# The Impact of an Early Career Recession on Schooling and Lifetime Welfare*

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**Abstract**

This paper evaluates the long-term welfare consequences from experiencing a recession as youths, taking into account the impact on schooling, future job mobility, human capital accumulation, labor supply and wages. The paper also explores the mechanisms that account for lifetime wage changes by decomposing those changes into different channels: changes from schooling, from work experience, and from job mobility. To achieve these goals, this paper develops and estimates a search equilibrium model with heterogenous agents and aggregate shocks. The model is an extension of a directed search model, the Block Recursive Equilibrium framework of Menzio and Shi (2010a), which remains tractable when it is solved outside of the steady state. The counterfactual analysis shows that experiencing the 1981-1982 recession at age 16-22 causes a 2.2% to 3.0% loss in lifetime welfare. Endogenizing schooling decision avoids overestimation of the welfare loss. The wage decomposition shows that the loss from job mobility explains the majority of the wage loss during the recession, and the loss in experience and tenure persists long after the recession.

**Keywords:** Business cycle, earning, schooling, directed search

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# 1 Introduction

The recent US recession has brought renewed interest in the relationship between individual careers and the business cycle. The impact of the business cycle on early careers is of particular concern because young workers are most vulnerable to economic shocks. During the recession, the youth unemployment rate (age 16-25) rose to a peak of 20.1% in 2010, compared to 11.5% in 2007, the year before the recession started. During the same period, young workers (age 16-25) faced an 8% decrease in wages relative to 2007.[1] In sum, individuals entering the labor market in a recession suffer from unemployment or work in a lower-paid job. More importantly, the occurrence of a recession during the period when youths are transiting from school to work affects human capital accumulation and job mobility decisions, therefore, may have a potential long-term impact on lifetime wages and employment opportunities. As suggested in many previous literature, students who enter the labor market in a recession suffer from persistent wage loss that can last for 5 to 20 years.[2]

Although youth unemployment increased during the recent recession, there was also an increase in the school enrollment rate, from 28.3% in 2007 to 32.6% in 2010.[3] Because the opportunity cost of schooling decreases during a recession, postponing entry into the laborforce by remaining in school may be an optimal response. The resulting increase in schooling may increase future wages and employment and, thus, partially offset the potential long-term negative impact of the recession. Moreover, there can be a change in the workforce composition across graduation cohorts if agents with different characteristics make different schooling decisions over the business cycle. To fully account for the lifetime consequences of a recession that occurs during the school-to-work transition period, it is necessary to conduct a welfare analysis that takes into account the impact on schooling, job mobility, human capital accumulation, labor supply and wages.

This paper evaluates the long-term welfare and labor market consequences of experiencing a recession as a youth. I also identify different channels that account for lifetime wage changes:

[1] Source: Current Population Survey.

[2] Adda et al. (2013), Oreopoulos et al. (2012), Kahn (2010), Oyer (2008), Oyer (2006), etc.

[3] Source: Current Population Survey. School enrollment rate is the proportion of people between age 16 to 25 who are full-time students. Although there was an upward trend in school enrollment rate during this period, the increase was 2% higher than the trend.

schooling, work experience, and job mobility. To achieve these goals, I develop and estimate a search equilibrium model with heterogenous agents and aggregate shocks. The model is an extension of the Block Recursive Equilibrium (BRE) model of a directed search model proposed by Menzio and Shi (2010a). Labor markets are divided into submarkets, which are defined by a piece-rate wage contract, as well as by the worker's characteristics (skill and age) at the time the firm and the worker meet. The contractual environment is such that the firm commits to a wage that remains a constant proportion of output throughout the duration of the employment relationship. On the labor supply side of the market, individuals make schooling, work and search decisions, including on-the-job search. Given their characteristics, individuals direct their search to a wage contract that maximizes expected utility. On the labor demand side of the market, firms decide whether to open a vacancy and in which submarket to open the vacancy. Workers and firms are brought into contact according to a constant returns to scale matching function. In equilibrium, the distribution of wages across workers and the level of unemployment are determined by the free entry condition of firms and workers in the search market.

In the model, individuals are heterogenous in their skill, utility value of leisure, utility value of schooling and the productivity shocks they experience during their lifetime. Skill is determined by an individual's schooling, experience, job tenure and initial human capital endowment. Upon meeting, a productivity level of the match is determined, and the match productivity stays constant throughout the lifetime of the match. A worker's output is determined by the current level of aggregate productivity, by the match productivity at the time the worker and the firm met and by the worker's skill. The model incorporates two types of driving forces of the business cycle, shocks to aggregate productivity and shocks to match productivity.[4]

Menzio and Shi (2010a) shows that solving a BRE model with heterogenous agents and productivity shocks is equivalent to solving a representative agent model. This allows me to avoid the analytical and computational difficulties involved in solving equilibrium labor market models with random search.[5] This is because in the BRE, agents' value functions and policy functions only de-

[4]Eyigungor (2010) shows that the addition of match productivity shocks solves the problem that shocks to aggregate productivity can account for only a small portion of the volatility in unemployment and vacancies.

[5]Robin (2011) proposes a random search model with heterogeneous agents and aggregate productivity shocks and shows that the model remains tractable when it is solved outside of the steady state. However, the tractability

pend on productivity shocks, and not on the endogenous distribution of workers across employment states. Hence, the model remains tractable when it is solved outside of the steady state where the distribution of workers across different wages and employment states is allowed to change over the business cycle.

I modify the framework of Menzio and Shi (2010a) to accommodate the research questions raised in this paper. First, I endogenize the schooling decision over the business cycle. In my model, an individual is allowed to decide when to enter the labor market, and whether and when to return to school. Second, I enrich agent heterogeneity by allowing agents to differ in characteristics that are potentially unobserved to researchers. For example, agents can have different initial human capital endowments, as well as different preferences for leisure and schooling. Third, I assume piece-rate wage contracts where workers are always paid a fixed fraction of their current period output while Menzio and Shi (2010a) use fixed-wage contracts and dynamic contracts. Using piece-rate contracts allows me to decompose the wage change into the gain from human capital accumulation, which is reflected by the change in the output, and the gain from on-the-job search, which is reflected by the change in the share given to workers.

I solve the equilibrium outside of the steady state where wages and the unemployment rate are endogenously determined over the business cycle. I estimate the model using simulated method of moments. Sample statistics used in the estimation are from the Monthly Current Population Survey (CPS) from 1982 to 2012 and the 1979 and 1997 National Longitudinal Surveys of Youth (NLSY). The CPS is primarily a cross-sectional data set that provides information on employment, income, school enrollment and demographics. The NLSY79 and NLSY97 are longitudinal data sets that provide supplementary information on experience and tenure. The sample is restricted to be white male individuals age 16 - 65.[6] The model is fit to the distributions of employment, income and labor market transitions over the life cycle and over the business cycle.

To quantify the lifetime impact of a recession, I perform a counterfactual analysis that evaluates the lifetime welfare loss from experiencing the 1981-1983 recession as a youth. On average, there

depends on several assumptions, including exogenous matching rate and full monopsony power of employers.

[6]The sample is restricted to the white male sample because their labor supply decisions are least sensitive to external factors such as childbearing or discrimination.

was a 3 percentage point (ppt) decrease in the employment rate and a 1.5% wage drop during the recession period. According to the estimation, it was caused by a 0.5% decrease in aggregate productivity and a 10% decrease in match productivity. For cohorts who turned age 16-22 in 1980, the 1980s recession leads to a 2.2% to 3.0% loss in lifetime welfare (in consumption dollars). Students with higher schooling level suffer from a larger welfare loss. This is because on average, there is a 0.25 year increase in schooling due to the recession and the increase is larger for the less educated. There is an initial 20% - 35% wage drop during the recession, which lasts for 5 - 10 years after the recession. The loss in the employment rate recovers right after the recession.

I also find that without endogenizing schooling decisions overestimates the welfare loss. Firstly, an early career recession leads an increase in schooling, which increases future wages by 4% and increases employment rate by 1 ppt. Therefore, acquiring more schooling can partially offset the negative impact from an early career recession. In addition, there is a change in the workforce composition with respect to initial human capital endowment. Those with higher initial human capital endowment are more likely to stay at school rather than enter the labor market in a recession. Hence, the wage loss in a recession is partly due to the fact that individuals with lower human capital are more likely to work in a recession.

The mechanisms underlying the life cycle wage changes are explored through a decomposition analysis. Individuals face a tradeoff when a recession occurs at their school-to-work transition period. On the one hand, they can start with a lower-paid job and catch up later through on-the-job search. However, it may take time to climb up the wage ladder due to search frictions. On the other hand, they can go to school or stay at home and wait until the recession is over. In either case, at the later time of labor market entry, they will lose experience and tenure. The wage decomposition shows that during the recession, 44.0% of the wage loss comes from job shopping, 40.7% comes from the decline in productivity, 19.1% comes from the loss in experience and tenure, and the increase in schooling offsets the wage loss by 3.8%. After the recession, the loss from job shopping recovers in 3 years. In contrast, the loss in experience and tenure recovers in 14 years. At the same time, the increase in schooling not only improves output, but also results in an increase in the share of output that workers can get from firms, which further increases wages.

This paper is related to several strands of literature, the literature on the long-term impact of early events on wages and employment, the literature on the effect of the business cycle on schooling and the literature on directed search. The first branch of literature explores whether a bad event occurring at an early career stage can cause a persistent loss in wages or employment opportunities, what is referred to as the "scarring effect". Ellwood (1982) was the first to examine the scarring effect from early unemployment and found persistent negative effects on wages, but no evidence on recurrent unemployment. His results have been confirmed by other follow-up studies in the US (Ruhm (1991), Jacobson et al. (1993), Stevens (1997), Davis and von Wachter (2011)). European studies (Franz et al. (1997), Gregg (2001), Gregg and Tominey (2005), von Wachter and Bender (2006), Skans (2010)) have found scarring effects from early unemployment on both wages and future unemployment in Britain, German and Sweden. At the same time, economists find that a recession occurring at labor market entry can also cause a long-term wage loss (Bowlus and Liu (2003), Raaum and Røed (2006), Oyer (2006), Oyer (2008), Genda et al. (2010), Oreopoulos et al. (2012)). For example, Kahn (2010) looks at American college graduates who entered the job market prior, during and after the 1980's recession and observes that a 1% increase in the unemployment rate at the time of graduation leads to an initial wage loss of 7% that recovers in 15 years. The most closely related paper is Adda et al. (2013) that analyzes how careers are affected by economic downturns for skilled and unskilled workers in Germany. They estimate a dynamic partial equilibrium model of vocational training choice, labor supply, and wage progression and find that exposure to an economic shock early in a worker's career leads to wage reductions that persist for 5 to 10 years. My paper contributes to this literature by developing and estimating a general equilibrium model to investigate and quantify the channels of the lifetime welfare loss of a recession that occurs at potential labor market entry. My paper also suggests that without endogenizing people's schooling decision during a recession, previous results overestimate the scarring effect.

This paper is also related to the literature that studies the cyclical pattern of schooling. Dellas and Koubi (2003) examine the cyclicality in school enrollment rates of various age groups in the US from 1950 to 1990. Betts and McFarland (1995) examine the impact of the business cycle on enrollments at individual community colleges between the late 1960s and the mid-1980s. They find that the overall pattern is countercyclical. This paper links the impact of the business cycle on schooling with the impact of schooling on lifetime wages and employment. In addition, I show that schooling decisions change the workforce composition with respect to initial human capital endowment during a recession.

