.

[15]I also estimate a two-regime descriptive model using the data on consumption and dividend growth together. The regime classification implied by the joint estimation is almost identical to that from the estimation using consumption growth only. Dividend growth is much more volatile than consumption growth. Hence, as a noisy signal, its marginal contribution is small in differentiating the regimes.

predictions by the price-dividend ratio. In both consumption-growth regimes, the slopes are negative, and the slope magnitude and the R^2 increase with the horizon. In addition, the magnitudes of the slopes and the R^2 values appear to be larger in the high volatility regime.

In summary, there are also substantive differences in the empirical properties of the asset market data across the two consumption-growth regimes. In the following, I present representative agent models with regime-switching dynamics of growth rates, and explore the capabilities of the calibrated models in explaining the regime-dependent empirical asset market phenomena.

3 Model setup

The regime $s_t \in \{\mathrm{L}, \mathrm{H}\}$ follows a Markov chain with a constant transition probability matrix

$$\Pi = \begin{bmatrix} \pi^{\mathrm{LL}} & \pi^{\mathrm{LH}} \\ \pi^{\mathrm{HL}} & \pi^{\mathrm{HH}} \end{bmatrix}, \qquad \pi^{i\mathrm{L}} + \pi^{i\mathrm{H}} = 1, \quad i \in \{\mathrm{L}, \mathrm{H}\}. \tag{1}$$

The regime switching process is conditionally independent of the consumption and dividend growth dynamics specified below. As in standard regime-switching dynamic term structure models,[16] the representative agent knows the true regime.

For convenience, define the indicator variable

$$\mathbb{1}^j_{t+1} = 1 \quad \text{if and only if} \quad s_{t+1} = j, \quad j \in \{\mathrm{L}, \mathrm{H}\}, \tag{2}$$

and the shorthand notation for the conditional expectation

$$E^i_t[\,\cdot\,] = E_t[\,\cdot\,|s_t = i]. \tag{3}$$

For a random variable whose value depends on s_{t+1},

$$y_{t+1} = y^j_{t+1}, \quad \text{when } s_{t+1} = j, \quad j \in \{\mathrm{L}, \mathrm{H}\}, \tag{4}$$

[16]See footnote 2 for regime-switching dynamic term structure studies.

it follows that

$$y_{t+1} = \sum_j \mathbb{1}^j_{t+1} y^j_{t+1}, \tag{5}$$

$$E^i_t[y_{t+1}] = \sum_j \pi^{ij} E^i_t[y^j_{t+1}]. \tag{6}$$

The agent exhibits Epstein-Zin preferences with regime-dependent risk aversion. Given $s_t = i$, the recursive utility function is

$$U_t = \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \Big(E_t[U_{t+1}^{1-\gamma^i}] \Big)^{\frac{1-\frac{1}{\psi}}{1-\gamma^i}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{7}$$

Here, C_t is consumption, $0 < \delta < 1$ is the time discount factor, γ^i is risk aversion, ψ is the elasticity of intertemporal substitution. The risk aversion parameter, which characterizes concerns over atemporal variations across states, depends on the current regime $s_t = i$. The other two parameters, both characterizing intertemporal choices, are the same across the regimes. Following derivations very similar to those in Epstein and Zin (1989), one can show that the log pricing kernel is [17]

$$m_{t+1} = \log M_{t+1} = \theta^i \log \delta - \frac{\theta^i}{\psi} \Delta c_{t+1} + (\theta^i - 1) r_{c,t+1}, \qquad \theta^i = \frac{1-\gamma^i}{1-\frac{1}{\psi}}. \tag{8}$$

The pricing kernel is driven by log consumption growth Δc_{t+1} and the log return on consumption wealth,

$$r_{c,t+1} = \log \frac{P_{c,t+1} + C_{t+1}}{P_{c,t}}, \tag{9}$$

where $P_{c,t}$ is the ex-dividend price of the entire future stream of consumption starting at C_{t+1} — in other words, the ex-dividend consumption wealth or the ex-dividend price of the consumption claim. With Epstein-Zin preferences, the agent not only cares about consumption at $t+1$, but also consumption wealth at $t+1$, which involves consumption of $t+2$, $t+3$, and so on. As shown later, regime-shift risk results from the agent's concerns over

[17] The derivations are available upon request. Two features are critical in obtaining the result in Eq. (8). First, the risk aversion parameter varies only with the regime, and is thus independent of the consumption process. As a result, the agent's utility is a homogeneous function of first degree with respect to wealth. Second, the time discount factor and the elasticity of intertemporal substitution, both characterizing intertemporal choices, are the same across the regimes.

future consumption wealth.

I obtain log-linear approximate analytical solutions to demonstrate the model mechanisms, in particular the intuition on regime-shift risk. The details of the approximate solutions are presented in the appendices. In single-regime studies, the analytical solutions build on the approximation that after log-linearization, the log pricing kernel and the log return are normal random variables. With regimes, they become mixtures of normals. Appendix A derives the results on the mean, the variance, and the covariance for mixtures of normals. In particular, the well-known result for a normal random variable,

$$\log E[\exp(X)] = E[X] + \frac{1}{2}\,\mathrm{var}[X],$$

is shown to be a second-order approximation when X is a mixture of normals.

To gauge the quantitative asset pricing performance, I calibrate the model parameters, obtain numerical solutions,[18] and conduct simulations. Following the convention in the literature, the calibration and the solution are at the monthly interval. The calibration of the growth rate parameters in the two models will be discussed separately below. For the regime process, the transition probability matrix Π is calibrated so that Π^{12}, which is the transition matrix over a 12-month horizon, matches the estimates obtained from the annual empirical consumption growth data.

For Esptein-Zin preferences, the risk aversion parameters are chosen to replicate the empirical equity premium. I incorporate the economic insight that investors are more risk averse during bad times, and set the risk aversion parameter in the high volatility regime to be 50% higher than that in the low volatility regime. Specifically, the parameters are 10 and 15, respectively, in the first model, and 6 and 9, respectively, in the second model. The time discount is smaller than 1, and the elasticity of intertemporal substitution is 1.5. As in Bansal and Yaron (2004), the models in this study rely on $\psi > 1$ to generate small and

[18] I use the projection method in Judd (1998) with cubic splines. Expectations are evaluated using Gaussian quadrature.

smooth risk-free rates.[19] Taken together, the agent prefers an early resolution of uncertainty.

To compare with the empirical results, I simulate 81×12 months of growth rates and asset market data. Growth rates are simulated following the calibrated dynamics, while asset market data are simulated following the numerical solutions. I time-aggregate the monthly simulated data to the annual frequency. For each simulated annual series, I estimate the two-regime descriptive model on consumption growth as is done on the empirical data. Then I use the smoothed regime probabilities to classify the low and high consumption volatility regimes. The maximum likelihood estimates of the simulated consumption growth and the regime-specific properties of the simulated data are compared with those of the empirical data. To facilitate the comparison, I compute the averages of the simulation results not only over all the 30,000 simulated samples, but also for about 1,000 simulated samples that end up with 65 years of the low volatility regime and 16 years of the high volatility regime, just like in the empirical data. Over the latter 1,000 samples, I also find the 5th and the 95th percentiles. These bounds are tighter than those over the entire 30,000 series, and are likely more relevant in assessing the model fit to the empirical data.

The models are solved and simulated with the perspective of the agent, who knows the true regime at the monthly interval. To compare the simulated data with the empirical data at the annual frequency, I take the perspective of the econometrician, and classify the regimes using the estimated regime probabilities. The simulations provide an opportunity to assess the accuracy of the econometrician's regime inference. For each year in the simulated annual consumption growth data, I define the "true" regime probabilities as proportional to the number of the months spent in the two regimes. For both models, the average correlations between the "true" and the estimated regime probabilities are about 0.74. Hence, the simple two-regime descriptive model appears to perform well.

[19]Bansal and Yaron (2004) present more detailed discussions regarding the debate whether ψ is above or below 1.

4 Model with i.i.d. growth rates

Conventional models in a single-regime setting often specify consumption and dividend growth as correlated i.i.d. shocks. In this section, I study a straightforward two-regime extension of the conventional models. In my model, within each regime, consumption and dividend growth are random walk shocks of constant volatilities, the correlation between the shocks is also constant, and the model parameters are different across the two regimes. As shown below, in the model the stock price co-varies positively with the agent's consumption wealth, and this generates a high equity premium.

4.1 Growth rate dynamics

Given $s_t = i$, consumption and dividend growth are

$$\Delta c_{t+1} = \log \frac{C_{t+1}}{C_t} = \mu_c^i + \sigma_c^i \varepsilon_{c,t+1}, \tag{10}$$

$$\Delta d_{t+1} = \log \frac{D_{t+1}}{D_t} = \mu_d^i + \sigma_d^i \varepsilon_{d,t+1}, \tag{11}$$

$$E_t^i[\varepsilon_{c,t+1}\varepsilon_{d,t+1}] = \chi^i, \tag{12}$$

$$\varepsilon_{c,t+1}, \varepsilon_{d,t+1}, \sim \text{i.i.d. } N(0,1).$$

Here, μ_c^i and μ_d^i are the mean parameters, σ_c^i and σ_d^i are the volatility parameters for the shocks, and χ^i is the correlation between the shocks.[20]

The calibrated parameters are reported in Table 4. Table 5 presents the maximum likelihood parameter estimates from the simulated consumption growth data, and Table 6 tabulates the regime-specific averages, standard deviations, and correlations of the simulated consumption and dividend growth data. The simulation results well replicate their counterparts in the empirical data.

[20] The parameters in the dynamics depend on $s_t = i$, but not on s_{t+1}. The same timing convention has been adopted in Cecchetti, Lam, and Mark (1993) and Dai, Singleton, and Yang (2007). As noted in the latter study, the s_t convention is more readily comparable with continuous-time models, and in the continuous time limit, the s_t and s_{t+1} formulations are equivalent.

4.2 Pricing kernel

Details of the approximate analytical solutions are in Appendix B. The only state variable is the regime s_t, and thus the valuation ratios are constant within each regime. Given $s_t = i$, the log price-consumption ratio is

$$\log \frac{P_{c,t}}{C_t} = z_{c,t} = A^i_{c0}. \tag{13}$$

The pricing kernel innovation is

$$m_{t+1} - E^i_t[m_{t+1}] \tag{14}$$

$$= -\gamma^i \sigma^i_c \varepsilon_{c,t+1} - (1-\theta^i)\textstyle\sum_j \left(\mathbb{1}^j_{t+1} - \pi^{ij}\right) \log(1 + e^{A^j_{c0}}) \tag{15}$$

$$= -\xi^i_c \sigma^i_c \varepsilon_{c,t+1} - \textstyle\sum_j \left(\mathbb{1}^j_{t+1} - \pi^{ij}\right) \xi^{ij}_\pi . \tag{16}$$

The first term is familiar from conventional models with power utility: The risk is the random walk shocks to consumption growth, and its market price is risk aversion γ^i. Both risk version and the volatility of the shocks are regime-dependent.

The second term is for regime-shift risk. It comes from the term $(\theta^i - 1) r_{c,t+1}$ in the pricing kernel, and thus results from the agent's concerns over future consumption wealth. The regime-shift shock is $\mathbb{1}^j_{t+1} - \pi^{ij}$. Intuitively, given the current regime $s_t = i$, the expectation for the next regime to be $s_{t+1} = j$ is the transition probability π^{ij}, while the realization is represented by the indicator function $\mathbb{1}^j_{t+1}$. Note that since one and only one regime will realize, the two shocks $\mathbb{1}^{\mathrm{H}}_{t+1} - \pi^{i\mathrm{H}}$ and $\mathbb{1}^{\mathrm{L}}_{t+1} - \pi^{i\mathrm{L}}$ are two sides of the same coin — one is the opposite of the other,

$$\mathbb{1}^{\mathrm{H}}_{t+1} - \pi^{i\mathrm{H}} = -(\mathbb{1}^{\mathrm{L}}_{t+1} - \pi^{i\mathrm{L}}). \tag{17}$$

As a result, the second term in the pricing kernel is equivalent to

$$(\mathbb{1}^{\mathrm{L}}_{t+1} - \pi^{i\mathrm{L}})(\xi^{i\mathrm{L}}_\pi - \xi^{i\mathrm{H}}_\pi), \tag{18}$$

and the market price of regime-shift risk is

$$\xi_\pi^{i\mathrm{L}} - \xi_\pi^{i\mathrm{H}} = (1-\theta^i)\Big(\log(1+e^{A_{c0}^{\mathrm{L}}}) - \log(1+e^{A_{c0}^{\mathrm{H}}})\Big) \approx (1-\theta^i)(A_{c0}^{\mathrm{L}} - A_{c0}^{\mathrm{H}}). \tag{19}$$

It is proportional to the difference in the price of the consumption claim between the low and high volatility regimes. In short, regime-shift risk arises because the agent cares about future consumption wealth, which is different depending on which regime is realized.

The risk-free rate, as presented in Appendix B.2, is constant within each regime, and is lower in the H regime mostly because of the low average consumption growth.