Lastly, the theoretical part of my paper is related to the literature on search models. Robin (2011) proposes a random search model with heterogeneous agents and aggregate productivity shocks and shows that the model remains tractable when it is solved outside of the steady state. However, the tractability depends on several assumptions: exogenous matching rates, full monopsony power of firms and no human capital accumulation. In contrast, the BRE model allows me to endogenize matching rates, incorporate learning by doing and get rid of the monopsony assumption. In Menzio and Shi (2010a), they formally prove existence and uniqueness of a BRE under various specifications of workers' preferences and contractual environments, including dynamic contracts and fixed-wage contracts. In Menzio and Shi (2011), they apply the BRE model to business cycle data and find that productivity shocks generate procyclical fluctuations in the rate at which unemployed workers become employed and countercyclical fluctuations in the rate at which employed workers become unemployed. In Menzio et al. (2012), they develop a life cycle BRE model of the labor market and show that their model correctly predicts the pattern of labor market transitions for workers of different ages. This paper extends their BRE framework by endogenizing schooling decisions, allowing for unobserved heterogeneity in agents, and proposing a different type of wage contract that facilitates the wage decomposition analysis. This is also the first paper that estimates a directed search model and shows that the BRE framework is able to fit the employment, wage and labor market transitions over the life cycle as well as over the business cycle.

The rest of the paper is organized as follows. Section 2 constructs and solves the model. Section 3 describes the data. Section 4 discusses the identification. Section 5 explains the estimation strategy. Section 6 shows the estimation results and the model fit. Section 7 presents results from counterfactual experiments. Section 8 concludes.

# 2 Model

This is a life-cycle model of the labor market with heterogenous agents and aggregate shocks. The model is an extension of a directed search model of Menzio and Shi (2010a). With directed search, I am able to focus on the Block Recursive Equilibrium where agents' value functions and policy functions depend on the labor market conditions only via productivity shocks, and not on the endogenous distribution of workers across employment states. Therefore, the model can be solved outside of the steady state where level of unemployment and distribution of wages are allowed to change over time. This property allows me to solve a BRE model with heterogeneous agents and productivity shocks as easily as solving a representative agent model and to avoid the analytical and computational difficulties involved in solving equilibrium labor market models with random search.

## 2.1 Agents and Markets

Labor markets are divided into submarkets, which are defined by a wage contract, as well as by the worker's characteristics (skill $s$ and age $a$) at the time the firm and the worker meet. The contractual environment is such that the firm commits to a wage that remains a constant proportion $\mu$ of output throughout the duration of the employment relationship. In submarket $(s, a, \mu)$, firms offer contracts $\mu$ to applicants of characteristics $(s, a)$ and firms offer unattractive contracts to applicants of characteristics $\notin (s, a)$.[7]

The economy is populated by a continuum of homogeneous firms with positive measure. Firms are risk-neutral and maximize the expected sum of profits with discount factor $\beta$. Firms choose whether to post a vacancy and in which submarket to post the vacancy. Each firm can only open one vacancy and pays a fixed cost for opening one vacancy. As each job consists of a single firm-worker pair, currently matched firms do not post new vacancies.

The economy is also populated by $A$ overlapping generations of agents. In every period, a new generation of agents is born into the economy and lives for $A$ periods. Agents are risk-neutral and

[7] Menzio and Shi (2010b) show that firms always find it optimal to attract exclusively one type of worker to each submarket.

maximize the expected sum of periodical consumption discounted at $\beta$. Individuals are born with different initial endowments, including their human capital $\alpha_0$, preference for schooling $b^s$, and preference for leisure $b^u$. These initial endowments are different by type $k$, common knowledge for individuals and firms, but unobserved to researchers.

Individuals have three mutually exclusive status: staying at school, employed, and unemployed.[8] Each period, individuals make schooling, work and search decisions, including on-the-job search. When searching, individuals direct their search to a submarket that corresponds with their skill and age by choosing a $\mu$ that maximizes their expected utility. A higher $\mu$ will result in a higher wage if matched, but a lower offer arrival rate. The worker's characteristics determine whether the worker will seek $\mu$ that offer him lower wages but are easier to find or $\mu$ that offer more generous wages but are harder to find.

Skill for individual $i$ at age $a$ is a vector of schooling, $h$, general experience, $X$, firm-specific experience (tenure), $R$, and initial human capital endowment, $\alpha_0$:

$$s_a^i = (h_a^i, X_a^i, R_a^i, \alpha_0^i)$$

Each match of a firm-worker pair embodies a productivity level $z_0$, determined at the time of its creation, and the embodied productivity of the match stays constant throughout the lifetime of the match. Each worker's output is $yz_0\phi(s)$, determined by the current aggregate productivity shock, $y$, the match productivity when the worker and the firm met, $z_0$, and worker production $\phi(s)$. Production function $\phi(s)$ is a function of worker skill:

$$\phi(s_a^i) \equiv \exp\left(\alpha_0^i + \alpha_1 h_a^i + \alpha_2 X_a^i + \alpha_3 R_a^i\right)$$

Worker's wage $w = \mu_0 y z_0 \phi(s)$, where $\mu_0$ is the share of output given to the worker determined at the time the worker and the firm met. Wage of an individual $i$ of age $a$ at time $t$ can be written

[8] Here I do not distinguish between unemployed workers and individuals out of the labor force but not in school because I focus on white-male in the empirical analysis.

as:

$$\log w^i_{a,t} = \log \mu^i_{0,a} + \log y_t + log z^i_{0,a} + \alpha^i_0 + \alpha_1 h^i_a + \alpha_2 X^i_a + \alpha_3 R^i_a$$

## 2.2 Environment

There are two driving forces of the business cycle: shocks to aggregate productivity, $y$, and shocks to match-specific productivity, $z$. Match productivity shocks only affect the values of new matches, not existing ones.[9] Thus, match productivity shocks have a larger impact of vacancy postings (and thus on the unemployment rate), compared to its impact on average productivity. Eyigungor (2010) shows that the addition of match productivity shocks solves the problem that shocks to aggregate productivity can account for only a small portion of the volatilities in unemployment and vacancies.

Time is discrete and continues forever. A period is three months. At the beginning of each period, the aggregate state of the economy can be summarized by $\psi = (y, z, n, u, e, \gamma)$. $y$ is the current aggregate productivity shock and $z$ is the current match productivity shock. $n(a)$ denotes the measure of students at age $a$. $u(s, a)$ denotes the measure of workers with $(s, a)$ who are in the labor market but are not employed. $e(\mu, s, a, z_0)$ denotes the measure of employed workers in submarket $(s, a, \mu)$ with match productivity $z_0$. $\gamma$ denotes the the measure of newly born workers. Although there are many elements in the state of economy $\psi$, I will show in Appendix Section A that the value functions and policy functions depend only on productivity shocks $y$ and $z$, and not on other state variables in $\psi$ that characterize the distribution of workers across employment states.

Every period is divided into five stages: entry-and-exit, separation, search, matching and production. In the first stage, nature draws a productivity shock $y$ and a match productivity shock $z$ from a distribution $\Gamma(y, z|\tilde{y}, \tilde{z})$, where $\tilde{y}$ and $\tilde{z}$ are aggregate shocks in the last period. Individuals make schooling decisions: students decide whether to leave school and enter the labor market while employed and unemployed workers decide whether to return to school. At the same time, any workers who are older than $A$ must leave the labor market. Any students who are older than the

[9] This is a plausible representation because investment opportunities available at the time of creation of a match may affect the technology of the match permanently (Eyigungor (2010)).

maximum schooling age $\tilde{A}$ must enter the labor market.

At the separation stage, an employed worker has probability $d \in \{\delta_h, 1\}$ of becoming unemployed. There are two types of separations, endogenous separation and exogenous separation. If the value of staying at home is greater than the value of being employed, the worker will choose $d = 1$, which is endogenous separation. Otherwise, workers will have an exogenous separation rate $\delta_h \in [0, 1]$, which can vary by schooling level $h$.

At the search stage, due to time constraints and other costs, agents may not search in every period. Therefore, each agent gets an exogenous probability to search in every period. Search probabilities depend on employment status, schooling level and age. In particular, a student has the opportunity to search with probability $\lambda_h^n \in (0, 1]$. An unemployed worker of age $a$ has the opportunity to search with probability $\lambda_{h,a}^u \in (0, 1]$. If a worker is employed at the beginning of the separation stage and has not lost his job, he has the opportunity to search with probability $\lambda_{h,a}^e \in (0, 1]$. If the worker lost his job during the separation stage, he cannot search in the current period. Whenever a worker has the opportunity to search, given his characteristics $(s, a)$, individuals direct their search to a wage contract $\mu$ that maximizes expected utility. Also, during the search stage, a firm chooses whether to open a vacancy and in which submarket $(s, a, \mu)$ to open the vacancy. The cost of maintaining a vacancy for one period is $\kappa > 0$.

At the matching stage, the vacancies and the workers who are searching in the same submarket come together through a frictional matching process. In submarket $(\mu, s, a)$, the ratio of firms searching for workers to workers searching for firms is $\theta_a(\mu, s, \psi)$, where $\psi$ is a summary of the state of the economy. Assuming constant returns to scale for the matching function, the probability that a worker searching in submarket $(\mu, s, a)$ meets a vacancy is $p(\theta_a(\mu, s, \psi))$, where $p$ is a twice-differentiable, strictly increasing and strictly concave function. Similarly, a vacancy-searching firm in submarket $(\mu, s, a)$ meets a worker with probability $q(\theta_a(x, s, \psi))$, where $q$ is a twice-differentiable, strictly decreasing, convex function such that $q(\theta) = p(\theta)/\theta$. When a firm and a worker meet in a submarket $(\mu, s, a)$, the match-specific productivity is determined and remains fixed for the lifetime of the match. If the individual does not get an offer, he returns to his previous employment position. Students who choose to stay in school at the first stage and don't get an offer will remain at school.

Those who choose to leave school and don't get an offer will be unemployed. In all cases, if the agent gets the offer, he enters a productive match with the firm.

At the production stage, a student gets $b_k^s$ as the non-pecuniary utility from schooling and pays $c_h$ as tuition. The utility from schooling can differ by type $k$ and the tuition can differ by schooling level $h$. An unemployed worker produces and consumes $b_{k,a}^u$ units of output, which differs by type $k$ and age $a$. A worker of $(s, a)$ who is employed produces $yz_0\phi(s)$ units of output and consumes $\mu_0$ fraction of them. At the end of the production stage, nature draws the measure of next period's entering cohort from the distribution $\Pi(\hat{\gamma}|\gamma)$. Throughout the paper, the caret indicates variables or functions in the next period.

## 2.3 Definition of Equilibrium

### 2.3.1 Worker's problem

Consider a worker whose lifetime utility is V and who has the opportunity to look for a job at the beginning of the search stage. His search decision is a choice of which $\mu$ to search. If the worker searches in submarket $(\mu, s, a)$, he succeeds in finding a job with probability $p(\theta_a(\mu, s, \psi))$, and fails with probability $1 - p(\theta_a(\mu, s, \psi))$. If he succeeds, he enters the production stage in a new employment relationship which gives him $H_a(\mu, s, z, \psi)$ lifetime utility and which always pays $\mu$ fraction of his output throughout the duration of the employment relationship. If he fails to find a new match or if he doesn't apply for a job, he enters the production stage by retaining his current employment position, which gives him a lifetime utility V. Therefore, the worker's lifetime utility at the beginning of the search stage is $V + R_a(V, s, \psi)$, where R is the search value function defined as

$$R_a(V, s, \psi) = \max\{0, \max_{\mu} p(\theta_a(\mu, s, \psi))[H_a(\mu, s, z, \psi) - V]\} \tag{1}$$

The worker maximizes the product of the probability of getting the offer and the extra utility generated by the offer by choosing the $\mu$. If there's no choice of $\mu$ that generates a higher value than V, the agent will remain in his current employment position. In this case, $R_a(V, s, \psi) = 0$. Denote $m(\mu, s, \psi)$ as the solution to the maximization problem in (1), and $\tilde{p}(\mu, s, \psi)$ as the composite function of $p(\theta(m(\mu, s, \psi), s, \psi))$.

Now, let's consider the search problem for an unemployed worker who has passed the maximum schooling age $\tilde{A}$ and who therefore cannot decide to return to school. For a worker of type $(s, a)$ in market condition $\psi$, who is unemployed at the beginning of the production stage, his lifetime utility at age $a$ is $U_a(s, \psi)$ such that

$$U_a(s, \psi) = b^u_{k,a} + \beta E_{\hat{\psi}|\psi}[U_{a+1}(s, \hat{\psi}) + \lambda^u_{h,a} R_{a+1}(U_{a+1}(s, \hat{\psi}), s, \hat{\psi})]$$

In the current period, the worker produces and consumes $b^u_{k,a}$ units of output, which can differ by different age $a$ and type $k$. In the next period, the worker's skill doesn't change because he was unemployed in the previous period (here I assume that there is no depreciation of skills). He gets the opportunity to search the labor market with probability $\lambda^u_{h,a}$. In this case, the worker's continuation utility is $U_{a+1}(s, \hat{\psi}) + R_{a+1}(U_{a+1}(s, \hat{\psi}), s, \hat{\psi})$. The worker has a probability of $1 - \lambda^u_{h,a}$ of not having the opportunity to search in the next period. In this case, the worker remains unemployed and his continuation utility is $U_{a+1}(s, \hat{\psi})$. I denote $x^u_{a+1}(s, \hat{\psi})$ as the policy function for the search problem.

For unemployed workers who are younger than the maximum schooling age, they make an additional decision about returning to school $I^u_a(s, \psi)$.

$$\begin{aligned} U_a(s, \psi) =& b^u_{k,a} + \beta E_{\hat{\psi}|\psi}[\max_{I^u \in \{0,1\}} \{I^u(W_{a+1}(s, \hat{\psi}) - c^u_h) \\ &+ (1 - I^u)(U_{a+1}(s, \hat{\psi}) + \lambda^u_{h,g} R_{a+1}(U_{a+1}(s, \hat{\psi}), s, \hat{\psi}))\}] \end{aligned} \tag{2}$$

If a worker decides to return to school, he gets the value of schooling $W_{a+1}(s, \hat{\psi})$ and pays a transition cost $c^u_h$ that varies by schooling level. If the value of returning to school is greater than the value of staying in the labor market, $I^u_a(s, \psi) = 1$, otherwise, $I^u_a(s, \psi) = 0$. The rest of the problem remains the same.

Next, consider a worker who is employed at the beginning of the production stage with wage contract $\mu_0$ and match productivity $z_0$. Denote $H_a(\mu_0, s, z_0, \psi)$ as his lifetime utility at age $a$.

Again, I first consider the worker's problem for those who are older than $\tilde{A}$.

$$H_a(\mu_0, s, z_0, \psi) = \mu_0 y z_0 \phi(s) + \beta E_{\hat{\psi}|\psi} \max_{d \in \{\delta_h, 1\}} \{dU_{a+1}(\tilde{s}, \hat{\psi})$$
$$+(1-d)[H_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi}) + \lambda^e_{h,a} R_{a+1}(H_{a+1}(\mu_0, \hat{s}, \hat{\psi}), \tilde{s}, \hat{\psi})]\}$$

where the current period skill is $s = (h, X, R, \alpha_0)$, and there two possible next period skills, $\tilde{s} = (h, \hat{X}, 0, \alpha_0)$ and $\hat{s} = (h, \hat{X}, \hat{R}, \alpha_0)$. $\tilde{s}$ are the workers' skills in the next period without taking into account firm-specific skills and $\hat{s}$ are the workers' skills in the next period including firm-specific skills. For unemployed workers and workers who switch jobs, only $\tilde{s}$ matters, while for workers who stay in their current jobs, $\hat{s}$ matters. $\hat{X}$ and $\hat{R}$ are experience and tenure in the next period, and they follow a law of motion specified in Section 2.3.4.

In the current period, the worker consumes $\mu_0 y z_0 \phi(s)$ units of output, where $z_0$ is the match-specific productivity determined at the time of the creation of the match. $z_0$ can be different from the current match productivity $z$, which is an element in $\psi$. At the separation stage of the next period, the worker has probability $d$ of becoming unemployed. If there's endogenous separation, that is, workers voluntarily separate from the firm because the value of non-employment is higher than the value of employment, $d = 1$, otherwise, $d = \delta_h$, which is the exogenous separation rate. If the worker separates from the firm, his continuation utility is $U_{a+1}(\tilde{s}, \hat{\psi})$ because he can not search in the next period. If there's no separation, at the search stage of the next period, the worker has probability $1 - \lambda^e_{h,g}$ of not having the opportunity to search in the labor market. In this case, the worker and firm remain matched and the continuation value is $H_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi})$. If the worker gets the opportunity to search the labor market in the search stage, the worker's continuation utility is $H_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi}) + R_{a+1}(H_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi}), \tilde{s}, \hat{\psi})$. I denote $x^e_{a+1}(\mu_0, s, z_0, \hat{\psi})$ as the policy function of the on-the-job search problem.

Employed workers who are younger than $\tilde{A}$ make an additional decision about whether to return to school $I^e_a(\mu_0, s, z_0, \hat{\psi})$. If individuals choose to return to school, they get the value of schooling

$W_{a+1}(s, \hat{\psi})$ and pay a transition cost $c_h^e$ that varies by schooling level.

$$H_a(\mu_0, s, z_0, \psi) = \mu_0 y z_0 \phi(s) + \beta E_{\hat{\psi}|\psi} \max_{I^e \in \{0,1\}, d \in \{\delta_h, 1\}} \{ I^e (W_{a+1}(\tilde{s}, \tilde{\psi}) - c_h^e)$$
$$+ (1 - I^e) d U_{a+1}(\tilde{s}, \hat{\psi})$$
$$+ (1 - I^e)(1 - d)[H_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi}) + \lambda_{h,g}^e R_{a+1}(H_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi}), \tilde{s}, \hat{\psi})]\} \quad (3)$$

### 2.3.2 Student's problem

Consider an agent who is still in school. The student's lifetime utility $W_a(s, \psi)$, is such that

$$W_a(s, \psi) = b_k^s - c_h + \beta E_{\hat{\psi}|\psi} \max_{I^n \in \{0,1\}} \{ I^n [W_{a+1}(\hat{s}, \hat{\psi}) + \lambda_h^n R_{a+1}(W_{a+1}(\hat{s}, \hat{\psi}), \hat{s}, \hat{\psi})]$$
$$+ (1 - I^n)[U_{a+1}(\hat{s}, \hat{\psi}) + \lambda_h^n R_{a+1}(U_{a+1}(\hat{s}, \hat{\psi}), \hat{s}, \hat{\psi})]\} \quad (4)$$

where $s = (h, X, 0, \alpha_0)$ and $\hat{s} = (h + 1, X, 0, \alpha_0)$.

In the current period, the student needs to pay $c_h$ tuition and gets $b_k^s$ non-pecuniary utility from schooling. I assume that the non-pecuniary utility for cohort $t$ is $b_k^s = b_k^0 + b_k^1 t$, where $b_k^1$ is an exogenous time trend in the consumption value of schooling. Both $b^0$ and $b^1$ are type-specific. As shown in Heckman and LaFontaine (2010), there has been a linear trend in the school enrollment rates for cohorts born from 1930 to 1980. The exogenous time trend in the consumption value of schooling allows me to fit the upward trend in the school enrollment rates observed in the data.

Students make decisions about whether to stay in school $I_{a+1}^n(s, \hat{\psi})$ in the next period. If they decide to stay in school ($I^n = 1$), in the next period, they get a search probability $\lambda_h^n$ and the continuation utility is $W_{a+1}(\hat{s}, \hat{\psi}) + \lambda_h^n R_{a+1}(W_{a+1}(\hat{s}, \hat{\psi}), \hat{s}, \hat{\psi})$. The reservation utility in the search problem is $W_{a+1}(\hat{s}, \hat{\psi})$ because if individuals do not get an offer in the next period, they will stay in school. If they decide to enter the labor market ($I^n = 0$), they will become unemployed if they do not get an offer in the next period. The continuation utility of individuals who choose to enter the labor market is $U_{a+1}(\hat{s}, \hat{\psi}) + \lambda_{h,g}^u R_{a+1}(U_{a+1}(\hat{s}, \hat{\psi}), \hat{s}, \hat{\psi})$. I denote $I_{a+1}^n(\hat{s}, \hat{\psi})$, $x_{a+1}^n(\hat{s}, \hat{\psi})$ (optimal market to search while in school) and $x_{a+1}^u(\hat{s}, \hat{\psi})$ (optimal market to search while in the labor market) as the policy function associated with (4).

### 2.3.3 Firm's problem

Consider a firm that employs a worker for a wage as a proportion $\mu_0$ of the output at the beginning of the production stage, and denotes as $J_a(\mu_0, s, z_0, \psi)$ its lifetime profit.

$$
\begin{aligned}
J_a(\mu_0, s, z_0, \psi) &= (1-\mu_0) y z_0 \phi(s) + \beta E_{\hat{\psi}|\psi}[(1 - I^e_{a+1}(\mu_0, s, z_0, \hat{\psi})) \\
&(1 - d_{a+1}(\mu_0, s, z_0, \hat{\psi}))(1 - \lambda^e_{h,a} \tilde{p}(H_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi}), \tilde{s}, \hat{\psi})) J_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi})]
\end{aligned} \tag{5}
$$

where $s$, $\hat{s}$, and $\tilde{s}$ are defined the same as in the employed worker's problem. In the current period, the firm's profit is given by $(1-\mu_0)yz_0\phi(s)$. The discounted sum of profits from the next period onward is $J_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi})$ times the probability that the worker remains with the firm in the next period, which is the product of the probability of not returning to school $(1 - I^e_{a+1}(\mu_0, s, z_0, \hat{\psi}))$, the probability of not being unemployed $(1 - d_{a+1}(\mu_0, s, z_0, \hat{\psi}))$, and the probability of not having an outside offer $(1 - \lambda^e_{h,a}\tilde{p}(H_{a+1}(\mu_0, \hat{s}, z_0, \hat{\psi}), \tilde{s}, \hat{\psi}))$.

During a search stage, a firm chooses whether and where to open a vacancy. The firm's benefit of creating a vacancy in a submarket $(s, a, \mu)$ is the product of the matching probability $q(\theta_a(\mu, s, \psi))$, and the value of meeting a worker, $J_a(\mu, s, z, \psi)$. Here the state space $z_0 = z$ because match-specific productivity $z_0$ is determined when the firm and the worker meet, and thus will be the current match productivity shock $z$ if the match is created. The firm's cost of creating a vacancy is $\kappa$. The tightness of the submarket is such that

$$
\kappa \geq q(\theta_a(\mu, s, \psi)) J_a(\mu, s, z, \psi) \tag{6}
$$

and $\theta_a(\mu, s, \psi) \geq 0$ with complementary slackness. The above condition guarantees that the tightness function $\theta$ is consistent with the firm's incentive to create vacancies. Condition (6) states that if the vacancy-to-applicant ratio in submarket $(\mu, s, a)$ is strictly positive, the cost of opening a vacancy must be equal to the benefit. Moreover, condition (6) states that if the vacancy-to-applicant ratio in the submarket is equal to zero, the cost to a firm of opening a vacancy must be larger than the benefit.