4.3 Stock return

Within each regime, the log price-dividend ratio is constant. Given $s_t = i$,

$$\log \frac{P_t}{D_t} = z_t = A_0^i. \tag{20}$$

The stock return innovation is

$$r_{t+1} - E_t^i[r_{t+1}] \tag{21}$$

$$= \sigma_d^i \varepsilon_{d,t+1} + \textstyle\sum_j (\mathbb{1}_{t+1}^j - \pi^{ij}) \log(1+e^{A_0^j}) \tag{22}$$

$$= \sigma_d^i \varepsilon_{d,t+1} + \textstyle\sum_j (\mathbb{1}_{t+1}^j - \pi^{ij}) \beta_\pi^j \tag{23}$$

$$= \sigma_d^i \varepsilon_{d,t+1} + (\mathbb{1}_{t+1}^{\mathrm{L}} - \pi^{i\mathrm{L}})(\beta_\pi^{\mathrm{L}} - \beta_\pi^{\mathrm{H}}). \tag{24}$$

This result demonstrates the exposure of the stock return to risks. The first term is i.i.d. dividend shocks, which are priced due to the correlation with consumption growth shocks. The second term is the exposure to regime-shift risk, and the loading is

$$\beta_\pi^{\mathrm{L}} - \beta_\pi^{\mathrm{H}} = \log(1+e^{A_0^{\mathrm{L}}}) - \log(1+e^{A_0^{\mathrm{H}}}) \approx A_0^{\mathrm{L}} - A_0^{\mathrm{H}}. \tag{25}$$

The exposure arises because the price-dividend ratio in the future is different across the two regimes.

The return variance is

$$\mathrm{var}_t^i[r_{t+1}] = (\sigma_d^i)^2 + \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}\Big(\beta_\pi^{\mathrm{L}} - \beta_\pi^{\mathrm{H}}\Big)^2 \approx (\sigma_d^i)^2 + \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}(A_0^{\mathrm{L}} - A_0^{\mathrm{H}})^2. \tag{26}$$

Regime-shift risk raises the return volatility beyond that generated by i.i.d. dividend growth shocks.

The contribution of regime-shift risk to the equity premium is

$$- \mathrm{cov}_t^i\Big[- \textstyle\sum_j (\mathbb{1}_{t+1}^j - \pi^{ij})\, \xi_\pi^{ij}\ ,\ \sum_j (\mathbb{1}_{t+1}^j - \pi^{ij})\beta_\pi^j\Big] \tag{27}$$

$$= \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}\Big(\xi_\pi^{i\mathrm{L}} - \xi_\pi^{i\mathrm{H}}\Big)\Big(\beta_\pi^{\mathrm{L}} - \beta_\pi^{\mathrm{H}}\Big) \tag{28}$$

$$\approx \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}(1-\theta^i)(A_{c0}^{\mathrm{L}} - A_{c0}^{\mathrm{H}})(A_0^{\mathrm{L}} - A_0^{\mathrm{H}}). \tag{29}$$

As discussed above, regime-shift risk arises because the agent worries about the uncertainty in future consumption wealth across regimes, and stocks are risky because the stock price in the future also depends on the regime. The subsequent model solution indicates that both the consumption wealth and the stock price are high in the L regime and low in the H regime. In other words, the stock price co-varies positively with the agent's consumption wealth across the regimes. In addition, the calibration, in particular $\psi = 1.5 > 1$, implies that $\theta^i < 0$. Taken together, regime-shifts contribute positively to the equity premium.

Finally, the results above also suggest that the wider the gaps between the valuation ratios across the two regimes, the larger the contributions to both the return volatility and the equity premium. Intuitively, the more different the two regimes are, the stronger the effects of regime-shift risk on asset prices.

4.4 Numerical solution and simulations

The numerical solution confirms that both valuation ratios are lower in the high volatility regime, consistent with the lower means of the growth rates. As discussed earlier, this leads the agent to demand positive compensation for holding regime-shift risk, increasing the equity premium for both regimes.

Table 7 reports the key properties of the simulated asset market data by the regime. The model replicates the empirical equity premium of more than 6% in the low consumption volatility regime. For the high volatility regime, the model-implied average equity premium is more than 8%. The average excess return of -2% realized in the high volatility regime in the empirical data is within the 5th- and 95th-percentile bounds obtained from the simulated data. Specifically, Table 7 shows that among the simulated samples that end up with 65 years of the low volatility regime and 16 years of the high volatility regime, the 5th percentile of the average excess return for the high volatility regime is about -3%.[21]

In the high volatility regime, the model also generates a lower risk-free rate, mostly due to the lower mean of consumption growth.[22] The higher stock return volatility is consistent with the higher dividend growth volatility. Finally, as noted already, the price-dividend ratio is lower. All of these model results are consistent with the empirical results. For both regimes, the model-implied return volatilities and price-dividend ratios are lower than the empirical values. The gap in the risk-free rate across the two regimes is wider in the simulated data. Overall, the model delivers a reasonable quantitative performance in matching the empirical results.

The model solution and the simulations suggest that regime-shift risk is the dominant risk, especially in the low volatility regime. Specifically, regime-shift risk accounts for 87% and 69% of the variance of the log pricing kernel in the low and high volatility regimes, respectively.[23] The equity premium in the low volatility regime generated by the model is almost entirely attributed to regime-shift risk. Without regime shifts, a single-regime model using the same L regime parameters yields an equity premium of about 0.5%.[24]

[21]Across all the simulated series, the 5th percentile is about -6%.

[22]The real yield curve is lower in the high volatility regime. In addition, the real yield curve is slightly upward-sloping in the low volatility regime, and downward-sloping in the high volatility regime.

[23]The remainders are from i.i.d. consumption growth shocks.

[24]This low value is indicative of the equity premium puzzle: Low volatilities of consumption and dividend

As a further illustration of the significance of regime-shift risk, I investigate the effect of the risk aversion parameters on the model-implied equity premium for the low volatility regime. As shown in Table 7, the average equity premium is 6.4% when the risk aversion parameters are 10 and 15 in the low and high volatility regimes, respectively. When risk version is raised to 15 in the low volatility regime, the equity premium in the low volatility regime increases to 7.3%. When risk version is lowered to 10 in the high volatility regime, the equity premium in the low volatility regime drops to 3.2%. In other words, for the equity premium in the low volatility regime, the risk aversion parameter in the high volatility regime exhibits a stronger impact than that in the low volatility regime. Such an effect necessarily arises through the regime-shift risk channel.

5 Model with expected growth components

Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007) suggest that consumption and dividend growth rates contain predictable components. Their empirical results are based on the implicit assumption that the data are of a single regime. In my study, the regime-specific empirical autocorrelations reported in Table 2 suggest that within each regime, consumption and dividend growth rates contain predictable components. In this section, I incorporate such components into the regime-switching dynamics. The expected components are homoscedastic within each regime; hence, the growth rate volatilities are constant within each regime, and only alternate between two discrete levels as the regime shifts. As shown below, both the market price of and the stock return exposure to regime-shift risk vary with the expected component, and this generates time-varying risk premiums within each regime.

growth and a low correlation between them, together with a low risk aversion, generate too small a equity premium.

5.1 Growth rate dynamics

Given $s_t = i$, consumption and dividend growth are

$$\Delta c_{t+1} = \log \frac{C_{t+1}}{C_t} = \mu_c^i + \lambda_c^i x_t + \sigma_c^i \varepsilon_{c,t+1}, \tag{30}$$

$$\Delta d_{t+1} = \log \frac{D_{t+1}}{D_t} = \mu_d^i + \lambda_d^i x_t + \sigma_d^i \varepsilon_{d,t+1}, \tag{31}$$

$$x_{t+1} = \phi_x^i x_t + \sigma_x^i \varepsilon_{x,t+1}, \tag{32}$$

$$E_t^i[\varepsilon_{c,t+1}\varepsilon_{d,t+1}] = \chi^i, \tag{33}$$

$$\varepsilon_{c,t+1}, \varepsilon_{d,t+1}, \varepsilon_{x,t+1} \sim \text{i.i.d. } N(0,1).$$

Here, μ_c^i and μ_d^i are the mean parameters, while the volatility parameters are σ_c^i, σ_d^i, σ_x^i for the shocks. Shocks are mutually independent except for the correlation χ^i between i.i.d. growth shocks.[25]

In the model, the two growth rates contain the common expected component x_t plus random walk shocks. Within each regime, the expected component is an AR(1) process with a persistence parameter $0 < \phi_x^i < 1$, driven by shocks of constant volatility σ_x^i. The loadings on x_t are λ_c^i for consumption growth, and λ_d^i for dividend growth.

The calibrated parameters are reported in Table 8. Following Bansal and Yaron (2004), the persistence parameters for the expected component are close to 1. Studies of regime-switching models generally conclude that in the high volatility regime, the underlying economic state variables mean-revert faster.[26] Accordingly, I set the persistence to 0.98 for the L regime, and 0.97 for the H regime.

In the L regime, the loading of consumption growth on the expected component is normalized to 1. The other parameters for consumption growth and the expected component are chosen to match the empirical properties of the annual data in the low volatility regime.

[25] See footnote 20 for the discussion on the timing convention.

[26] As supporting evidence, Panel A of Table 3 shows that the empirical autocorrelations of the price-dividend ratio are lower in the high consumption volatility regime.

The empirical results in Section 2.2 indicate that the volatilities of economic fundamentals in the high volatility regime are about 2 to 4 times those in the low volatility regime.[27] Consequently, I calibrate the H regime dynamics for the expected component so that its standard deviation is 3 times that in the L regime. The other H regime parameters for consumption growth are subsequently chosen to match the empirical properties of the annual data in the high volatility regime. In particular, the loading of consumption growth on the expected component is 3.5.

The parameters for the exposure of dividend growth to the expected component are set to 1 in the L regime, and 5 in the H regime. The low exposure in the L regime is sufficient to replicate the autocorrelation of dividend growth in the low volatility regime. The high exposure in the H regime is consistent with the higher empirical correlation between consumption and dividend growth in the high volatility regime.

The simulated growth rates well replicate the empirical maximum likelihood parameter estimates from the annual consumption growth data, as well as the regime-specific empirical properties of both consumption and dividend growth rates. The results on the maximum likelihood estimation, the means, the volatilities, and the correlations are very similar to those for the model with i.i.d. growth rates in Tables 5 and 6. The results on autocorrelations are reported in Table 9.

5.2 Pricing kernel

Details of the approximate analytical solutions are in Appendix C. There are two state variables in the model: the regime s_t and the expected component x_t. Given $s_t = i$, the log

[27]Specifically, the volatilities of the annual growth rates for consumption, GDP, and industrial production in the high volatility regime are 3.6, 2.1, and 2.5 times those in the low volatility regime, respectively.

price-consumption ratio is

$$\log \frac{P_{c,t}}{C_t} = z_{c,t} \approx A^i_{c0} + A^i_{c1} x_t, \tag{34}$$

$$\frac{A^i_{c1}}{1-\frac{1}{\psi}} \approx \frac{\lambda^i_c}{1-\pi^{ii}\kappa^i_{c1}\phi^i_x} + \frac{\pi^{ij}\kappa^j_{c1}\phi^i_x}{1-\pi^{ii}\kappa^i_{c1}\phi^i_x}\frac{A^j_{c1}}{1-\frac{1}{\psi}}\ . \tag{35}$$

Here, κ^i_{c1} and κ^j_{c1} are constants very close to 1, and A^i_{c0}, A^i_{c1}, and A^j_{c1} are constants.

If $s_t = i$, then the regime at $t+1$ either stays in the same regime i, or switches into the other regime j. These two possibilities are reflected in the two terms in the sensitivity A^i_{c1}. The first term is proportional to λ^i_c, the exposure of consumption growth to x_t in the current regime. The second term, accounting for the switch, is proportional to the transition probability π^{ij} and A^j_{c1}, the sensitivity in the other regime. A high persistence of x_t magnifies the response of the price-consumption ratio to the expected component. Note, however, with regime shifts, the effective persistence of x_t becomes $\pi^{ii}\phi^i_x$.

The pricing kernel innovation is

$$\begin{aligned}
& m_{t+1} - E^i_t[m_{t+1}] \\
\approx\ & -\gamma^i\sigma^i_c\varepsilon_{c,t+1} - (1-\theta^i)\textstyle\sum_j \mathbb{1}^j_{t+1}\kappa^j_{c1}A^j_{c1}\sigma^i_x\varepsilon_{x,t+1} \\
& - (1-\theta^i)\textstyle\sum_j \left(\mathbb{1}^j_{t+1} - \pi^{ij}\right)\left(\log(1+e^{A^j_{c0}}) + \kappa^j_{c1}A^j_{c1}\phi^i_x x_t\right) \\
=\ & -\xi^i_c\sigma^i_c\varepsilon_{c,t+1} - \textstyle\sum_j \mathbb{1}^j_{t+1}\xi^j_x\sigma^i_x\varepsilon_{x,t+1} - \sum_j \left(\mathbb{1}^j_{t+1} - \pi^{ij}\right)\xi^{ij}_\pi\ .
\end{aligned}$$

The first term, similar to that in conventional models with power utility, is for i.i.d. consumption growth shocks, and its market price is risk aversion γ^i. The second term is for long-run growth risk: The risk is the shocks to the expected growth component, and the market price of risk is proportional to A^j_{c1}, the sensitivity coefficient in the $t+1$ regime. This is a two-regime extension of the long-run growth risk in Bansal and Yaron (2004).

The third term is for regime-shift risk. The shock $\mathbb{1}^j_{t+1} - \pi^{ij}$, as discussed earlier, is the innovation in the regime. Because

$$\xi_\pi^{ij} = (1-\theta^i)\big(\log(1+e^{A_{c0}^j}) + \kappa_{c1}^j A_{c1}^j \phi_x^i x_t\big) \tag{36}$$

$$\approx (1-\theta^i)(A_{c0}^j + A_{c1}^j \phi_x^i x_t) \approx (1-\theta^i)E_t^i[z_{c,t+1}^j], \tag{37}$$

the market price of regime-shift risk is

$$\xi_\pi^{i\mathrm{L}} - \xi_\pi^{i\mathrm{H}} \approx (1-\theta^i)\Big(A_{c0}^{\mathrm{L}} - A_{c0}^{\mathrm{H}} + (A_{c1}^{\mathrm{L}} - A_{c1}^{\mathrm{H}})\phi_x^i x_t\Big) \tag{38}$$

$$\approx (1-\theta^i)E_t^i[z_{c,t+1}^{\mathrm{L}} - z_{c,t+1}^{\mathrm{H}}], \tag{39}$$

which is proportional to the difference in the expected price of the consumption claim across the two regimes. In the model solution, $A_{c0}^{\mathrm{L}} > A_{c0}^{\mathrm{H}}$: the price-consumption ratio is higher in the L regime, consistent with the higher consumption growth. In addition, $A_{c1}^i > 0$: the price-consumption ratio varies positively with x_t. Further, $A_{c1}^{\mathrm{L}} < A_{c1}^{\mathrm{H}}$: the slope is steeper in the H regime, consistent with the larger loading of consumption growth on the expected component. Hence, the market price of regime-shift risk varies negatively with x_t : as x_t increases, the gap $\xi_\pi^{i\mathrm{L}} - \xi_\pi^{i\mathrm{H}}$ shrinks and the market price of regime-shift risk falls.

Both long-run growth risk and regime-shift risk arise as a result of Epstein-Zin preferences. They both come from the term $(1-\theta^i)r_{c,t+1}$ in the pricing kernel, and thus have origins in the agent's concerns over future consumption wealth. The wealth will vary with the shock to the expected component; this gives rise to long-run growth risk. The wealth will be different depending on which regime is realized; this gives rise to regime-shift risk.

The risk-free rate, as presented in Appendix C.2, varies positively with x, and is lower in the H regime mostly because of the low average consumption growth.

5.3 Stock return

Given $s_t = i$, the log price-dividend ratio is

$$\log \frac{P_t}{D_t} = z_t \approx A_0^i + A_1^i x_t, \tag{40}$$

$$A_1^i \approx \frac{\lambda_d^i - \frac{1}{\psi}\lambda_c^i}{1 - \pi^{ii}\kappa_1^i \phi_x^i} + \frac{\pi^{ij}\kappa_1^j \phi_x^i}{1 - \pi^{ii}\kappa_1^i \phi_x^i} A_1^j. \tag{41}$$

Here, κ_1^i and κ_1^j are constants very close to 1, and A_0^i, A_1^i, and A_1^j are constants. Similar to the earlier result for the log price-consumption ratio, the two terms in the sensitivity A_1^i correspond to the two subsequent possibilities: The regime either remains the same or switches.

The stock return innovation is

$$r_{t+1} - E_t^i[r_{t+1}] \tag{42}$$

$$\approx \sigma_d^i \varepsilon_{d,t+1} + \sum_j \mathbb{1}_{t+1}^j \kappa_1^j A_1^j \sigma_x^i \varepsilon_{x,t+1} + \sum_j (\mathbb{1}_{t+1}^j - \pi^{ij})\big(\log(1 + e^{A_0^j}) + \kappa_1^j A_1^j \phi_x^i x_t\big) \tag{43}$$

$$= \sigma_d^i \varepsilon_{d,t+1} + \sum_j \mathbb{1}_{t+1}^j \beta_x^j \sigma_x^i \varepsilon_{x,t+1} + \sum_j (\mathbb{1}_{t+1}^j - \pi^{ij})\beta_\pi^{ij}. \tag{44}$$

This result demonstrates the exposure of the stock return to risks. The first term is i.i.d. dividend shocks. The second term is the exposure to long-run growth risk, similar to that in Bansal and Yaron (2004). The final term is the exposure to regime-shift risk. Since

$$\beta_\pi^{ij} = \log(1 + e^{A_0^j}) + \kappa_1^j A_1^j \phi_x^i x_t \approx A_0^j + A_1^j \phi_x^i x_t \approx E_t^i[z_{t+1}^j], \tag{45}$$

the loading of the stock return on regime-shift risk is

$$\beta_\pi^{i\mathrm{L}} - \beta_\pi^{i\mathrm{H}} \approx A_0^{\mathrm{L}} - A_0^{\mathrm{H}} + (A_1^{\mathrm{L}} - A_1^{\mathrm{H}})\phi_x^i x_t \approx E_t^i[z_{t+1}^{\mathrm{L}} - z_{t+1}^{\mathrm{H}}]. \tag{46}$$

The exposure arises because the stock price is regime-dependent, and the loading is the gap in the expected price-dividend ratio between the two future regimes. In the model solution, $A_0^{\mathrm{L}} > A_0^{\mathrm{H}}$: the price-dividend ratio is higher in the L regime, consistent with the higher dividend growth. In addition, $A_1^i > 0$: the price-dividend ratio varies positively with the expected growth x_t. Further, $A_1^{\mathrm{L}} < A_1^{\mathrm{H}}$: the slope is much steeper in the H regime, consistent with the larger exposure of dividend growth to the expected component. Hence, the loading of the return on regime-shift risk varies negatively with x_t : as x_t increases, the gap $\beta_\pi^{i\mathrm{L}} - \beta_\pi^{i\mathrm{H}}$ shrinks and the return loading on regime-shift risk falls.

The return variance is

$$\mathrm{var}_t^i[r_{t+1}] \approx (\sigma_d^i)^2 + \sum_j \pi^{ij}(\kappa_1^j A_1^j \sigma_x^i)^2 + \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}(\beta_\pi^{i\mathrm{L}} - \beta_\pi^{i\mathrm{H}})^2. \tag{47}$$

The first two terms, reflecting the contributions from i.i.d. dividend growth shocks and the exposure to long-run growth risk, are constant within each regime. In the third term, the contribution by regime-shift risk drives the return volatility to vary negatively with the expected component.

The contributions of i.i.d. growth shocks and long-run growth risk to the equity premium are also constant. The contribution by long-run regime-shift risk,

$$-\,\mathrm{cov}_t^i\left[-\sum_j (\mathbb{1}_{t+1}^j - \pi^{ij})\,\xi_\pi^{ij}\,,\ \sum_j (\mathbb{1}_{t+1}^j - \pi^{ij})\beta_\pi^{ij}\right] \tag{48}$$

$$= \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}\left(\xi_\pi^{i\mathrm{L}} - \xi_\pi^{i\mathrm{H}}\right)\left(\beta_\pi^{i\mathrm{L}} - \beta_\pi^{i\mathrm{H}}\right) \tag{49}$$

$$\approx \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}(1-\theta^i)E_t^i[z_{c,t+1}^{\mathrm{L}} - z_{c,t+1}^{\mathrm{H}}]E_t^i[z_{t+1}^{\mathrm{L}} - z_{t+1}^{\mathrm{H}}], \tag{50}$$

results from the co-variation between the price-consumption and price-dividend ratios across the two regimes. Since both valuation ratios are higher in the L regime, the positive co-variation implies a positive contribution to the level of the equity premium. Further, as discussed in the above, both the market price of and the stock return exposure to regime-shift risk vary negatively with the expected component x_t, and so does the equity premium. In contrast, the price-dividend ratio increases with x_t. Taken together, as x_t increases, the price-dividend ratio rises, but the equity premium falls — the price-dividend ratio and the equity premium are negatively correlated.

The model also generates the volatility feedback effect [Campbell and Hentschel (1992) and Glosten, Jaganathan, and Runkle (1993)]. As discussed above, an increase in the expected component raises the stock price but lowers the return volatility. In addition, since the stock price is high and the return volatility is low in the L regime, a switch to the L regime is associated with a positive return shock and a negative volatility shock. Both mechanisms arise through the regime-shift channel, and contribute to the negative correlation between the return innovation and the volatility innovation. The details are in Appendix C.3.

5.4 Numerical solution and simulations

Many aspects of the numerical solution and the simulations of the model with expected components are similar to those for the model with i.i.d. growth rates discussed earlier. As reported in Table 10, the simulations match the empirical equity premium for the low volatility regime, and bracket the empirical average excess stock return for the high volatility regime within the 5th- and 95th-percentile bounds. Regime-shift risk is the dominant risk, accounting for 90% of the variance of the log pricing kernel in the low volatility regime, and 63% in the high volatility regime.[28] The equity premium in the low volatility regime is almost entirely attributed to regime-shift risk. The model also generates a lower risk-free rate,[29] a higher return volatility, and a lower price-dividend ratio in the high volatility regime. The model performance is comparable to that of the model with i.i.d. growth rates.

The simulated data also indicate a higher volatility for the risk-free rate in the high volatility regime, qualitatively consistent with the empirical results. Quantitatively, for both regimes, the volatilities of the risk-free rate in the model are lower than those in the empirical data. The empirical risk-free rates, however, are ex post realized rates. They contain unexpected inflation, and thus are more volatile than the ex ante risk-free rates, which the model simulation generates.

[28]In addition, in the low volatility regime, i.i.d. consumption growth shocks contribute 3% and long-run growth risk contributes 7%; in the high volatility regime, i.i.d. consumption growth shocks contribute 5% and long-run growth risk contributes 32%.

[29]With expected consumption growth, the real yield curves are downward-sloping within each regime. This is the same as the single-regime result in Bansal and Yaron (2004).

In the high volatility regime, the model-implied volatility for the log price-dividend ratio is lower than, but largely consistent with the empirical value. In the low volatility regime, the model-implied value is much lower than the empirical value. Due to the small volatility of consumption growth and consequently that calibrated for the expected component, the model is limited in its capability in generating volatile price-dividend ratios. Relaxing some of the model assumptions and incorporating additional sources of time variations in future research may improve the model fit.

Table 10 also indicates that in the model, as in the empirical data, the autocorrelations for the price-dividend ratio are larger in the low volatility regime, and the match is particularly good in the high volatility regime.

The model solution also confirms that both valuation ratios increase with the expected component, and the slopes are steeper in the high volatility regime. As discussed earlier, this mechanism generates a negative relation between the price-dividend ratio and the equity premium. This result arises through the regime-shift risk channel. For comparison, in single-regime models, the equity premium is essentially flat or even slightly increases with the price-dividend ratio.

The return predictability results obtained from the simulated data are presented in Table 11. The model generates more negative slopes and higher R^2 for the predictive regressions in the high volatility regime, consistent with the empirical results. The model-implied slopes for the high volatility regime are somewhat lower in magnitude than the empirical slopes, while the match is better in the low volatility regime.

6 Concluding remarks

In this paper I report that key empirical properties of consumption growth, dividend growth, and asset market data differ across the two regimes identified in the consumption growth data. I present regime-switching asset pricing models to explain these empirical results. Regime-shift risk generates risk premiums because the valuation ratios differ across the regimes, and risk premiums vary over time because the sensitivities of the valuation ratios differ across the regimes. The high volatility regime is associated with deep recessions. My study thus suggests that concerns over slumping into recessions wield strong influence on the asset prices during economic expansions.

The parsimonious models in my paper are capable of explaining major regime-dependent asset market phenomena. To focus on regime shifts, the models abstract away from a number of mechanisms employed in existing studies, such as time-varying transition probabilities and hidden regimes that the agent infers by updating the regime probabilities.[30] Incorporating these flexibilities may lead to new insights and improve the fit to the empirical data.

The nature of regime-switching descriptive models is such that the estimation results and the identified regimes most probably depend on the choice of sample period and data frequency. When I estimate the two-regime descriptive model on quarterly consumption growth of 1947–2010, the simulated likelihood ratio test rejects the single-regime null hypothesis with a p-value less than 10^{-4}.[31] Among the 255 quarters of the consumption growth data, 73 quarters are in the high volatility regime. All NBER-dated recessions during the sample period of 1947–2010, except those of 1969–1970 and 2001, are identified as in the high volatility regime. When measured by the peak-to-trough GDP decline, the two excluded recessions are the shallowest. Hence, the association of the high volatility regime with deep recessions appears to be a stable feature of the consumption growth data.