### 2.3.4 Law of Motion

In order to reduce the size of the state space, I assume that experience $X$ and tenure $R$ each take on P values, so that the possible values of experience and tenure arranged in ascending order are

$$X \in XC = \{x(1), ..., x(P)\}$$

$$R \in RC = \{r(1), ..., r(P)\}$$

After each quarter of work experience, human capital increases to the next level with probability $p$, and with probability $(1-p)$ human capital does not increase. There are separate skill increase probabilities for experience and tenure, and the rates of skill increase are also allowed to vary by skill level. The skill increase parameters are $\{p_X^l, p_R^l, l = 1, ..., P\}$, where the subscripts $X$ and $R$ refer to experience and tenure, and $l$ indexes levels. When experience improves to the next level, the agent's skill increases by $\alpha_2$. When tenure reaches the next level, the agent's skill increases by $\alpha_3$. The human capital transition probabilities are known by agents in the model. The size of the state space is significantly reduced when $P$ is a small number relative to the possible values of years of experience and tenure, but the model still captures the human capital improvement process. In this work, $P = 4$.

For the law of motion of aggregate shocks $(y_t, z_t)$, I follow Eyigungor (2010) and assume a first order Markov process of the form below:

$$\begin{bmatrix} \log(y_{t+1}) \\ \log(z_{t+1}) \end{bmatrix} = \begin{bmatrix} \rho_y & 0 \\ 0 & \rho_z \end{bmatrix} \begin{bmatrix} \log(y_t) \\ \log(z_t) \end{bmatrix} + \begin{bmatrix} \epsilon_{y,t} \\ \epsilon_{z,t} \end{bmatrix}$$

Innovation $(\epsilon_{y,t}, \epsilon_{z,t})$ are serially independent, multivariate normal random variables distributed as below:

$$\begin{bmatrix} \epsilon_{y,t} \\ \epsilon_{z,t} \end{bmatrix} \sim N\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_y^2 & \rho_{y,z}\sigma_y\sigma_z \\ \rho_{y,z}\sigma_y\sigma_z & \sigma_z^2 \end{bmatrix} \right)$$

### 2.3.5 Definition of equilibrium

Following Menzio and Shi (2010a) and Menzio et al. (2012), I define a Block Recursive Equilibrium.

**Definition 1.** A Block Recursive Equilibrium (BRE) consists of a market tightness function $\theta_a$, a value function for the firm $J_a$, a value function for the unemployed worker, $U_a$, policy functions for the unemployed worker, $x_a^u$ and $I_a^u$, a value function for the employed worker, $H_a$, policy functions for the employed worker, $d_a$, $x_a^e$, and $I_a^u$, a value function for students $W_a$, and policy functions for the student, $x_a^n$ and $I_a^n$. These functions satisfy the following conditions:

(i) $\theta_a$ satisfies (6) for all submarkets $(\mu, s, a)$ and market conditions $\psi$.

(ii) $U_a$, $x_a^u$ and $I_a^u$ satisfy (1) and (2) for all types of workers $(s, a)$ and market conditions $\psi$;

(iii) $H_a$, $d_a$, $x_a^e$ and $I_a^e$ satisfy (1) and (3) for all submarkets $(\mu, s, a)$ with match productivity z and market condition $\psi$;

(iv) $W_a$, $I_a^n$ and $x_a^n$ satisfy (1) and (4) for all types of workers $(s, a)$ and market conditions $\psi$;

(v) $J_a$ satisfies (5) for all submarkets $(\mu, s, a)$ with match productivity z and market condition $\psi$.

A Block Recursive Equilibrium is a recursive equilibrium in which the agents' value and policy functions depend on the aggregate state of the economy $\psi$ only through productivity shocks $y$ and $z$. The equilibrium is block recursive because the search process is directed. In fact, with directed search, workers with different characteristics choose to search in different submarkets. As a result of this self-selection process, a firm that opens a vacancy in a particular submarket knows that it will meet only one type of worker. Hence, the firm's expected value from meeting a worker does not depend on the distribution of workers across employment states, and because of firms' free entry, the probability that the firm meets an applicant must have the same property. Therefore, agents' value and policy functions will also be independent of the distribution.

In Appendix A, I present the solution to the equilibrium and formally prove that there exists a Block Recursive Equilibrium. Furthermore, I formally prove that policy functions and value functions depend on the aggregate state of the economy only through aggregate shocks, and not on the endogenous distribution of workers across employment states.

# 3 Data

The data sets used in this paper are Monthly Current Population Survey (CPS) from 1980 to 2012, and National Longitudinal Surveys of Youth (NLSY) 1979 (from 1979 to 2010) and 1997 (from 1997 to 2010).

The Current Population Survey, a monthly household survey conducted by the Bureau of the Census for the Bureau of Labor Statistics, provides a comprehensive body of information on the employment and unemployment experience of the US population, classified by age, sex, race, and a variety of other characteristics. In addition, individual weekly income information becomes available in the monthly survey beginning in 1982.[10] I use the CPS to construct quarterly wage and employment data conditioned on age and schooling. Quarterly wage is calculated by mutiplying weekly income by 13 (13 weeks per quarter) and has been inflation adjusted to 2012 dollars using the Consumer Price Index. An agent is defined as employed if he worked more than 30 hours last week. An agent is defined as in school if his school enrollment status is full time. The CPS is surveyed via a 4-8-4 sampling scheme. Households are in the survey for four consecutive months, out for eight, and then return for another four months before leaving the sample permanently. This design ensures a high degree of continuity from one month to the next (as well as over the year) and allows me to match individuals across quarters. By matching the households surveyed in the first month and fourth month, I am able to calculate quarterly transition rates between employment status (schooling, employment and unemployment) and transition rates between employers. In this paper, the sample is restricted to the white-male sample because their labor supply decisions are least sensitive to external factors such as childbearing or discrimination. The CPS data provides a larger variation in the labor market conditions at the time of labor market entry compared to longitudinal surveys that only follow a certain number of cohorts. The total sample size from the CPS used in the estimation is 13 million.

Although monthly CPS can be used to calculate the distributions of income, employment and labor market transitions by cohort and age, being primarily a cross-sectional data set, it does not

[10] I also use individual annual income data from March CPS for 1980 and 1981 because I perform a counterfactual analysis to evaluate the welfare loss from the 1981-1983 recession.

contain a history of employment choices that would enable the calculation of work experience and tenure. The National Longitudinal Survey of Youth 1979 and 1997 are longitudinal data sets that can be used to calculate statistics that are conditioned on experience and tenure. The NLSY79 is a nationally representative sample of 12,686 young men and women who were 14-22 years old when they were first surveyed in 1979. The NLSY97 consists of a nationally representative sample of approximately 9,000 youths who were 12 to 16 years old in 1996. In both surveys, youths continue to be interviewed on an annual or bi-annual basis. The latest data I can get is from 2010. Anyone working more than 390 hours in a quarter (30 hours per week for 13 weeks) is considered employed and anyone less than that is not. Using the weekly working hours data, I am able to construct the whole working history and calculate work experience. Then, I calculate the hourly wage (total earnings divided by hours worked) and multiply by 390 to get a quarterly earnings equivalent, which is what I use for their quarterly wage. The NLSY data sets also report the start and stop date for each job. Using the employer history data, I collect information on tenure and transitions between employers. I only keep the sample including those who were 16 years old or less at the time of the first survey because I need to track the whole working history. I also restrict the sample to white males only, which leaves me with 32,000 observations.

# 4 Identification

There are four groups of parameters to be identified: search parameters, aggregate shocks, wage parameters and utility parameters. Search parameters include vacancy cost, $\kappa$, exogenous separation rate, $\delta_h$, the probability that a student/unemployed worker/employed worker gets to search, $\lambda_h^n/\lambda_{h,a}^u/\lambda_{h,a}^e$, and matching function parameter. Aggregate shocks include aggregate productivity shocks $y_t$ and match productivity shocks $z_t$. Wage parameters include type-specific human capital endowment $\alpha_0$, return to schooling $\alpha_1$, return to experience $\alpha_2$, and return to tenure $\alpha_3$. Utility parameters include the preference for schooling $b_k^s$, preference for leisure $b_{k,a}^u$ and cost of schooling $c_h$.

The identification of search parameters follows Menzio and Shi (2011). For the matching function, I use the functional form introduced by den Haan et al. (2000):

$$m(u, v) = \frac{uv}{(u^\rho + v^\rho)^{1/\rho}}$$

This function ensures that the probability of finding a job and filling a vacancy always lies between 0 and 1. Define $\theta$ as market tightness ratio, which is the ratio of firms searching for workers to workers searching for firms, $v/u$. The probability of a searching worker matching a firm is $p(\theta) = (x^{-\rho} + 1)^{-1/\rho}$. The parameter in the matching function, $\rho$, can be identified from the job-to-job transition rates of workers with different tenure values, holding other things equal. The offer arrival rates observed by the researchers are the product of search probability $\lambda^e_{h,a}$ and the match probability $p(\theta)$. On-the-job search probability will be the same for workers having the same age and schooling, but they will face different $p(\theta)$ because they choose to search in different $\mu$ due to their different skills. This allows me to trace down the matching function and identify $\rho$.

I normalize the search probability for unemployed worker having the lowest age and schooling level to be 1. Therefore, vacancy cost $\kappa$ can be identified using average transition rates from unemployment to employment for this group. Exogenous separation rates $\delta_h$ can be identified using the average transition rates from employment to unemployment of workers with different schooling levels. The search probability for students $\lambda^n_h$ can be identified using transition rates from schooling to employment. Transition rates from schooling to employment equal search probability $\lambda^n_h$ times the probability of matching $p(\theta)$. The matching probability is endogenously determined in the equilibrium and can be calculated from the student's search problem. Therefore, I am able to back up $\lambda^n_h$ using transition rates divided by $p(\theta)$. Similarly, transition rates from unemployment to employment by age and schooling can be used to identify the search probability for unemployed workers, $\lambda^u_{h,a}$. Job-to-job transition rates by age and schooling can be used to identify the search probability for employed workers, $\lambda^e_{h,a}$.

Productivity shocks $y_t$ and $z_t$ can be identified from the wage fluctuations and unemployment fluctuations over the business cycle. First, aggregate productivity shocks $y_t$ can be directly identified from the wage profile:

$$\log w^i_{a,t} = \log \mu^i_{0,a} + \log y_t + log z^i_{0,a} + \alpha^i_0 + \alpha_1 h^i_a + \alpha_2 X^i_a + \alpha_3 R^i_a + \eta^i_t$$

where $\eta^i_t$ is the measurement error. $y_t$ is the only element in the wage profile that changes with calendar time. Match productivity shocks $z_t$ can be identified from the unemployment rate at calendar time $t$. If there's a positive match productivity shock $z_t$, the payoff to a firm of hiring a worker is higher, thus more firms would like to post vacancies. Therefore, market tightness $\theta$ will increase, job offer arrival rates will increase and the unemployment rate will decrease. Since the unemployment rate is decreasing in $z_t$, $z_t$ can be identified. I normalize the mean of $y_t$ and mean of $z_t$ to be 0.

For wage parameters, the wage profile conditional on schooling, experience and tenure can help to identify $\alpha_1$, $\alpha_2$, and $\alpha_3$, respectively. However, I only observe accepted wages, not wage offers. So I use age as an exclusion restriction. Age does not directly enter the wage profile, but will affect the observed wage because it affects what submarket individuals can search in and what $\mu$ they will search.