[30]For example, regime transition probabilities are time-varying in Whitelaw (2000), and the agent updates the regime probabilities in Lettau, Ludvigson, and Wachter (2008), Ozoguz (2009), and Ghosh and Constantinides (2010).

[31]See Section 2.1 for more discussions of the simulated likelihood ratio test.

Appendices

A Mixture of normals

A.1 Mean and variance

Let X be a mixture of normal random variables. Its value depends on the state, which can be either L or H, with the probabilities $p^{\mathrm{L}} + p^{\mathrm{H}} = 1$.

$$X = \begin{cases} X^{\mathrm{L}} = \mu^{\mathrm{L}} + \sigma^{\mathrm{L}}\varepsilon, & \text{in state L, with the probability } p^{\mathrm{L}}, \\ X^{\mathrm{H}} = \mu^{\mathrm{H}} + \sigma^{\mathrm{H}}\varepsilon, & \text{in state H, with the probability } p^{\mathrm{H}} = 1 - p^{\mathrm{L}}. \end{cases}$$

Define the indicator variable

$$\mathbb{1}^j = 1, \quad \text{if and only if the state is } j, \quad j \in \{\mathrm{L}, \mathrm{H}\}.$$

Hence,

$$X = \textstyle\sum_j \mathbb{1}^j X^j = \sum_j \mathbb{1}^j (\mu^j + \sigma^j \varepsilon).$$

Then,

$$\begin{aligned} E[X] &= p^{\mathrm{L}}\mu^{\mathrm{L}} + p^{\mathrm{H}}\mu^{\mathrm{H}}, \\ E[X^2] &= p^{\mathrm{L}}\big((\mu^{\mathrm{L}})^2 + (\sigma^{\mathrm{L}})^2\big) + p^{\mathrm{H}}\big((\mu^{\mathrm{H}})^2 + (\sigma^{\mathrm{H}})^2\big). \end{aligned}$$

The variance is

$$\begin{aligned} \mathrm{var}[X] &= E[X^2] - E[X]^2 \\ &= p^{\mathrm{L}}\big((\mu^{\mathrm{L}})^2 + (\sigma^{\mathrm{L}})^2\big) + p^{\mathrm{H}}\big((\mu^{\mathrm{H}})^2 + (\sigma^{\mathrm{H}})^2\big) - \big(p^{\mathrm{L}}\mu^{\mathrm{L}} + p^{\mathrm{H}}\mu^{\mathrm{H}}\big)^2 \\ &= p^{\mathrm{L}}(\sigma^{\mathrm{L}})^2 + p^{\mathrm{H}}(\sigma^{\mathrm{H}})^2 \\ &\quad + \big(p^{\mathrm{L}} - (p^{\mathrm{L}})^2\big)(\mu^{\mathrm{L}})^2 + \big(p^{\mathrm{H}} - (p^{\mathrm{H}})^2\big)(\mu^{\mathrm{H}})^2 - 2p^{\mathrm{L}}p^{\mathrm{H}}\mu^{\mathrm{L}}\mu^{\mathrm{H}} \\ &= p^{\mathrm{L}}(\sigma^{\mathrm{L}})^2 + p^{\mathrm{H}}(\sigma^{\mathrm{H}})^2 + p^{\mathrm{L}}p^{\mathrm{H}}(\mu^{\mathrm{L}} - \mu^{\mathrm{H}})^2. \end{aligned}$$

Note that

$$\begin{aligned} X - E[X] &= \textstyle\sum_j \mathbb{1}^j X^j - \sum_j p^j \mu^j = \sum_j \mathbb{1}^j(\mu^j + \sigma^j\varepsilon) - \sum_j p^j\mu^j \\ &= \textstyle\sum_j \mathbb{1}^j \sigma^j \varepsilon + \sum_j (\mathbb{1}^j - p^j)\mu^j. \end{aligned}$$

Hence, in var$[X]$,

$$\text{var}\left[\sum_j \mathbb{1}^j \sigma^j \varepsilon\right] = p^{\text{L}}(\sigma^{\text{L}})^2 + p^{\text{H}}(\sigma^{\text{H}})^2$$

is contributed by the variation within the state, and

$$\text{var}\left[\sum_j (\mathbb{1}^j - p^j)\mu^j\right] = p^{\text{L}} p^{\text{H}} (\mu^{\text{L}} - \mu^{\text{H}})^2$$

reflects the variation across the two states.

A.2 Exponential function

For a normal random variable $Y = \mu + \sigma\varepsilon,\ \varepsilon \sim N(0,1)$,

$$E[e^Y] = e^{\mu + \frac{1}{2}\sigma^2}.$$

For X as a mixture of normals

$$X = \sum_j \mathbb{1}^j(\mu^j + \sigma^j \varepsilon),$$

with the Taylor expansion

$$\begin{aligned} & \log(ae^x + be^y) \\ = \ & \log(a+b) + \frac{ax+by}{a+b} + \frac{1}{2}\frac{ab}{(a+b)^2}(x^2 + y^2 - 2xy) + \cdots, \end{aligned}$$

it can be shown that

$$\begin{aligned} & \log E[e^X] = \log\Big(p^L E\big[\exp(X^{\text{L}})\big] + p^{\text{H}} E\big[\exp(X^{\text{H}})\big]\Big) \\ = \ & \log\Big(p^{\text{L}} \exp\big(\mu^{\text{L}} + \frac{1}{2}(\sigma^{\text{L}})^2\big) + p^{\text{H}} \exp\big(\mu^{\text{H}} + \frac{1}{2}(\sigma^{\text{H}})^2\big)\Big) \\ \approx \ & p^{\text{L}}\big(\mu^{\text{L}} + \frac{1}{2}(\sigma^{\text{L}})^2\big) + p^{\text{H}}\big(\mu^{\text{H}} + \frac{1}{2}(\sigma^{\text{H}})^2\big) + \frac{1}{2}p^{\text{L}}p^{\text{H}}\Big(\mu^{\text{L}} - \mu^{\text{H}} + \frac{1}{2}(\sigma^{\text{L}})^2 - \frac{1}{2}(\sigma^{\text{H}})^2\Big)^2 \\ \approx \ & p^{\text{L}}\mu^{\text{L}} + p^{\text{H}}\mu^{\text{H}} + \frac{1}{2}p^{\text{L}}(\sigma^{\text{L}})^2 + \frac{1}{2}p^{\text{H}}(\sigma^{\text{H}})^2 + \frac{1}{2}p^{\text{L}}p^{\text{H}}\big(\mu^{\text{L}} - \mu^{\text{H}}\big)^2. \end{aligned}$$

In the last line above, only linear and quadratic terms are retained.

A comparison with the mean and the variance of X suggests that, for X as a mixture of normal random variables,

$$\log E[e^X] \approx E[X] + \frac{1}{2}\,\text{var}[X]$$

provides a second-order approximation.

A.3 Covariance

Consider two random variables, both mixtures of normals,

$$\begin{aligned} X &= \textstyle\sum_j \mathbb{1}^j(\mu_x^j + \sigma_x^j\,\varepsilon_x), \\ Y &= \textstyle\sum_j \mathbb{1}^j(\mu_y^j + \sigma_y^j\,\varepsilon_y), \\ E[\varepsilon_x\varepsilon_y] &= \chi, \qquad \varepsilon_x, \varepsilon_y \sim N(0,1). \end{aligned}$$

Then

$$\begin{aligned} E[X] &= p^{\mathrm{L}}\mu_x^{\mathrm{L}} + p^{\mathrm{H}}\mu_x^{\mathrm{H}}, \\ E[Y] &= p^{\mathrm{L}}\mu_y^{\mathrm{L}} + p^{\mathrm{H}}\mu_y^{\mathrm{H}}, \\ E[XY] &= p^{\mathrm{L}}\big(\mu_x^{\mathrm{L}}\mu_y^{\mathrm{L}} + \sigma_x^{\mathrm{L}}\sigma_y^{\mathrm{L}}\chi\big) + p^{\mathrm{H}}\big(\mu_x^{\mathrm{H}}\mu_y^{\mathrm{H}} + \sigma_x^{\mathrm{H}}\sigma_y^{\mathrm{H}}\chi\big). \end{aligned}$$

The covariance is

$$\begin{aligned} \mathrm{cov}[X,Y] &= E[XY] - E[X]E[Y] \\ &= p^{\mathrm{L}}\big(\mu_x^{\mathrm{L}}\mu_y^{\mathrm{L}} + \sigma_x^{\mathrm{L}}\sigma_y^{\mathrm{L}}\chi\big) + p^{\mathrm{H}}\big(\mu_x^{\mathrm{H}}\mu_y^{\mathrm{H}} + \sigma_x^{\mathrm{H}}\sigma_y^{\mathrm{H}}\chi\big) \\ &\quad - (p^{\mathrm{L}}\mu_x^{\mathrm{L}} + p^{\mathrm{H}}\mu_x^{\mathrm{H}})(p^{\mathrm{L}}\mu_y^{\mathrm{L}} + p^{\mathrm{H}}\mu_y^{\mathrm{H}}) \\ &= p^{\mathrm{L}}\sigma_x^{\mathrm{L}}\sigma_y^{\mathrm{L}}\chi + p^{\mathrm{H}}\sigma_x^{\mathrm{H}}\sigma_y^{\mathrm{H}}\chi \\ &\quad + (p^{\mathrm{L}} - (p^{\mathrm{L}})^2)\mu_x^{\mathrm{L}}\mu_y^{\mathrm{L}} + (p^{\mathrm{H}} - (p^{\mathrm{H}})^2)\mu_x^{\mathrm{H}}\mu_y^{\mathrm{H}} \\ &\quad - p^{\mathrm{L}}\mu_x^{\mathrm{L}}p^{\mathrm{H}}\mu_y^{\mathrm{H}} - p^{\mathrm{H}}\mu_x^{\mathrm{H}}p^{\mathrm{L}}\mu_y^{\mathrm{L}} \\ &= p^{\mathrm{L}}\sigma_x^{\mathrm{L}}\sigma_y^{\mathrm{L}}\chi + p^{\mathrm{H}}\sigma_x^{\mathrm{H}}\sigma_y^{\mathrm{H}}\chi + p^{\mathrm{L}}p^{\mathrm{H}}(\mu_x^{\mathrm{L}} - \mu_x^{\mathrm{H}})(\mu_y^{\mathrm{L}} - \mu_y^{\mathrm{H}}). \end{aligned}$$

Note that

$$\begin{aligned} X - E[X] &= \textstyle\sum_j \mathbb{1}^j\sigma_x^j\varepsilon_x + \sum_j (\mathbb{1}^j - p^j)\mu_x^j, \\ Y - E[Y] &= \textstyle\sum_j \mathbb{1}^j\sigma_y^j\varepsilon_y + \sum_j (\mathbb{1}^j - p^j)\mu_y^j. \end{aligned}$$

Hence, in $\mathrm{cov}[X,Y]$,

$$\mathrm{cov}\left[\textstyle\sum_j \mathbb{1}^j\sigma_x^j\varepsilon_x,\ \sum_j \mathbb{1}^j\sigma_y^j\varepsilon_y\right] = p^{\mathrm{L}}\sigma_x^{\mathrm{L}}\sigma_y^{\mathrm{L}}\chi + p^{\mathrm{H}}\sigma_x^{\mathrm{H}}\sigma_y^{\mathrm{H}}\chi$$

is contributed by the co-movement within the state, and

$$\mathrm{cov}\left[\textstyle\sum_j \mu_x^j(\mathbb{1}^j - p^j),\ \sum_j \mu_y^j(\mathbb{1}^j - p^j)\right] = p^{\mathrm{L}}p^{\mathrm{H}}(\mu_x^{\mathrm{L}} - \mu_x^{\mathrm{H}})(\mu_y^{\mathrm{L}} - \mu_y^{\mathrm{H}})$$

reflects the co-movement across the two states.