The type-dependent initial human capital endowments $\alpha_0$ are identified from the first and second moments of wage over age. Note that what's left to be identified in the constant of wage is $\log \mu^i + \alpha^i_0$. Although both $\mu^i$ and $\alpha^i_0$ are unobserved to researchers, $\mu^i$ is not a parameter, but an equilibrium outcome that is determined in the individuals' search problem. Given other parameters, I can calculate the equilibrium $\mu^i$, which is a function of $\alpha^i_0$ and other observables. Below is a simple example to identify the type-specific initial endowment with two types. Suppose there are two types of individuals in the economy: $\pi$ fraction of the population is type 1 with $\alpha^1_0$ and $1-\pi$ fraction of the population is type 2 with $\alpha^2_0$. At age 1, type 1 individuals with initial endowment $\alpha^1_0$ will choose $\mu^1_1$, and type 2 individuals will choose $\mu^2_1$. At age 2, type 1 individuals will choose $\mu^1_2$, and type 2 individuals will choose $\mu^2_2$. Let's refer $\log \mu^i + \alpha^i_0$ to wage residual for the simplicity of the argument. Mean of wage residual at age 1 is $\pi(\alpha^1_0 + \log \mu^1_1) + (1-\pi)(\alpha^2_0 + \log \mu^2_1)$. Square mean of wage residual at age 1 is $\pi(\alpha^1_0 + \log \mu^1_1)^2 + (1-\pi)(\alpha^2_0 + \log \mu^2_1)^2$. And mean of wage residual at age 2 is $\pi(\alpha^1_0 + \log \mu^1_2) + (1-\pi)(\alpha^2_0 + \log \mu^2_2)$. With three equations and three unknowns, I can solve for $\alpha_0^1$, $\alpha_0^2$ and $\pi$.

Lastly, the identification of utility parameters follows Keane and Wolpin (1997). Preference for leisure can be identified from the distribution of employment by age. Preference for schooling can be identified from the distribution of highest grade completed. Costs of high school are assumed to be 0 and costs of college and graduate school are identified from school enrollment rates by schooling level.

# 5 Estimation Method

When estimating the model, I assume that the discount factor $\beta$ equals 0.99. To reduce the number of parameters that need to be estimated, I divide schooling level into four groups, including high school dropouts (years of schooling $< 12$), high school graduates (years of schooling $= 12$), some college ($12 <$ years of schooling $< 16$), and college graduates (years of schooling $\geq 16$). I also divide age into five groups, including 16-25, 26-35, 36-45, 46-55, and 56-65. I impose the assumption that the search probabilities ($\lambda_h^n$, $\lambda_{h,a}^u$, $\lambda_{h,a}^e$) are the same for individuals in the same schooling group and age group. Values of leisure $b_{k,a}^u$ are restricted to be the same for people in the same age group. Similarly, costs of schooling $c_h$ are restricted to be the same for people in the same schooling group.

Estimation is done by simulated method of moments. In particular, a weighted squared deviation between sample aggregate statistics and their simulated analogs is minimized with respect to the model's parameters. The weights are the inverses of the estimated variances of the sample statistics.

The simulated aggregate statistics are generated from samples starting with cohorts that turned age 16 in 1933, and thus turned 65 in 1982, and ending with cohorts that turned 16 in 2012. I simulate the behavior of samples of 800 individuals per cohort. Cross sectional simulated statistics, therefore, contain 40,000 observations.

The estimation contains two steps: an inner loop and an outer loop. In the inner loop, I first simulate a series of aggregate shocks ($y_t$ and $z_t$) from 1933 to 1979 given a guess of the parameters in the Markov process. Then I simulate idiosyncratic shocks 100 times for each individual during this period. Given a series of simulated aggregate shocks and idiosyncratic shocks, I simulate the behaviors of all cohorts from 1933 to 1979. I repeat the above step 100 times by drawing 100 different series of aggregate shocks and idiosyncratic shocks from year 1933 to year 1979. Starting in 1980, I simulate 100 series of idiosyncratic shocks for each individual up to year 2012. Because the quarterly wage and employment data are available during this period [11], given the simulated idiosyncratic shocks, I can identify aggregate shocks $y_t$ and $z_t$ for each period by searching for a combination of $y_t$ and $z_t$ for every quarter that best fits the wage and unemployment rate from CPS data. Given the identified aggregate shocks and simulated idiosyncratic shocks in 1980-2012, I simulate the behaviors of all cohorts and compute the simulated moments.

In the outer loop, I estimate the Markov process using the identified $y_t$ and $z_t$ from 1980 to 2012 from the inner loop. Then I update the parameters in the Markov process using the estimates and do the inner loop again. The above step is repeated until the parameters in the Markov process converge, that is, rational expectation holds and the estimated $y_t$ and $z_t$ follow the Markov process.

The aggregate productivity shocks and match productivity shocks used to match the wage fluctuation and employment fluctuation are shown in Figure 1. In particular, my model is able to simulate the 1981-82 recession, the 1991-1992 recession, the 2001 recession, and the recent Great Recession. As shown in Figure 1, wage and employment rate can have different patterns and they do not necessarily move together. Using two productivity shocks allows me to fit both wage and employment fluctuations over the business cycle very well. The green lines show the underlying shocks used in each quarter to fit the data, which are plotted in the secondary axis. In general, a positive match productivity shock will increase the value of new matches and the number of openings, hence increasing employment rate. Since the match productivity only affects the values of new matches, not existing matches, it has a smaller impact on wages than on employment. At the same time, a positive aggregate productivity shock will increase wages as well as employment. Overall, wages fluctuate less than employment over the business cycle.

I compute the simulated moments and compare them with sample aggregate statistics, which are listed as follows:

[11] Quarterly wages for 1980 and 1981 are not available from monthly CPS, so I use March CPS to obtain annual wage information and divided it by 4.

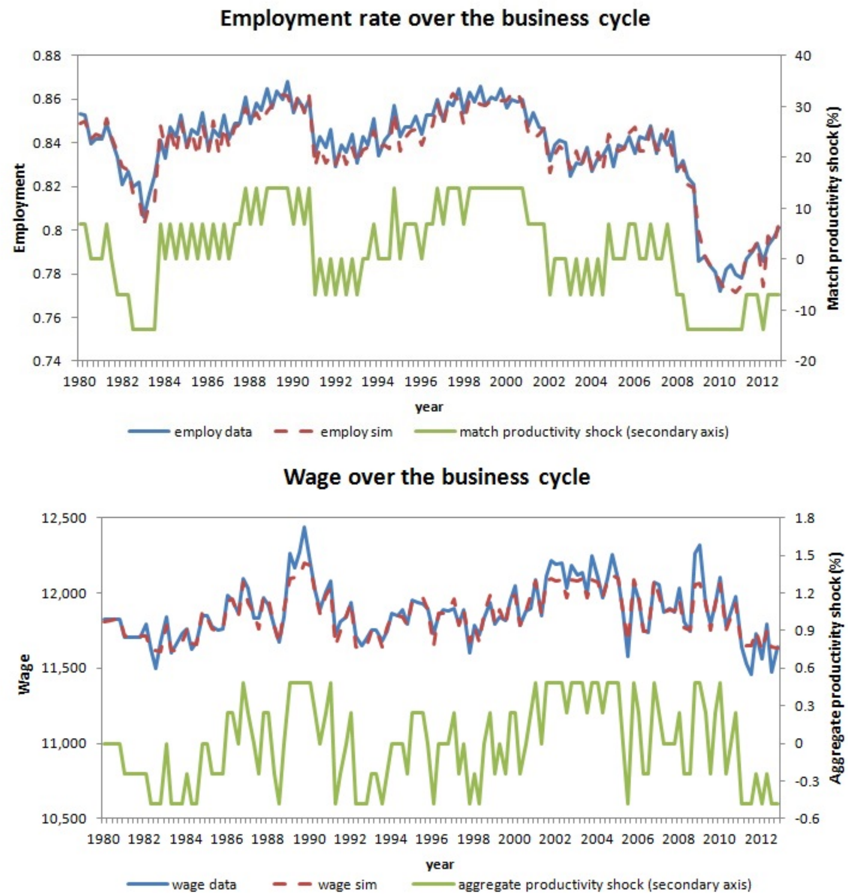



Figure 1: Actual and predicted employment and wages in 1982 - 2012

1. Employment

   (a) CPS: Proportion of employment by year, age and schooling.

   (b) NLSY: Proportion of employment by experience and tenure.

2. Wages

   (a) CPS: Mean and variance of quarterly wages by year and schooling; mean of quarterly wages by year and age.

   (b) NLSY: Mean of quarterly wages by experience and tenure.

3. Schooling

(a) CPS: Distribution of highest grade completed by year and age; Proportion of individuals in school by year and age.

4. Transitions

   (a) CPS: Quarterly transition of schooling to unemployment (SU), schooling to employment (SE), unemployment to employment (UE), employment to unemployment (EU), and between jobs (EE) by year, age and schooling.

   (b) NLSY: Quarterly transition rates between jobs (EE) by tenure.

# 6 Results

## 6.1 Parameter Estimates and Moment Fit

The parameter estimates, and their asymptotic standard errors (in parentheses) are shown in Tables 1 - 4. Table 1 shows the parameters in the wage function. Table 2 shows the parameters in the flow utility of agents. Table 3 shows the search parameters. Table 4 shows the aggregate shocks evolvement process.

Table 1: Parameter estimates: wage parameters

| | |
|---|---|
| Return to schooling ($\alpha_1$) | 0.1308(0.0042) |
| Return to experience ($\alpha_2$) | 0.3784(0.0068) |
| Return to tenure ($\alpha_3$) | 0.0525(0.0004) |
| Transition probability from level 1 to 2 for experience ($p_X^1$) | 0.0630(0.0022) |
| Transition probability from level 2 to 3 for experience ($p_X^2$) | 0.0712(0.0038) |
| Transition probability from level 3 to 4 for experience ($p_X^3$) | 0.0548(0.0023) |
| Transition probability from level 1 to 2 for tenure ($p_T^1$) | 0.0224(0.0016) |
| Transition probability from level 2 to 3 for tenure ($p_T^2$) | 0.0116(0.0010) |
| Transition probability from level 3 to 4 for tenure ($p_T^3$) | 0.0071(0.0008) |
| Initial human capital endowment for type 1 ($\alpha_{01}$) | 6.3925(0.4802) |
| Initial human capital endowment for type 2 ($\alpha_{02}$) | 6.5980(0.3994) |
| Initial human capital endowment for type 3 ($\alpha_{03}$) | 6.5235(0.6673) |
| Fraction of type 1 individuals ($\pi_1$) | 0.2779(0.0265) |
| Fraction of type 2 individuals ($\pi_2$) | 0.4834(0.0588) |
| Standard error of wage measurement error ($\sigma_\eta$) | 0.2481(0.0072) |

Table 2: Parameter estimates: utility parameters

| Utility from schooling | | | | | |
|---|---|---|---|---|---|
| Type | | Constant ($b_k^0$) | | Trend ($b_k^1$) | |
| 1 | | $739(85)$ | | $15.01(6.59)$ | |
| 2 | | $1,538(189)$ | | $21.79(10.7)$ | |
| 3 | | $3,388(247)$ | | $39.46(12.4)$ | |
| | | HS dropout | High school | Some college | College |
| Tuition ($c$) | | 0 | 0 | 3,578 (126) | 4,483 (148) |
| US transition cost ($c^u$) | | 21,473(389) | 20,148 (435) | 40,482 (410) | 23,102 (372) |
| ES transition cost ($c^e$) | | 9,692(157) | 13,089 (302) | 25,389 (224) | 10,122 (134) |
| Utility from unemployment ($b_{k,a}^u$) | | | | | |
| | Age group | | | | |
| Type | 16-25 | 26-35 | 36-45 | 46-55 | 56-65 |
| 1 | $1,684(187)$ | $2,382(82)$ | $4,727(128)$ | $6,427(207)$ | $7,899(311)$ |
| 2 | $3,182(214)$ | $2,467(90)$ | $4,833(133)$ | $6,589(216)$ | $12,924(462)$ |
| 3 | $6,342(393)$ | $2,525(95)$ | $4,964(145)$ | $6,530(224)$ | $20,614(683)$ |

The model aims to match the distributions of wages, employment, and labor market transition transitions over the life cycle and over the business cycle. Figure 2 and Figure 3 show the fit over the life cycle and Figure 4, Figure 5 and Table 5 show the fit over the business cycle.