B Model with i.i.d. growth rates

Given $s_t = i$, consumption and dividend growth are

$$\Delta c_{t+1} = \log \frac{C_{t+1}}{C_t} = \mu_c^i + \sigma_c^i \varepsilon_{c,t+1},$$
$$\Delta d_{t+1} = \log \frac{D_{t+1}}{D_t} = \mu_d^i + \sigma_d^i \varepsilon_{d,t+1},$$
$$E_t^i[\varepsilon_{c,t+1}\varepsilon_{d,t+1}] = \chi^i.$$

B.1 Log price-consumption ratio

Within each regime, the price-consumption ratio is constant. Assume that given $s_t = i$,

$$\log \frac{P_{c,t}}{C_t} = z_{c,t} = A_{c0}^i.$$

Then,

$$\begin{aligned} r_{c,t+1} &= \log(1 + e^{z_{c,t+1}}) + \Delta c_{t+1} - z_{c,t} \\ &= \textstyle\sum_j \mathbb{1}_{t+1}^j \log(1 + e^{A_{c0}^j}) + \mu_c^i + \sigma_c^i \varepsilon_{c,t+1} - A_{c0}^i. \end{aligned}$$

Hence, $r_{c,t+1}$ is a mixture of normals, and

$$E_t^i[r_{c,t+1}] = \textstyle\sum_j \pi^{ij} \log(1 + e^{A_{c0}^j}) + \mu_c^i - A_{c0}^i,$$

which is constant within each regime. The consumption return innovation is

$$r_{c,t+1} - E_t^i[r_{c,t+1}] = \textstyle\sum_j (\mathbb{1}_{t+1}^j - \pi^{ij}) \log(1 + e^{A_{c0}^j}) + \sigma_c^i \varepsilon_{c,t+1}.$$

The pricing kernel is

$$\begin{aligned} m_{t+1} &= \theta^i \log \delta - \frac{\theta^i}{\psi} \Delta c_{t+1} + (\theta^i - 1) r_{c,t+1} \\ &= \theta^i \log \delta - \frac{\theta^i}{\psi} \left(\mu_c^i + \sigma_c^i \varepsilon_{c,t+1}\right) + (\theta^i - 1) r_{c,t+1}. \end{aligned}$$

Hence, m_{t+1} is also a mixture of normals, and the pricing kernel innovation is

$$\begin{aligned} & m_{t+1} - E_t^i[m_{t+1}] \\ = & -\frac{\theta^i}{\psi}\sigma_c^i\varepsilon_{c,t+1} + (\theta^i - 1)\Big(\textstyle\sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right)\log(1+e^{A_{c0}^j}) + \sigma_c^i\varepsilon_{c,t+1}\Big) \\ = & -\gamma^i\sigma_c^i\varepsilon_{c,t+1} - (1-\theta^i)\textstyle\sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right)\log(1+e^{A_{c0}^j}) \\ = & -\xi_c^i\sigma_c^i\varepsilon_{c,t+1} - \textstyle\sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right)\xi_\pi^{ij}\,. \end{aligned}$$

B.2 Risk-free rate

Given $s_t = i$,

$$1 = E_t^i[e^{m_{t+1}+r_{f,t}}].$$

Since m_{t+1} is a mixture of normals,

$$-r_{f,t} = \log E_t^i[e^{m_{t+1}}] \approx E_t^i[m_{t+1}] + \frac{1}{2}\,\mathrm{var}_t^i[m_{t+1}].$$

That is,

$$r_{f,t} \approx -\theta^i \log\delta + \frac{\theta^i}{\psi}\mu_c^i - (\theta^i - 1)E_t^i[r_{c,t+1}] - \frac{1}{2}\,\mathrm{var}_t^i[m_{t+1}].$$

Add $(\theta^i - 1)r_{f,t}$ to both sides, and divide by θ^i (assume $\theta^i \neq 0$), then

$$r_{f,t} \approx -\log\delta + \frac{1}{\psi}\mu_c^i - \frac{\theta^i - 1}{\theta^i}E_t^i[r_{c,t+1} - r_{f,t}] - \frac{1}{2\theta^i}\,\mathrm{var}_t^i[m_{t+1}],$$

which is constant within each regime.

B.3 Log price-dividend ratio

Within each regime, the price-dividend ratio is constant. Assume that given $s_t = i$,

$$\log\frac{P_t}{D_t} = z_t = A_0^i.$$

Then,

$$\begin{aligned} r_{t+1} = & \log(1+e^{z_{t+1}}) + \Delta d_{t+1} - z_t \\ = & \textstyle\sum_j \mathbb{1}_{t+1}^j \log(1+e^{A_0^j}) + \mu_d^i + \sigma_d^i\varepsilon_{d,t+1} - A_0^i, \end{aligned}$$

Hence, r_{t+1} is a mixture of normals, and

$$E_t^i[r_{t+1}] = \sum_j \pi^{ij} \log(1 + e^{A_0^j}) + \mu_d^i - A_0^i.$$

The return innovation is

$$\begin{aligned} & r_{t+1} - E_t^i[r_{t+1}] \\ &= \sigma_d^i \varepsilon_{d,t+1} + \sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right) \log(1 + e^{A_0^j}) \\ &= \sigma_d^i \varepsilon_{d,t+1} + \sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right) \beta_\pi^j. \end{aligned}$$

The return variance is constant within each regime,

$$\mathrm{var}_t^i[r_{t+1}] = (\sigma_d^i)^2 + \pi^{i\mathrm{L}} \pi^{i\mathrm{H}} \Big(\beta_\pi^{\mathrm{L}} - \beta_\pi^{\mathrm{H}}\Big)^2 = V_0^i.$$

For the equity premium,

$$E_t^i[r_{t+1} - r_{f,t}] + \frac{1}{2} \mathrm{var}_t^i[r_{t+1}] \approx -\mathrm{cov}_t^i[m_{t+1}, r_{t+1}].$$

The contribution of i.i.d. growth shocks is

$$-\mathrm{cov}_t^i \Big[-\gamma^i \sigma_c^i \varepsilon_{c,t+1},\ \sigma_d^i \varepsilon_{d,t+1} \Big] = \gamma^i \chi^i \sigma_c^i \sigma_d^i.$$

The contribution from regime-shift risk is

$$\begin{aligned} & -\mathrm{cov}_t^i \Big[-\sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right) \xi_\pi^{ij},\ \sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right) \beta_\pi^j \Big] \\ &= \pi^{i\mathrm{L}} \pi^{i\mathrm{H}} \Big(\xi_\pi^{i\mathrm{L}} - \xi_\pi^{i\mathrm{H}}\Big) \Big(\beta_\pi^{\mathrm{L}} - \beta_\pi^{\mathrm{H}}\Big). \end{aligned}$$

The innovation to the return variance is

$$\mathrm{var}_{t+1}[r_{t+2}] - E_t^i[\mathrm{var}_{t+1}[r_{t+2}]] = \sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right) V_0^j.$$

Then

$$\begin{aligned} & \mathrm{cov}[r_{t+1} - E_t^i[r_{t+1}], \mathrm{var}_{t+1}[r_{t+2}] - E_t^i[\mathrm{var}_{t+1}[r_{t+2}]]] \\ &= \pi^{i\mathrm{L}} \pi^{i\mathrm{H}} \Big(\beta_\pi^{\mathrm{L}} - \beta_\pi^{\mathrm{H}}\Big) \Big(V_0^{\mathrm{L}} - V_0^{\mathrm{H}}\Big). \end{aligned}$$

C Model with expected growth components

Given $s_t = i$, the consumption and dividend growth are

$$\Delta c_{t+1} = \log \frac{C_{t+1}}{C_t} = \mu_c^i + \lambda_c^i x_t + \sigma_c^i \varepsilon_{c,t+1},$$
$$\Delta d_{t+1} = \log \frac{D_{t+1}}{D_t} = \mu_d^i + \lambda_d^i x_t + \sigma_d^i \varepsilon_{d,t+1},$$
$$x_{t+1} = \phi_x^i x_t + \sigma_x^i \varepsilon_{x,t+1}.$$

The following derivations will use the Taylor expansion that, for $z = z_0 + u$ and small u,

$$\log(1+e^z) \approx \log(1+e^{z_0}) + \frac{e^{z_0}}{1+e^{z_0}} u = \log(1+e^{z_0}) + \kappa_1 u,$$

in which the constant κ_1 is smaller than but close to 1.

C.1 Log price-consumption ratio

Assume the log price-consumption ratio given $s_t = i$ is

$$\log \frac{P_{c,t}}{C_t} = z_{c,t} \approx A_{c0}^i + A_{c1}^i x_t.$$

Then,

$$\begin{aligned} r_{c,t+1} = & \log(1+e^{z_{c,t+1}}) + \Delta c_{t+1} - z_{c,t} \\ \approx & \textstyle\sum_j \mathbb{1}_{t+1}^j \big(\log(1+e^{A_{c0}^j}) + \kappa_{c1}^j A_{c1}^j x_{t+1} \big) + \Delta c_{t+1} - z_{c,t} \\ = & \textstyle\sum_j \mathbb{1}_{t+1}^j \big(\log(1+e^{A_{c0}^j}) + \kappa_{c1}^j A_{c1}^j \phi_x^i x_t + \kappa_{c1}^j A_{c1}^j \sigma_x^i \varepsilon_{x,t+1} \big) \\ & + \mu_c^i + \lambda_c^i x_t + \sigma_c^i \varepsilon_{c,t+1} - A_{c0}^i - A_{c1}^i x_t. \end{aligned}$$

Hence, $r_{c,t+1}$ is a mixture of normals, and

$$\begin{aligned} r_{c,t+1}^j \approx & \log(1+e^{A_{c0}^j}) + \kappa_{c1}^j A_{c1}^j \phi_x^i x_t + \kappa_{c1}^j A_{c1}^j \sigma_x^i \varepsilon_{x,t+1} \\ & + \mu_c^i + \lambda_c^i x_t + \sigma_c^i \varepsilon_{c,t+1} - A_{c0}^i - A_{c1}^i x_t. \end{aligned}$$

Then,

$$\begin{aligned} E_t^i[r_{c,t+1}] &= \textstyle\sum_j \pi^{ij} E_t^i[r_{c,t+1}^j] \\ &\approx \textstyle\sum_j \pi^{ij} \big(\log(1+e^{A_{c0}^j}) + \kappa_{c1}^j A_{c1}^j \phi_x^i x_t \big) + \mu_c^i + \lambda_c^i x_t - A_{c0}^i - A_{c1}^i x_t, \end{aligned}$$

and the consumption return innovation is

$$
\begin{aligned}
& r_{c,t+1} - E_t^i[r_{c,t+1}] \\
\approx & \textstyle\sum_j (\mathbb{1}_{t+1}^j - \pi^{ij})\big(\log(1+e^{A_{c0}^j}) + \kappa_{c1}^j A_{c1}^j \phi_x^i x_t\big) \\
& + \textstyle\sum_j \mathbb{1}_{t+1}^j \kappa_{c1}^j A_{c1}^j \sigma_x^i \varepsilon_{x,t+1} + \sigma_c^i \varepsilon_{c,t+1}.
\end{aligned}
$$

The pricing kernel is

$$
\begin{aligned}
m_{t+1} &= \theta^i \log\delta - \frac{\theta^i}{\psi}\Delta c_{t+1} + (\theta^i - 1) r_{c,t+1} \\
&= \theta^i \log\delta - \frac{\theta^i}{\psi}\big(\mu_c^i + \lambda_c^i x_t + \sigma_c^i \varepsilon_{c,t+1}\big) + (\theta^i - 1) r_{c,t+1}.
\end{aligned}
$$

Hence, m_{t+1} is also a mixture of normals, and

$$
m_{t+1}^j = \theta^i \log\delta - \frac{\theta^i}{\psi}\big(\mu_c^i + \lambda_c^i x_t + \sigma_c^i \varepsilon_{c,t+1}\big) + (\theta^i - 1) r_{c,t+1}^j.
$$

From the pricing equation,

$$
\begin{aligned}
0 &= \log E_t^i[e^{m_{t+1} + r_{c,t+1}}] \\
&\approx E_t^i[m_{t+1} + r_{c,t+1}] + \frac{1}{2}\operatorname{var}_t^i[m_{t+1} + r_{c,t+1}] \\
&= \textstyle\sum_j \pi^{ij} E_t^i[m_{t+1}^j + r_{c,t+1}^j] + \frac{1}{2}\operatorname{var}_t^i[m_{t+1} + r_{c,t+1}].
\end{aligned}
$$

Collect the linear x_t terms and, for tractability, ignore quantitatively small contributions from the variance term,

$$
\begin{aligned}
0 &\approx \textstyle\sum_j \pi^{ij}\Big(-\frac{\theta^i}{\psi}\lambda_c^i x_t + \theta^i\big(\kappa_{c1}^j A_{c1}^j \phi_x^i x_t + \lambda_c^i x_t - A_{c1}^i x_t\big)\Big), \\
0 &\approx -\frac{1}{\psi}\lambda_c^i + \Big(\textstyle\sum_j \pi^{ij}\kappa_{c1}^j \phi_x^i A_{c1}^j\Big) + \lambda_c^i - A_{c1}^i.
\end{aligned}
$$

That is,

$$
A_{c1}^i - \pi^{ii}\kappa_{c1}^i \phi_x^i A_{c1}^i - \pi^{ij}\kappa_{c1}^j \phi_x^i A_{c1}^j \approx (1 - \frac{1}{\psi})\lambda_c^i,
$$

or

$$
\frac{A_{c1}^i}{1-\frac{1}{\psi}} \approx \frac{\lambda_c^i}{1 - \pi^{ii}\kappa_{c1}^i \phi_x^i} + \frac{\pi^{ij}\kappa_{c1}^j \phi_x^i}{1 - \pi^{ii}\kappa_{c1}^i \phi_x^i}\frac{A_{c1}^j}{1-\frac{1}{\psi}}\ .
$$

Now the pricing kernel innovation is

$$
\begin{aligned}
& m_{t+1} - E_t^i[m_{t+1}] \\
\approx & -\frac{\theta^i}{\psi}\sigma_c^i \varepsilon_{c,t+1} \\
& + (\theta^i - 1)\Big(\textstyle\sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right)\big(\log(1+e^{A_{c0}^j}) + \kappa_{c1}^j A_{c1}^j \phi_x^i x_t\big) \\
& \qquad\qquad + \textstyle\sum_j \mathbb{1}_{t+1}^j \kappa_{c1}^j A_{c1}^j \sigma_x^i \varepsilon_{x,t+1} + \sigma_c^i \varepsilon_{c,t+1}\Big) \\
= & -\gamma^i \sigma_c^i \varepsilon_{c,t+1} - (1-\theta^i)\textstyle\sum_j \mathbb{1}_{t+1}^j \kappa_{c1}^j A_{c1}^j \sigma_x^i \varepsilon_{x,t+1} \\
& - (1-\theta^i)\textstyle\sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right)\big(\log(1+e^{A_{c0}^j}) + \kappa_{c1}^j A_{c1}^j \phi_x^i x_t\big) \\
= & -\xi_c^i \sigma_c^i \varepsilon_{c,t+1} - \textstyle\sum_j \mathbb{1}_{t+1}^j \xi_x^{ij} \sigma_x^i \varepsilon_{x,t+1} - \textstyle\sum_j \left(\mathbb{1}_{t+1}^j - \pi^{ij}\right) \xi_\pi^{ij} \,.
\end{aligned}
$$

In addition,

$$
\begin{aligned}
E_t^i[m_{t+1}] &= \theta^i \log\delta - \frac{\theta^i}{\psi}(\mu_c^i + \lambda_c^i x_t) + (\theta^i - 1)E_t^i[r_{c,t+1}] \\
&\approx -\frac{1}{\psi}\lambda_c^i x_t + (\theta^i - 1)\Big(-\frac{1}{\psi}\lambda_c^i x_t + E_t^i[r_{c,t+1}]\Big) + \text{const} \\
&\approx -\frac{1}{\psi}\lambda_c^i x_t + \text{const}.
\end{aligned}
$$

C.2 Risk-free rate

Given $s_t = i$,

$$
1 = E_t^i[e^{m_{t+1}+r_{f,t}}].
$$

Since m_{t+1} is a mixture of normals,

$$
-r_{f,t} = \log E_t^i[e^{m_{t+1}}] \approx E_t^i[m_{t+1}] + \frac{1}{2}\operatorname{var}_t^i[m_{t+1}].
$$

Hence,

$$
r_{f,t} \approx -\theta^i \log\delta + \frac{\theta^i}{\psi}(\mu_c^i + \lambda_c^i x_t) - (\theta^i - 1)E_t^i[r_{c,t+1}] - \frac{1}{2}\operatorname{var}_t^i[m_{t+1}].
$$

Add $(\theta^i - 1)r_{f,t}$ to both sides, and divide by θ^i (assume $\theta^i \neq 0$), then

$$
r_{f,t} \approx -\log\delta + \frac{1}{\psi}(\mu_c^i + \lambda_c^i x_t) - \frac{\theta^i - 1}{\theta^i}E_t^i[r_{c,t+1} - r_{f,t}] - \frac{1}{2\theta^i}\operatorname{var}_t^i[m_{t+1}].
$$

C.3 Log price-dividend ratio

Assume the log price-dividend ratio given $s_t = i$ is

$$\log \frac{P_t}{D_t} = z_t \approx A_0^i + A_1^i x_t.$$

Then,

$$\begin{aligned} r_{t+1} =& \log(1 + e^{z_{t+1}}) + \Delta d_{t+1} - z_t \\ &\approx \textstyle\sum_j \mathbb{1}_{t+1}^j \big(\log(1 + e^{A_0^j}) + \kappa_1^j A_1^j x_{t+1}\big) + \Delta d_{t+1} - z_t \\ &= \textstyle\sum_j \mathbb{1}_{t+1}^j \big(\log(1 + e^{A_0^j}) + \kappa_1^j A_1^j \phi_x^i x_t + \kappa_1^j A_1^j \sigma_x^i \varepsilon_{x,t+1}\big) \\ &\quad + \mu_d^i + \lambda_d^i x_t + \sigma_d^i \varepsilon_{d,t+1} - A_0^i - A_1^i x_t. \end{aligned}$$

Hence, r_{t+1} is a mixture of normals, and

$$\begin{aligned} r_{t+1}^j \approx& \log(1 + e^{A_0^j}) + \kappa_1^j A_1^j \phi_x^i x_t + \kappa_1^j A_1^j \sigma_x^i \varepsilon_{x,t+1} \\ &+ \mu_d^i + \lambda_d^i x_t + \sigma_d^i \varepsilon_{d,t+1} - A_0^i - A_1^i x_t. \end{aligned}$$

Then,

$$\begin{aligned} E_t^i[r_{t+1}] &= \textstyle\sum_j \pi^{ij} E_t^i[r_{t+1}^j] \\ &\approx \textstyle\sum_j \pi^{ij} \big(\log(1 + e^{A_0^j}) + \kappa_1^j A_1^j \phi_x^i x_t\big) + \mu_d^i + \lambda_d^i x_t - A_0^i - A_1^i x_t. \end{aligned}$$

From the pricing equation,

$$\begin{aligned} 0 &= \log E_t^i[e^{m_{t+1} + r_{t+1}}] \\ &\approx E_t^i[m_{t+1} + r_{t+1}] + \frac{1}{2} \operatorname{var}_t^i[m_{t+1} + r_{t+1}] \\ &= \textstyle\sum_j \pi^{ij} E_t^i[m_{t+1}^j + r_{t+1}^j] + \frac{1}{2} \operatorname{var}_t^i[m_{t+1} + r_{t+1}]. \end{aligned}$$

Collect the linear x_t terms and, for tractability, ignore quantitatively small contributions from the variance term,

$$\begin{aligned} 0 &\approx \textstyle\sum_j \pi^{ij} \Big(-\frac{1}{\psi} \lambda_c^i x_t + \kappa_1^j A_1^j \phi_x^i x_t + \lambda_d^i x_t - A^i x_t \Big), \\ 0 &\approx -\frac{1}{\psi} \lambda_c^i + \Big(\textstyle\sum_j \pi^{ij} \kappa_1^j A_1^j \phi_x^i \Big) + \lambda_d^i - A^i. \end{aligned}$$

That is,

$$A_1^i - \pi^{ii} \kappa_1^i A_1^i \phi_x^i - \pi^{ij} \kappa_1^j A_1^j \phi_x^i \approx \lambda_d^i - \frac{1}{\psi} \lambda_c^i,$$

or

$$A_1^i \approx \frac{\lambda_d^i - \frac{1}{\psi}\lambda_c^i}{1-\pi^{ii}\kappa_1^i\phi_x^i} + \frac{\pi^{ij}\kappa_1^j\phi_x^i}{1-\pi^{ii}\kappa_1^i\phi_x^i}A_1^j.$$

Now, the return innovation is

$$\begin{aligned}
&r_{t+1} - E_t^i[r_{t+1}] \\
&\approx \sigma_d^i\varepsilon_{d,t+1} + \sum\nolimits_j \mathbb{1}_{t+1}^j\kappa_1^j A_1^j\sigma_x^i\varepsilon_{x,t+1} + \sum\nolimits_j (\mathbb{1}_{t+1}^j - \pi^{ij})\big(\log(1+e^{A_0^j}) + \kappa_1^j A_1^j\phi_x^i x_t\big) \\
&= \sigma_d^i\varepsilon_{d,t+1} + \sum\nolimits_j \mathbb{1}_{t+1}^j\beta_x^j\sigma_x^i\varepsilon_{x,t+1} + \sum\nolimits_j (\mathbb{1}_{t+1}^j - \pi^{ij})\beta_\pi^{ij}.
\end{aligned}$$

The return variance is

$$\begin{aligned}
\mathrm{var}_t^i[r_{t+1}] &\approx (\sigma_d^i)^2 + \sum\nolimits_j \pi^{ij}(\kappa_1^j A_1^j\sigma_x^i)^2 + \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}(\beta_\pi^{i\mathrm{L}} - \beta_\pi^{i\mathrm{H}})^2 \\
&= (\sigma_d^i)^2 + \sum\nolimits_j \pi^{ij}(\kappa_1^j A_1^j\sigma_x^i)^2 \\
&\quad + \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}\Big(\log(1+e^{A_0^{\mathrm{L}}}) - \log(1+e^{A_0^{\mathrm{H}}}) + (\kappa_1^{\mathrm{L}}A_1^{\mathrm{L}} - \kappa_1^{\mathrm{H}}A_1^{\mathrm{H}})\phi_x^i x_t\Big)^2 \\
&\approx (\sigma_d^i)^2 + \sum\nolimits_j \pi^{ij}(\kappa_1^j A_1^j\sigma_x^i)^2 + \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}(A_0^{\mathrm{L}} - A_0^{\mathrm{H}})^2 \\
&\quad + 2\pi^{i\mathrm{L}}\pi^{i\mathrm{H}}(A_0^{\mathrm{L}} - A_0^{\mathrm{H}})(A_1^{\mathrm{L}} - A_1^{\mathrm{H}})\phi_x^i x_t \\
&= V_0^i + V_1^i x_t.
\end{aligned}$$

For the equity premium,

$$E_t^i[r_{t+1} - r_{f,t}] + \frac{1}{2}\,\mathrm{var}_t^i[r_{t+1}] \approx -\,\mathrm{cov}_t^i[m_{t+1}, r_{t+1}].$$

The contribution from i.i.d. growth shocks is the same as in the previous model. The contribution from long-run growth risk is

$$\begin{aligned}
&-\,\mathrm{cov}_t^i\Big[-\sum\nolimits_j \mathbb{1}_{t+1}^j\xi_x^{ij}\sigma_x^i\varepsilon_{x,t+1},\ \sum\nolimits_j \mathbb{1}_{t+1}^j\beta_x^j\sigma_x^i\varepsilon_{x,t+1}\Big] \\
&= \pi^{i\mathrm{L}}\xi_x^{i\mathrm{L}}\beta_x^{\mathrm{L}}(\sigma_x^i)^2 + \pi^{i\mathrm{H}}\xi_x^{i\mathrm{H}}\beta_x^{\mathrm{H}}(\sigma_x^i)^2.
\end{aligned}$$

The contribution from regime-shift risk is

$$\begin{aligned}
&-\,\mathrm{cov}_t^i\Big[-\sum\nolimits_j (\mathbb{1}_{t+1}^j - \pi^{ij})\,\xi_\pi^{ij}\,,\ \sum\nolimits_j (\mathbb{1}_{t+1}^j - \pi^{ij})\beta_\pi^{ij}\Big] \\
&= \pi^{i\mathrm{L}}\pi^{i\mathrm{H}}\Big(\xi_\pi^{i\mathrm{L}} - \xi_\pi^{i\mathrm{H}}\Big)\Big(\beta_\pi^{i\mathrm{L}} - \beta_\pi^{i\mathrm{H}}\Big).
\end{aligned}$$

The innovation to the return variance is

$$\begin{aligned}
&\mathrm{var}_{t+1}[r_{t+2}] - E_t^i[\mathrm{var}_{t+1}[r_{t+2}]] \\
&= \sum\nolimits_j \mathbb{1}_{t+1}^j V_1^j\sigma_x^i\varepsilon_{x,t+1} + \sum\nolimits_j (\mathbb{1}_{t+1}^j - \pi^{ij})V_0^j.
\end{aligned}$$

Then for

$$\text{cov}[r_{t+1} - E_t^i[r_{t+1}], \text{var}_{t+1}[r_{t+2}] - E_t^i[\text{var}_{t+1}[r_{t+2}]]],$$

the contribution from long-run growth risk is

$$\begin{aligned} & \text{cov}_t^i \left[\textstyle\sum_j \mathbb{1}_{t+1}^j \beta_x^j \sigma_x^i \varepsilon_{x,t+1},\ \sum_j \mathbb{1}_{t+1}^j V_1^j \sigma_x^i \varepsilon_{x,t+1} \right] \\ = & \textstyle\sum_j \pi^{ij} \beta_x^j V_1^j (\sigma_x^i)^2, \end{aligned}$$

and the contribution from regime-shift risk is

$$\begin{aligned} & \text{cov}_t^i \left[\textstyle\sum_j (\mathbb{1}_{t+1}^j - \pi^{ij}) \beta_\pi^{ij},\ \sum_j (\mathbb{1}_{t+1}^j - \pi^{ij}) V_0^j \right] \\ = & \pi^{i\text{L}} \pi^{i\text{H}} \left(\beta_\pi^{i\text{L}} - \beta_\pi^{i\text{H}} \right) \left(V_0^{\text{L}} - V_0^{\text{H}} \right). \end{aligned}$$

References

Abel, A. B. (1994). Exact solutions for expected rates of return under markov regime switching: Implications for the equity premium puzzle. *Journal of Money, Credit and Banking 26*, 345–361.