Figure 2 shows the moment fit of quarterly wages and employment by schooling and years after labor market entry. Employment probability slightly increases in the first ten years, then becomes flat, and decreases 30 years after entry. In addition, people with higher education levels are more likely to be employed. My model is able to predict the employment patterns by schooling and potential experience mainly because of the type- and age-dependent values of leisure. Wages rise rapidly in the first 15 years and become flat afterwards. People with higher education levels get higher wages and receive a larger wage increase as they work longer. The wage profile simulated from the model is able to capture the differences between education groups and the upward trend corresponding with potential experience.

Figure 3 shows the labor market transition by years after labor market entry. The upper left corner shows transition rates from unemployment to employment (UE). The rate decreases as potential experience increases, starting from 50% at the time of entry and dropping to 5% 40 years after entry. The decline in the UE rate is partly due to the decline in workers' work-life expectancy,

Table 3: Parameter estimates: search parameters

| | | | | |
|---|---|---|---|---|
| Vacancy posting cost ($k$) | | | $1,400(153)$ | |
| Matching function parameter ($\rho$) | | | 1.000(0.016) | |
| Exogenous separation rate for high school dropout ($\delta_1$) | | | 0.062(0.003) | |
| Exogenous separation rate for high school graduate ($\delta_2$) | | | 0.044(0.002) | |
| Exogenous separation rate for some college ($\delta_3$) | | | 0.029(0.002) | |
| Exogenous separation rate for college graduate ($\delta_4$) | | | 0.021(0.001) | |
| Search probability – unemployed workers ($\lambda^u_{h,a}$) | | | | |
| | Education | | | |
| Age | HS dropout | High school | Some college | College |
| 16-25 | 1.000 | 0.553 (0.034) | 0.798 (0.078) | 0.404 (0.051) |
| 26-35 | 0.924 (0.053) | 0.456 (0.020) | 0.335 (0.029) | 0.322 (0.005) |
| 36-45 | 0.775 (0.037) | 0.366 (0.009) | 0.384 (0.015) | 0.288 (0.006) |
| 46-55 | 0.410 (0.022) | 0.394 (0.018) | 0.252 (0.018) | 0.144 (0.008) |
| 56-65 | 0.362 (0.010) | 0.178 (0.004) | 0.067 (0.004) | 0.055 (0.003) |
| Search probability – employed workers ($\lambda^e_{h,a}$) | | | | |
| | Education | | | |
| Age | HS dropout | High school | Some college | College |
| 16-25 | 1.000 (0.015) | 1.000 (0.011) | 1.000 (0.011) | 1.000 (0.009) |
| 26-35 | 1.000 (0.044) | 1.000 (0.026) | 1.000 (0.022) | 1.000 (0.008) |
| 36-45 | 1.000 (0.042) | 1.000 (0.032) | 1.000 (0.034) | 1.000 (0.006) |
| 46-55 | 1.000 (0.046) | 1.000 (0.030) | 1.000 (0.027) | 1.000 (0.007) |
| 56-65 | 1.000 (0.039) | 1.000 (0.033) | 1.000 (0.025) | 1.000 (0.009) |
| Search probability – students ($\lambda^n_h$) | | | | |
| | HS dropout | High school | Some college | College |
| | 0.054 (0.012) | 0.997 (0.026) | 0.129 (0.039) | 0.972 (0.088) |

Table 4: Parameter estimates: aggregate shocks evolvement process

| | |
|---|---|
| Autoregressive coefficient of aggregate productivity ($\rho_y$) | 0.9904(0.0324) |
| Standard error of shocks to aggregate productivity ($\sigma_y$) | 0.0032(0.0003) |
| Autoregressive coefficient of match productivity ($\rho_z$) | 0.8110(0.0268) |
| Standard error of shocks to match productivity ($\sigma_z$) | 0.0924(0.0007) |
| Correlation coefficient of shocks to aggregate and match productivity ($\rho_{yz}$) | 0.1033(0.0492) |

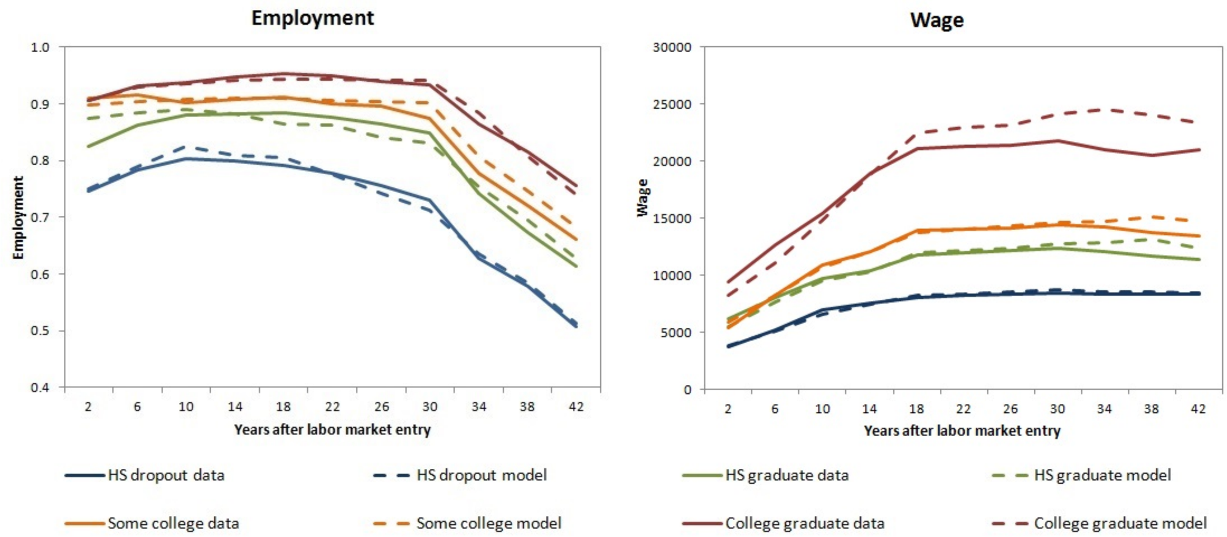



Figure 2: Actual and predicted employment and wages by schooling and potential experience

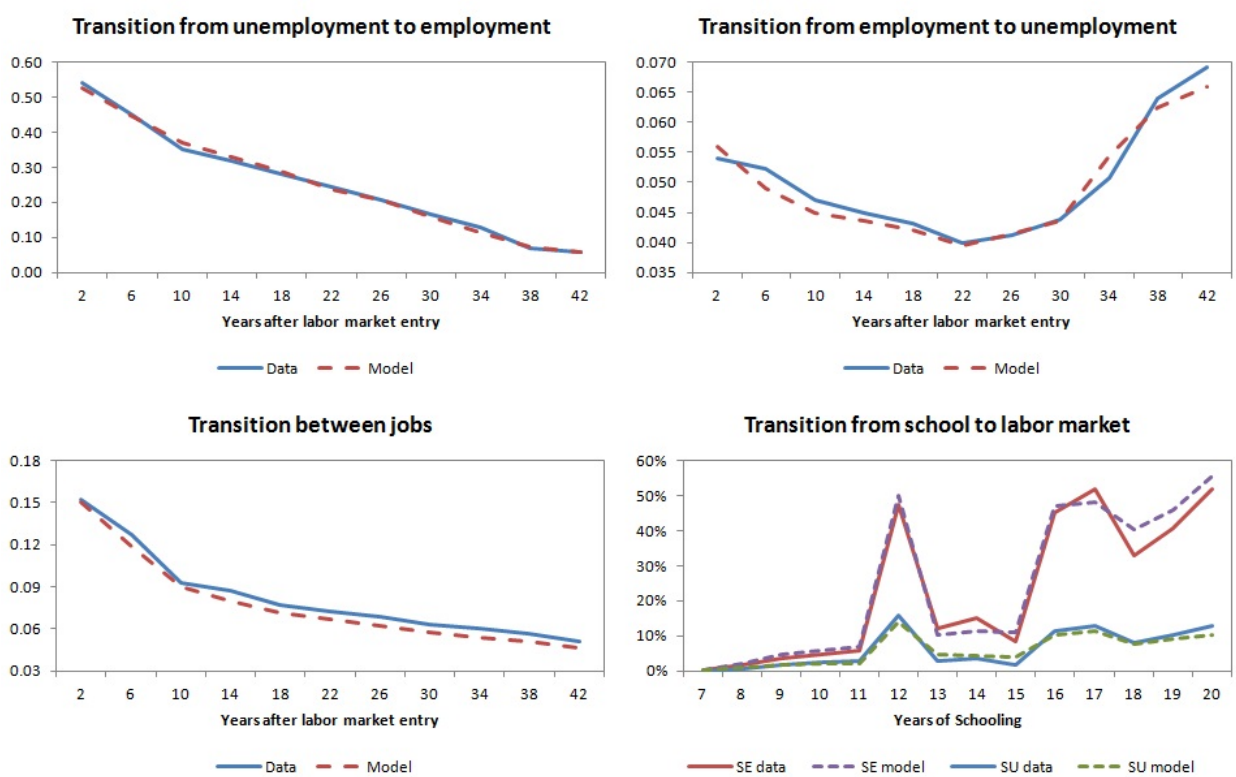



Figure 3: Actual and predicted labor market transitions by potential experience

which reduces their value to the firms as production partners. It's also because value of leisure increases and search probability for unemployed workers declines as age increases. The upper right corner shows the transition rates from employment to unemployment (EU) by schooling and potential experience. EU rates decline in the first 20 years, and start to increase 20 years after labor market entry. The decline in the EU in the first 20 years is due to the increase in $\mu$ through on-the-job search, and the decline after 20 years is due to the increase in value of leisure. The bottom left corner shows the job-to-job transition rates (EE) by schooling and potential experience. There's a sharp decrease in EE transitions in the first 10 years, and the decrease slows down afterwards. The decline in the EE rate is caused initially by the increase in $\mu$ and later by the decline in the workers' work-life expectancy. The bottom right corner shows the transition from school to employment (SE) and transitions from school to unemployment (SU) by schooling. Both SE and SU rates are low before grade 12, and spike at grade 12, when many people leave high school. After grade 12, both transition rates become low again until people reach grade 16, when many people leave college. This is because search probabilities are high for students at grade 12 and after grade 16.

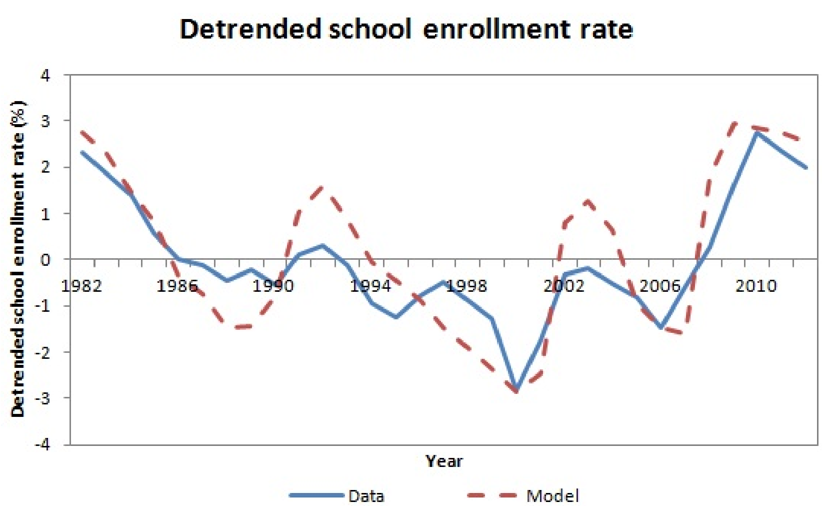



Figure 4: Actual and predicted school enrollment rate in 1982 - 2012

Figure 4 shows the fit of the detrended school enrollment rate between 1980 and 2012.[12] The

[12]Here I take a linear trend.

figure shows that school enrollment rates increased in a bust and decreased in a boom. For example, in the recent recession, school enrollment rates were 2% - 3% larger than the trend. My model is able to capture the countercyclical pattern of the school enrollment rate over the business cycle.