Ang, A. and G. Bekaert (2002a). Regime switches in interest rates. *Journal of Business and Economic Statistics 20*, 163–182.

Ang, A. and G. Bekaert (2002b). Short rate nonlinearities and regime switches. *Journal of Economic Dynamics and Control 26*, 1243–1274.

Ang, A., G. Bekaert, and M. Wei (2008). The term structure of real rates and expected inflation. *Journal of Finance 63*, 797–849.

Balke, N. and R. J. Gordon (1986). Appendix B: Historical data. In R. J. Gordon (Ed.), *The American Business Cycle: Continuity and Change*, pp. 781–850. Chicago, IL: University of Chicago Press.

Bansal, R., D. Kiku, and A. Yaron (2007). Long run risks: Estimation and inference. Working paper. Duke University and University of Pennsylvania.

Bansal, R., D. Kiku, and A. Yaron (2010). Long run risks, the macroeconomy, and asset prices. *American Economic Review: Papers and Proceedings 100*, 542–546.

Bansal, R. and I. Shaliastovich (2011). Learning and asset-price jumps. *Review of Financial Studies 24*, 2738–2780.

Bansal, R., G. Tauchen, and H. Zhou (2004). Regime shifts, risk premiums in the term structure, and the business cycle. *Journal of Business and Economic Statistics 22*, 396–409.

Bansal, R. and A. Yaron (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance 59*, 1481–1509.

Bansal, R. and H. Zhou (2002). Term structure of interest rates with regime shifts. *Journal of Finance 57*, 1997–2043.

Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics 121*, 823–866.

Bekaert, G. and E. Engstrom (2010). Asset return dynamics under bad environment-good environment fundamentals. Working paper. Columbia University.

Burns, A. F. and W. C. Mitchell (1946). *Measuring Business Cycles*. New York, NY: National Bureau of Economic Research.

Cai, J. (1994). A Markov model of switching-regime ARCH. *Journal of Business and Economic Statistics 12*, 309–16.

Campbell, J. Y. and J. H. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy 107*, 205–251.

Campbell, J. Y. and L. Hentschel (1992). No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of Financial Economics 31*, 281–318.

Cecchetti, S. G., P. Lam, and N. C. Mark (1990). Mean reversion in equilibrium asset prices. *American Economic Review 80*, 398–418.

Cecchetti, S. G., P. Lam, and N. C. Mark (1993). The equity premium and the risk-free rate: Matching the moments. *Journal of Monetary Economics 31*, 21–45.

Dai, Q., K. J. Singleton, and W. Yang (2007). Regime shifts in a dynamic term structure model of U.S. Treasury bond yields. *Review of Financial Studies 20*, 1669–1706.

Drechsler, I. and A. Yaron (2011). What's vol got to do with it. *Review of Financial Studies 24*, 1–45.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*, 937–969.

Filardo, A. J. (1994). Business-cycle phases and their transitional dynamics. *Journal of Business and Economic Statistics 12*, 299–308.

Garcia, R. (1998). Asymptotic null distribution of the likelihood ratio test in markov switching models. *International Economic Review 39*, 763–788.

Garcia, R. and P. Perron (1996). An analysis of the real interest rate under regime shifts. *Review of Economics and Statistics 78*, 111–125.

Ghosh, A. and G. M. Constantinides (2010). The predictability of returns with regime shifts in consumption and dividend growth. Working paper. Carnegie Mellon University and University of Chicago.

Glosten, L., R. Jaganathan, and D. E. Runkle (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance 48*, 1779–1801.

Goodwin, T. H. (1993). Business-cycle analysis with a markov-switching model. *Journal of Business and Economic Statistics 11*, 331–339.

Gray, S. F. (1996). Modeling the conditional distribution of interest rates as a regime switching process. *Journal of Financial Economics 42*, 27–62.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica 57*, 357–384.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton, NJ: Princeton University Press.

Hamilton, J. D. and G. Lin (1996). Stock market volatility and the business cycle. *Journal of Applied Econometrics 11*, 573–93.

Hamilton, J. D. and R. Susmel (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics 64*, 307–333.

Hansen, B. E. (1992). The likelihood ratio test under nonstandard conditions: Testing the markov switching model of GNP. *Journal of Applied Econometrics 7*, S61–S82.

Judd, K. (1998). *Numerical Methods in Economics*. Cambridge, MA: MIT Press.

Kandel, S. and R. F. Stambaugh (1990). Expectations and volatility of consumption and asset returns. *Review of Financial Studies 3*, 207–232.

Kandel, S. and R. F. Stambaugh (1991). Asset returns and intertemporal preferences. *Journal of Monetary Economics 27*, 39–71.

Lam, P. (1990). The Hamilton model with a general autoregressive component. *Journal of Monetary Economics 26*, 409–432.

Lettau, M., S. C. Ludvigson, and J. A. Wachter (2008). The declining equity premium: What role does macroeconomic risk play? *Review of Financial Studies 21*, 1653–1687.

Maheu, J. M. and T. H. McCurdy (2000). Identifying bull and bear markets in stock returns. *Journal of Business and Economic Statistics 18*, 100–112.

Ozoguz, A. (2009). Good times or bad times? Investors' uncertainty and stock returns. *Review of Financial Studies 22*, 4377–4422.

Perez-Quiros, G. and A. Timmermann (2001). Business cycle asymmetries in stock returns: Evidence from higher order moments and conditional densities. *Journal of Econometrics 103*, 259–306.

Rietz, T. A. (1988). The equity risk premium: A solution. *Journal of Monetary Economics 22*, 117–131.

Turner, C. M., R. Startz, and C. R. Nelson (1989). A Markov model of heteroskedasticity, risk, and learning in the stock market. *Journal of Financial Economics 25*, 3–22.

Veronesi, P. (2004). The peso problem hypothesis and stock market returns. *Journal of Economic Dynamics and Control 28*, 707–725.

Wachter, J. A. (2011). Can time-varying risk of rare disasters explain aggregate stock market volatility? Working paper. University of Pennsylvania.

Whitelaw, R. F. (2000). Stock market risk and return: An equilibrium approach. *Review of Financial Studies 13*, 521–547.

Table 1: Two-regime descriptive model on empirical annual consumption growth

	Two regimes		Single regime
	Low vol	High vol	
Mean (%)	2.22 (0.14)	0.47 (1.01)	1.86 (0.29)
Vol (%)	1.16 (0.10)	3.90 (0.88)	2.16 (0.11)
Tran prob	0.966 (0.035)	0.852 (0.142)	
$\frac{1}{T}\log f$	2.6933		2.4148

Likelihood ratio: $-2\log(f^{1\mathrm{R}}/f^{2\mathrm{R}})$

Data	Simulation			p-value
	Mean	95%	99%	
45.1	4.6	14.2	19.5	$< 10^{-4}$

The top panel presents the maximum likelihood estimation of a two-regime descriptive model on the empirical annual consumption growth rates. The parameters include the means, the volatilities, and the transition probabilities within the same regime. A single-regime descriptive model is also estimated as a comparison. The standard errors are reported in parentheses. The likelihood function is denoted by f. The empirical sample period is 1929–2010, and thus for the growth rate data $T = 81$ years.

The bottom panel presents the simulated likelihood ratio test on the null hypothesis of a single-regime specification. I simulate 10,000 samples using the single-regime parameter estimates. For each sample, I estimate both the two-regime and the single-regime models and compute the likelihood ratio. I report the mean and the percentiles of the 10,000 likelihood ratios obtained from the simulated samples. All the simulated likelihood ratios are lower than that obtained from the empirical data. I thus conclude that the single-regime null hypothesis is strongly rejected with a p-value smaller than 10^{-4}.

Table 2: Empirical properties of growth rate data

	Low vol	High vol
	Est (Std Err)	Est (Std Err)
$\mu[\Delta c]$ (%)	2.22 (0.14)	0.39 (1.00)
$\sigma[\Delta c]$ (%)	1.11 (0.09)	3.99 (0.58)
$AC(1)[\Delta c]$	0.32 (0.10)	0.47 (0.17)
$AC(2)[\Delta c]$	0.06 (0.13)	0.10 (0.22)
$AC(3)[\Delta c]$	-0.03 (0.10)	-0.16 (0.19)
$\mu[\Delta d]$ (%)	1.94 (1.3)	-1.87 (5.1)
$\sigma[\Delta d]$ (%)	10.8 (1.0)	20.4 (2.1)
$AC(1)[\Delta d]$	0.22 (0.13)	0.24 (0.34)
$AC(2)[\Delta d]$	-0.04 (0.12)	-0.14 (0.47)
Correlation	0.27 (0.10)	0.61 (0.13)

This table presents the properties of empirical annual consumption and dividend growth rates within the low and high consumption volatility regimes. The regimes are classified using the smoothed regime probabilities computed following the maximum likelihood estimation of a two-regime descriptive model on the empirical annual consumption growth rates.

In the table, Δc is consumption growth, Δd is dividend growth, $\mu[\,]$ is the mean, $\sigma[\,]$ is the standard deviation, and $AC(1)$ to $AC(3)$ are the first- to third-order autocorrelations.

The empirical sample period is 1929–2010. The estimates and the standard errors (in parentheses) are obtained with GMM.

Table 3: Empirical properties of asset market data

A. Asset prices

	Low vol	High vol
	Est (Std Err)	Est (Std Err)
$\mu[r-r_f]$ (%)	6.36 (1.88)	-2.03 (7.87)
$\sigma[r]$	0.157 (0.015)	0.294 (0.034)
$\mu[r_f]$ (%)	1.03 (0.38)	0.03 (1.67)
$\sigma[r_f]$ (%)	3.05 (0.41)	6.68 (1.34)
$\mu[p-d]$	3.31 (0.05)	3.15 (0.07)
$\sigma[p-d]$	0.393 (0.030)	0.299 (0.052)
$AC(1)[p-d]$	0.94 (0.04)	0.63 (0.21)
$AC(2)[p-d]$	0.89 (0.06)	0.23 (0.30)

B. Return predictability

Horizon	Low vol		High vol	
(Year)	Slope (Std Err)	R^2	Slope (Std Err)	R^2
1	-0.10 (0.04)	0.07	-0.37 (0.28)	0.10
2	-0.18 (0.07)	0.10	-0.67 (0.25)	0.16
3	-0.23 (0.08)	0.13	-1.79 (0.81)	0.39

This table presents the properties of the empirical annual asset market data within the low and high consumption volatility regimes. The regimes are classified using the smoothed regime probabilities computed following the maximum likelihood estimation of a two-regime descriptive model on the empirical annual consumption growth rates.

In Panel A, r is the stock return, r_f is the risk-free rate, $p-d$ is the log price-dividend ratio, $\mu[\,]$ is the mean, $\sigma[\,]$ is the standard deviation, and $AC(1)$ to $AC(2)$ are the first- to second-order autocorrelations.

In Panel B, the cumulative excess stock return from the end of year t to the end of year $t+J$ is regressed on the log price-dividend ratio at the end of year t, and J is the horizon.

The empirical sample period is 1929–2010. The estimates and the standard errors (in parentheses) are obtained with GMM.

Table 4: Calibrated parameters for the model with i.i.d. growth rates

Consumption and dividend

Regime	μ_c^i	σ_c^i	μ_d^i	σ_d^i	χ^i	π^{ii}
L	0.00183	0.0042	0.0015	0.037	0.27	0.9969
H	0.0003	0.0155	-0.0009	0.081	0.61	0.9865

Preferences

Regime	δ	γ^i	ψ
L	0.9986	10	1.5
H		15	

This table presents the calibrated parameters for the model with i.i.d. consumption and dividend growth within each regime. The growth rate dynamics are in Eqs. (10) to (12), and the preferences are in Eq. (8). The calibration is at the monthly interval.

Table 5: Maximum likelihood estimation on annual consumption growth simulated from the model with i.i.d. growth rates

	Data	Model			
		All series	Low vol = 65, High vol = 16		
		Mean	Mean	5%	95%
		Low volatility regime			
Mean (%)	2.22	2.20	2.21	1.87	2.58
Vol (%)	1.16	1.15	1.16	0.93	1.40
Tran prob	0.966	0.960	0.959	0.891	0.984
		High volatility regime			
Mean (%)	0.47	0.58	0.50	-1.93	2.87
Vol (%)	3.90	3.62	3.82	1.29	5.31
Tran prob	0.852	0.841	0.853	0.673	0.940

This table compares the maximum likelihood estimation of a two-regime descriptive model on annual consumption growth in the empirical data versus that in the simulated data. The parameters include the means, the volatilities, and the transition probabilities within the same regime.

The empirical sample period is 1929–2010. The simulations are based on the model with i.i.d. consumption and dividend growth within each regime. Each simulated sample is 81×12-month long, and is then time-aggregated to the annual frequency. The parameters are estimated using maximum likelihood. The regimes are classified using the smoothed regime probabilities computed following the maximum likelihood estimation. The means and percentiles of the estimates are reported for the entire 30,000 simulated series and for about 1,000 simulated series that contain 65 years of the low consumption volatility regime and 16 years of the high consumption volatility regime.