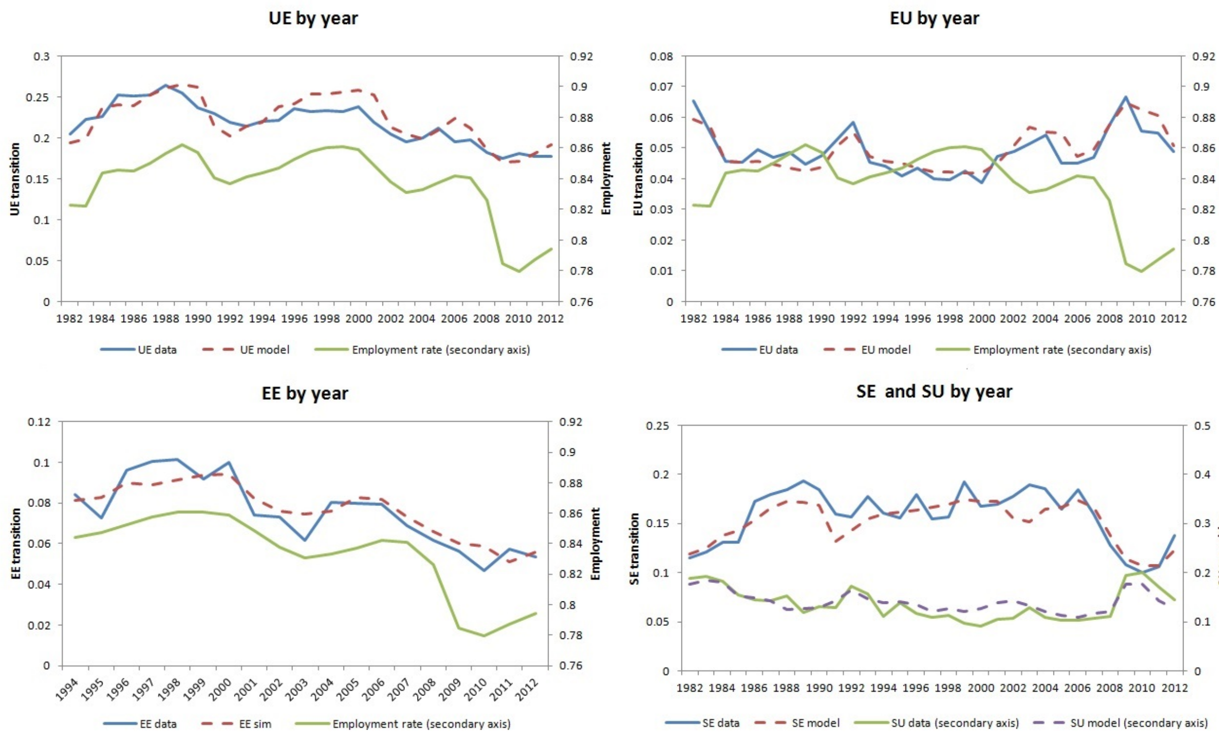



Figure 5: Actual and predicted labor market transitions in 1982 - 2012

Figure 5 shows labor market transitions over the business cycle. Employment rate is also plotted in this figure on the secondary axis as an indicator of the business cycle. The upper left figure shows the UE transitions in 1980 - 2012. My model is able to predict the procyclical pattern. For example, during the 2008 recession, the predicted UE transition rates dropped from 22% to 17%. The upper right figure shows EU transitions over the business cycle. My model is able to capture the countercyclical pattern. During the recent recession, the predicted EU transition rates increased from 5% to 6.5%. The lower left figure shows EE transitions over the business cycle. Since employer information is not available in CPS until 1994, I can only compute the EE transitions after 1994. My model is able to capture the procyclical pattern. During the recent recession, the predicted EE transition rates dropped from 8% to 5%. The lower right figure shows SE and SU transitions over the business cycle. My model can predict the procyclical pattern of SE transition. During the recent recession, the predicted SE transition rates dropped from 15% to 11%. In addition, my model is able to predict the counter-cyclical pattern of SU transition. During the recent recession, the predicted SU transition rates increased from 11% to 17%.

Table 5: Moment fit: AR(1) regression on employment and earnings

| AR(1) regression on log employment rate ($\log(E_t)$) | | |
|---|---|---|
| | Data | Model |
| $\log(E_{t-1})$ | 0.929** | 0.925** |
| | [0.035] | [0.013] |
| constant | -0.013** | -0.014** |
| | [0.006] | [0.003] |
| AR(1) regression on log wage ($\log(W_t)$) | | |
| | Data | Model |
| $\log(W_{t-1})$ | 0.950** | 0.948** |
| | [0.028] | [0.017] |
| constant | 0.471* | 0.489** |
| | [0.216] | [0.135] |

Standard errors in brackets. + significant at 10%; * significant at 5%; ** significant at 1%.

I also run an AR(1) regression on employment rate using my simulated employment rates across quarters. I compare my regression coefficients with the coefficients from an AR(1) regression using the actual employment rate from CPS. The autoregressive coefficient from the data is 0.929, compared to 0.925 from the model. I run the same regressions for average wages across quarters using both data from CPS and simulated data from the estimated model. The autoregressive coefficient of wage from the data is 0.950, pretty close to the number I calculate with the model, which is 0.948. The details of the regressions are shown in Table 5.

### 6.2 Discussion

Table 6 tests the correlation between unemployment rate at the time of labor market entry and future wages for while-male college graduates. I run a regression of log wage on unemployment rate at college graduation and the interaction between college unemployment rate and potential experience, which mimics the work of Kahn (2010). The results also include controls for a quadratic in potential experience, contemporaneous year effects and the contemporaneous unemployment rate. In Kahn (2010), she used NLSY79 white males with at least a BA/BS and found a negative correlation between wages and the unemployment rate at the time of college graduation ($U_{\text{College}}$). The coefficient of the interaction between college unemployment rate and potential experience was found to be insignificant, indicating that the wage loss was persistent. In a robustness check, she also used March CPS data 1988-2007, restricting the sample to workers who turned 22 between 1986 and 1996. The magnitude of the initial wage loss was smaller and the loss was less persistent. The regression results using simulated data from my model are similar to Kahn's findings.

Table 6: "Scarring effect" on wages

| | Kahn (2010) | | Model |
|---|---|---|---|
| | NLSY79 | CPS(1988-2007) | |
| $U_{\text{college}}$ | -0.062* | -0.040* | -0.058** |
| | [0.021] | [0.011] | [0.010] |
| $U_{\text{college}}$ * Exp | 0.000 | 0.005* | 0.006** |
| | [0.002] | [0.002] | [0.001] |

Kahn (2010) used NLSY79 white males with at least a BA/BS and March CPS data 1988-2007, restricting the sample to workers who turned 22 between 1986 and 1996. I mimic her regression using the simulated samples with college degree.
Standard errors in brackets. + significant at 10%; * significant at 5%; ** significant at 1%.
The results also include controls for a quadratic in potential experience, age adjusted AFQT score (only for NLSY), contemporaneous year effects and the contemporaneous unemployment rate.

However, this OLS regression only shows correlation between unemployment rate and wages, not causality. Firstly, running OLS regression ignores the schooling decisions during the business cycle. The regression treats college unemployment rate and schooling as exogenous, but in fact, they are endogenously determined because students can choose to avoid bad economic condition at labor market entry by postponing their entry into the labor market and staying at school during a recession. Therefore, OLS regression ignores the increase in schooling due to the recession, which increases future wages and labor supply. In addition, graduation cohorts can be differently selected. For example, if a recession induces some people to complete college that they otherwise would not, there can be a change in the workforce composition for college graduates across cohorts.

Moreover, longer term projections may be confounded by other economic shocks. It is difficult to isolate the impact of an early career shock on future careers from other possible determinants. Therefore, to analyze the impact of a recession on lifetime wages, we need to conduct a counterfactual analysis.

# 7 Counterfactual Analysis

I conduct a counterfactual experiment to study the impact of experiencing the 1981-1982 recession as a youth.[13] On average, there was a 3% decrease in the employment rate and 1.5% wage drop during the recession period.[14] According to the estimation, it was caused by a 0.5% decrease in aggregate productivity and 10% decline in match productivity (on average). In the counterfactual, I first simulate the behaviors for cohorts who turned age 16 - 22 in year 1980 based on actual aggregate productivity shocks and match productivity shocks. Then I simulate the behaviors for these cohorts again by replacing the negative aggregate productivity shocks and match productivity shocks by 0 in 1981-1982. I compare the difference in terms of lifetime welfare, wages and labor supply.

## 7.1 Welfare Analysis

Table 7 shows the impact of the 1981-82 recession by cohort. The table shows lifetime welfare loss for cohorts who turned age 16-22 in 1980. Welfare is measured by the discounted lifetime utility at age 16, which is measured in consumption dollars because utility function is linear in consumption. In general, the impact of the 1981-82 recession is smaller for cohorts who were younger when the recession started, especially for those who were only 16 and 17 in 1980. This is because a large proportion of these younger cohorts would stay at school anyway during the recession and they are not directly affected by the recession. Moreover, the recession has a larger impact for those who were in the labor market (workers) than those who stay at school (students) by the time the recession started. This is again because attending school can shield agents from the direct impact

[13] The reason of picking the recession in 1981-82 is because the ones in 1990s and 2000s are small and the recent recession is too severe. In addition, the data allows me to observe the labor market performance of individuals who entered the labor market during the 1981-82 recession for 30 years.

[14] According to NBER, the recession started in the third quarter of 1981 and ended in the last quarter of 1982.

Table 7: Welfare loss from the 1981-82 recession by cohort

| Age in 1980 | Total | Workers | Students |
|---|---|---|---|
| 16 | 2.16% | 3.92% | 1.98% |
| 17 | 2.33% | 3.85% | 2.05% |
| 18 | 2.85% | 3.71% | 2.09% |
| 19 | 2.96% | 3.49% | 2.17% |
| 20 | 2.87% | 3.15% | 2.26% |
| 21 | 2.82% | 2.98% | 2.28% |
| 22 | 2.76% | 2.79% | 2.37% |

Table 8: Welfare loss from the 1981-82 recession by schooling level

| Schooling level in 1980 | Welfare loss | Max welfare loss* | Change in schooling |
|---|---|---|---|
| 10 | 2.02% | 4.73% | 0.34 |
| 12 | 2.14% | 4.89% | 0.25 |
| 14 | 2.27% | 4.95% | 0.17 |
| 16 | 2.39% | 5.01% | 0.04 |

* Maximum welfare loss is the welfare loss when people stay at home for 2 years when they were hit by the recession at the time of potential entry.

of the recession. The welfare loss is smaller for older workers because they had accumulated some experience when the recession occurred.