Table 6: Properties of growth rates simulated from the model with i.i.d. growth rates

	Data	Model			
		All series	Low vol = 65 High vol = 16		
		Mean	Mean	5%	95%
		Low volatility regime			
$\mu[\Delta c]$ (%)	2.22	2.20	2.21	1.85	2.59
$\sigma[\Delta c]$ (%)	1.11	1.15	1.16	0.91	1.40
$\mu[\Delta d]$ (%)	1.94	1.94	1.95	-0.70	4.50
$\sigma[\Delta d]$ (%)	10.8	10.8	10.9	9.1	13.0
Correlation	0.27	0.26	0.26	0.06	0.46
		High volatility regime			
$\mu[\Delta c]$ (%)	0.39	0.35	0.33	-2.26	3.02
$\sigma[\Delta c]$ (%)	3.99	3.90	3.93	1.26	5.69
$\mu[\Delta d]$ (%)	-1.87	-1.62	-1.97	-14.0	8.67
$\sigma[\Delta d]$ (%)	20.4	19.7	20.4	10.8	28.9
Correlation	0.61	0.61	0.62	0.15	0.84

This table compares the properties of annual consumption and dividend growth rates within the low and high consumption volatility regimes in the empirical data versus those in the simulated data.

In the table, Δc is consumption growth, Δd is dividend growth, $\mu[\ ]$ is the mean, and $\sigma[\ ]$ is the standard deviation.

The empirical sample period is 1929–2010. The simulations are based on the model with i.i.d. consumption and dividend growth within each regime. Each simulated sample is 81 × 12-month long, and is then time-aggregated to the annual frequency. The regimes are classified using the smoothed regime probabilities computed following the maximum likelihood estimation of a two-regime descriptive model on annual consumption growth. The means and percentiles of the estimates are reported for the entire 30,000 simulated series and for about 1,000 simulated series that contain 65 years of the low consumption volatility regime and 16 years of the high consumption volatility regime.

Table 7: Properties of asset market data simulated from the model with i.i.d. growth rates

	Data	Model			
		All series	Low vol = 65 High vol = 16		
		Mean	Mean	5%	95%
		Low volatility regime			
$\mu[r-r_f]$ (%)	6.36	6.35	6.37	3.43	9.05
$\sigma[r]$	0.157	0.147	0.149	0.120	0.186
$\mu[r_f]$ (%)	1.03	1.82	1.81	1.38	2.00
$\mu[p-d]$	3.31	2.72	2.72	2.68	2.75
		High volatility regime			
$\mu[r-r_f]$ (%)	-2.03	8.59	8.71	-3.09	20.38
$\sigma[r]$	0.294	0.269	0.276	0.147	0.373
$\mu[r_f]$ (%)	0.03	-0.84	-0.97	-1.87	1.71
$\mu[p-d]$	3.15	2.51	2.50	2.39	2.71

This table compares the properties of annual asset market data within the low and high consumption volatility regimes in the empirical data versus those in the simulated data.

In the table, r is the stock return, r_f is the risk-free rate, $p-d$ is the log price-dividend ratio, $\mu[\,]$ is the mean, and $\sigma[\,]$ is the standard deviation.

The empirical sample period is 1929–2010. The simulations are based on the numerical solution of the model with i.i.d. consumption and dividend growth within each regime. Each simulated sample is 81×12-month long, and is then time-aggregated to the annual frequency. The regimes are classified using the smoothed regime probabilities computed following the maximum likelihood estimation of a two-regime descriptive model on annual consumption growth. The means and percentiles of the estimates are reported for the entire 30,000 simulated series and for about 1,000 simulated series that contain 65 years of the low consumption volatility regime and 16 years of the high consumption volatility regime.

Table 8: Calibrated parameters for the model with expected growth components

Consumption and dividend

Regime	ϕ_x^i	σ_x^i	μ_c^i	σ_c^i	λ_c^i	μ_d^i	σ_d^i	χ^i	λ_d^i	π^{ii}
L	0.98	0.00008	0.00183	0.0036	1	0.0015	0.036	0.27	1	0.9969
H	0.97	0.000293	0.0003	0.0114	3.5	-0.0009	0.079	0.61	5	0.9865

Preferences

Regime	δ	γ^i	ψ
L	0.9984	6	1.5
H		9	

This table presents the calibrated parameters for the model with expected growth components. The growth rate dynamics are in Eqs. (30) to (32), and the preferences are in Eq. (8). The calibration is at the monthly interval.

Table 9: Properties of growth rates simulated from the model with expected growth components

	Data	Model			
		All series	Low vol = 65 High vol = 16		
		Mean	Mean	5%	95%
		Low volatility regime			
$AC(1)[\Delta c]$	0.32	0.27	0.27	0.04	0.48
$AC(2)[\Delta c]$	0.06	0.05	0.05	-0.18	0.31
$AC(3)[\Delta c]$	-0.03	0.04	0.05	-0.20	0.28
$AC(1)[\Delta d]$	0.22	0.22	0.22	0.01	0.40
$AC(2)[\Delta d]$	-0.04	-0.03	-0.03	-0.26	0.20
		High volatility regime			
$AC(1)[\Delta c]$	0.47	0.39	0.42	-0.16	0.78
$AC(2)[\Delta c]$	0.10	0.04	0.04	-0.51	0.63
$AC(3)[\Delta c]$	-0.16	-0.01	-0.04	-0.66	0.63
$AC(1)[\Delta d]$	0.24	0.19	0.18	-0.25	0.56
$AC(2)[\Delta d]$	-0.14	-0.09	-0.10	-0.58	0.41

This table compares the properties of annual consumption and dividend growth rates within the low and high consumption volatility regimes in the empirical data versus those in the simulated data.

In the table, Δc is consumption growth, Δd is dividend growth, and $AC(1)$ to $AC(3)$ are the first- to third-order autocorrelations.

The empirical sample period is 1929–2010. The simulations are based on the model with expected growth components. Each simulated sample is 81×12-month long, and is then time-aggregated to the annual frequency. The regimes are classified using the smoothed regime probabilities computed following the maximum likelihood estimation of a two-regime descriptive model on annual consumption growth. The means and percentiles of the estimates are reported for the entire 30,000 simulated series and for about 1,000 simulated series that contain 65 years of the low consumption volatility regime and 16 years of the high consumption volatility regime.

Table 10: Properties of asset market data simulated from the model with expected growth components

	Data	Model			
		All series	Low vol = 65 High vol = 16		
		Mean	Mean	5%	95%
Low volatility regime					
$\mu[r-r_f]$ (%)	6.36	6.47	6.46	3.46	9.18
$\sigma[r]$	0.157	0.146	0.147	0.113	0.189
$\mu[r_f]$ (%)	1.03	1.05	1.05	0.63	1.42
$\sigma[r_f]$ (%)	3.05	0.71	0.74	0.43	1.22
$\mu[p-d]$	3.31	2.81	2.81	2.77	2.84
$\sigma[p-d]$	0.393	0.098	0.101	0.067	0.148
$AC(1)[p-d]$	0.94	0.81	0.81	0.55	0.89
$AC(2)[p-d]$	0.89	0.65	0.64	0.11	0.75
High volatility regime					
$\mu[r-r_f]$ (%)	-2.03	8.40	8.41	-3.10	20.03
$\sigma[r]$	0.294	0.269	0.275	0.134	0.377
$\mu[r_f]$ (%)	0.03	-0.83	-0.85	-3.95	1.69
$\sigma[r_f]$ (%)	6.68	2.12	2.28	0.58	3.91
$\mu[p-d]$	3.15	2.61	2.60	2.43	2.81
$\sigma[p-d]$	0.299	0.193	0.196	0.082	0.255
$AC(1)[p-d]$	0.63	0.64	0.64	0.10	0.89
$AC(2)[p-d]$	0.23	0.25	0.21	-0.40	0.78

This table compares the properties of annual asset market data within the low and high consumption volatility regimes in the empirical data versus those in the simulated data.

In the table, r is the stock return, r_f is the risk-free rate, $p-d$ is the log price-dividend ratio, $\mu[\,]$ is the mean, $\sigma[\,]$ is the standard deviation, and $AC(1)$ to $AC(2)$ are the first- to second-order autocorrelations.

The empirical sample period is 1929–2010. The simulations are based on the numerical solution of the model with expected growth components. Each simulated sample is 81×12-month long, and is then time-aggregated to the annual frequency. The regimes are classified using the smoothed regime probabilities computed following the maximum likelihood estimation of a two-regime descriptive model on annual consumption growth. The means and percentiles of the estimates are reported for the entire 30,000 simulated series and for about 1,000 simulated series that contain 65 years of the low consumption volatility regime and 16 years of the high consumption volatility regime.

Table 11: Return predictability in asset market data simulated from the model with expected growth components

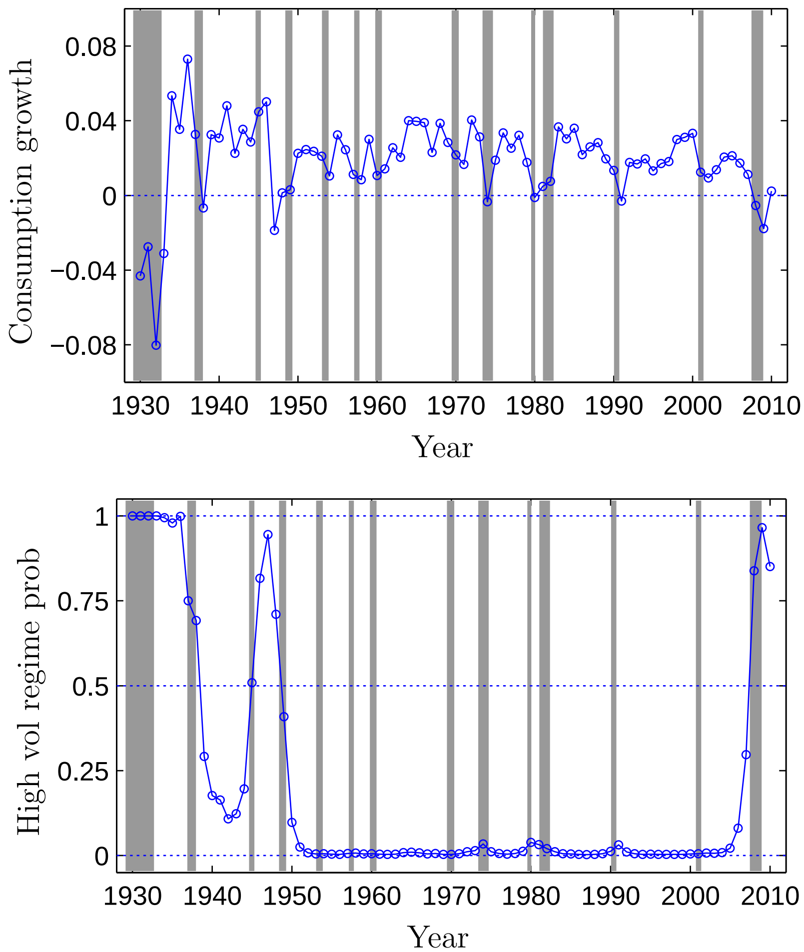
Horizon (Year)		Data	Model: All series	Model: Low vol = 65, High vol = 16
		Low volatility regime		
1	Slope	-0.10	-0.17	-0.18
	R^2	0.04	0.04	0.03
2	Slope	-0.18	-0.24	-0.24
	R^2	0.07	0.05	0.04
3	Slope	-0.23	-0.31	-0.30
	R^2	0.08	0.06	0.05
		High volatility regime		
1	Slope	-0.37	-0.25	-0.22
	R^2	0.10	0.12	0.09
2	Slope	-0.67	-0.39	-0.40
	R^2	0.16	0.19	0.16
3	Slope	-1.79	-0.53	-0.57
	R^2	0.39	0.25	0.24

This table compares the excess stock return predictability in annual asset market data within the low and high consumption volatility regimes in the empirical data versus those in the simulated data.

The cumulative excess stock return from the end of year t to the end of year $t + J$ is regressed on the log price-dividend ratio at the end of year t, and J is the horizon.

The empirical sample period is 1929–2010. The simulations are based on the numerical solution of the model with expected growth components. Each simulated sample is 81×12-month long, and is then time-aggregated to the annual frequency. The regimes are classified using the smoothed regime probabilities computed following the maximum likelihood estimation of a two-regime descriptive model on annual consumption growth. The means of the estimates are reported for the entire 30,000 simulated series and for about 1,000 simulated series that contain 65 years of the low consumption volatility regime and 16 years of the high consumption volatility regime.

Figure 1: Regimes in consumption growth



The top panel plots the empirical annual consumption growth. The bottom panel plots the smoothed probability for the high consumption volatility regime computed following the maximum likelihood estimation of a two-regime descriptive model on annual consumption growth.

The empirical sample period is 1929–2010, and thus 1930–2010 for the growth rates. The shaded regions mark the NBER dated recessions.