Table 8 shows the impact of the 1981-82 recession for students with different schooling level by the time the recession started. Here I present the welfare loss for four schooling levels: grade 10, 12, 14, and 16. In general, higher educated groups suffer from a large welfare loss. This is because on average, there is a 0.23-year increase in schooling due to the recession, and the increase is large for lower educated groups, as shown in the last column. For students with lower schooling levels, their tuition costs are lower, so they are more likely to obtain more schooling during a recession.

In the second column of Table 8, I show the maximum welfare loss in the worst case scenario, when individuals are forced to stay at home for two years until the recession ends. In this case, after the recession, these unlucky cohort behaves the same as a lucky cohort just leaving school, except that these unlucky cohorts have lost two years of earnings. In this worst case scenario, individuals suffer a 5% welfare loss. When individuals have the choice of going to school or working during a recession, welfare loss is reduced by half.

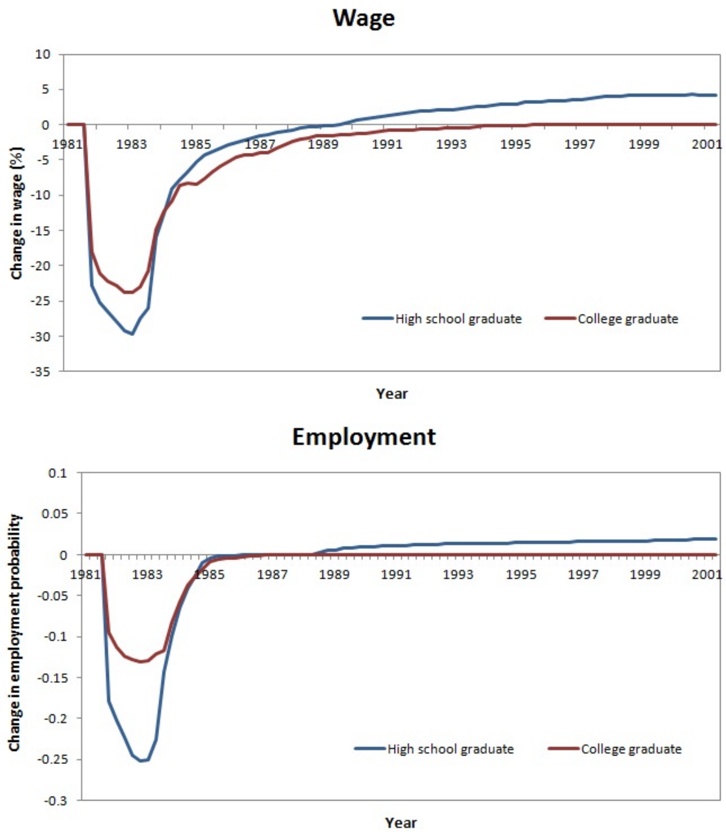



Figure 6: Impact of the 1981-82 recession on wage and employment

Figure 6 presents the percentage change in wages from the 1981-82 recession for those who have completed high school and those who have completed college in 1980. For both groups of students, there was a large initial wage loss of 20% to 30% during the recession. The wage drop was larger for high school graduates. After the recession, the wage loss of high school graduates recovers in five years and there is even a 4% wage increase after the recession because some students increase their schooling due to the recession. In contrast, since college graduates merely increase their schooling during the recession, their wage loss recovers much slowly, which fades in 10 years. These findings about college graduates are similar to those of Kahn (2010), where she finds that a 1% increase in unemployment leads to a 7% to 9% initial wage loss that recovers in 15 years.

Figure 6 also presents the change in employment probability from the 1981-82 recession for high school graduates and college graduates. Individuals suffer from a 10% to 25% decline in employment probability during the recession, and high school graduates suffer more severely than

Table 9: Impact on schooling for potential labor market entrants from the 1981-82 recession

| Highest grade completed | Change in years of schooling | Fraction of attending school | Change in laborforce composition ($\alpha_0$) |
|---|---|---|---|
| High School Dropout | 0.37 | 18.1% | $-0.0249$ |
| High School Graduate | 0.29 | 15.7% | $-0.0134$ |
| Some College | 0.19 | 12.5% | $-0.0066$ |
| College Graduate | 0.05 | 4.2% | $-0.0015$ |

college students because a large proportion of high school graduates choose to attend school instead of working in the recession. After the recession, the employment rate quickly recovers, which is consistent with findings in the literature that there is no scarring effect on future unemployment in the US (e.g., Genda et al. (2010)). For high school graduates, there is an increase in employment rate of one ppt, while college grduates does not see an increase. This is because high school graduates acquire 0.29-year more schooling than they would have without a recession, and the increase in schooling for college graduates is subtle.

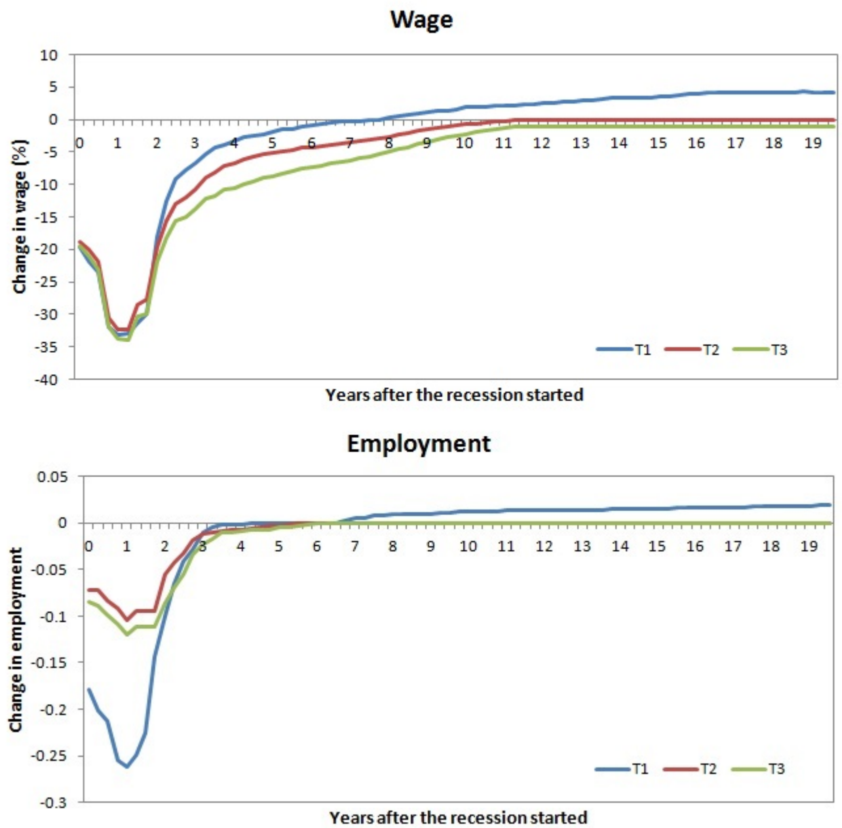
## 7.2 Importance of endogenizing schooling decisions

The above findings suggest that schooling plays an important role in the welfare analysis. Schooling decisions have two potential effects in the welfare analysis, the insurance effect and the composition effect. First, acquiring more schooling can partially shield agents from the negative impact of a recession. This is because the flow utility of schooling is not affected by the business cycle and acquiring more schooling can increase future wages and employment opportunities. Second, there can be selection problem if agents with different characteristics make different schooling decisions over the business cycle. As a result, there will be a change in the workforce composition in a recession.

Table 9 shows the impact on schooling from the 1981-82 recession for potential labor market entrants who would have entered the labor market if the recession did not occur. Here I present the impact on labor market entrants with different education levels. For high school graduates, there is a 0.29-year increase in schooling due to the recession, while there is only a 0.05-year increase for college graduates. The same pattern holds for fraction of attending school. When the 1980s recession occurred, 15.7% of high school graduates acquire more schooling due to the recession, while only 4.2% of college graduates do so. The increase in schooling partially offsets the negative impact from a recession. Therefore, those who undergo a larger increase in schooling incur a smaller welfare loss.

The last column of Table 9 shows a change in the laborforce composition with respect to initial human capital ($\alpha_0$) during a recession. In general, those who choose to stay in school during a recession have higher initial human capital endowment compared to those who choose to enter the labor market. The difference is larger for lower educated groups. For example, the difference for high school graduates is 0.0134, while the difference for college graduates is 0.0015. The selection in terms of initial human capital is mainly because individuals with higher initial human capital also have higher values in schooling, as shown in the estimates in Table 1 and Table 2. Therefore, although workers with higher initial human capital are less affected by a recession, they are more likely to stay at school because they value school more. This selection problem from the schooling decision suggests that it is important to endogenize the schooling decision over the business cycle because the workforce composition with respect to unobserved initial human capital is different across graduation cohorts.

To further illustrate the importance of endogenizing the schooling decision, Figure 7 presents the changes in wages and employment of high school students who would have graduated from high school and entered the labor market without the 1981-82 recession. T1 represents the case when these high school graduates are allowed to make schooling decisions during the recession, which captures the correct treatment effect. T2 represents the case when high school graduates are prohibited from staying at school during the recession and are forced to either work or stay at home. T2 ignores the insurance effect but takes into account the composition effect. T3 is part of the treatment group who still chooses to enter the labor market in a recession. In other words, T3 represents the high school graduates in the recession. T3 reflects what is measured in the OLS regression. The OLS regression compares the wages of students graduated in a boom with the wages of students graduated in a bust, so it ignores both the insurance effect and composition effect.




T1: baseline; T2: ignore insurance effect; T3: ignore insurance effect and composition effect.

Figure 7: Insurance effect and composition effect of the schooling decision

For all groups, there is a significant wage drop during the recession. For T1, the wage loss quickly recovers after the recession. Five years after the recession, wages recover to the original level. Indeed, there is even a 4% wage increase 15 years after the recession. However, if we ignore the schooling decisions made during a recession, T2 shows that we will observe a much slower recovery and we won't be able to get the future wage growth. In this case, wage loss recovers in 8 years. If we further ignore the composition effect, T3 has the slowest recovery rate and there is a permanent wage loss because those with lower initial human capital are more likely to enter the labor market in a recession.

The bottom figure shows the change in employment opportunities. For T2 and T3, there is an initial decrease in employment rate of 10 ppt during the recession. For T1, there is a larger initial decrease in employment rate compared to T2 and T3 because 15.7% of the sample in T1 attend school during the recession. After the recession, all three groups recover quickly and there is a 1-ppt increase seven years after the recession only for T1. This is because workers with higher schooling face a higher UE transition rate and a lower EU transition rate than workers with less education.

Therefore, when examining the impact of a recession on wages and employment, it is important to endogenize the schooling decisions over the business cycle. If we ignore the insurance effect or composition effect, we may underpredict the changes in wages and employment and overestimate the scarring effect on wages.

Table 10 shows the present values of wages, earnings and utility for high school graduates at age 18 for the three treatment groups. For all three measures of the welfare loss, ignoring the insurance effect (T2) overestimates the welfare loss, and ignoring both effects (T3) causes a further overestimation. Note here using the present value of wages as the welfare measure will cause underestimation of the welfare loss from a recession compared to the present value of utility, because the calculation ignores those who were unemployed and those who went to school in a recession and only accounts for those who were working. If we treat those who are not working as having a zero earning and calculate the present value of earnings, the measure is still not accurate because people enjoy the value of leisure or value of schooling when they are not working. The correct measure of welfare should be the present value of utility. The calculation of present values shows that the present value of wages causes underestimation of the welfare loss and the present value of earnings causes overestimation of the welfare loss. This finding suggests that it is necessary to develop and estimate a life cycle model to recover the actual welfare loss from a recession.

Table 10: Changes in the lifetime values for high school graduates from the 1981-82 recession

| | Wages | Earnings | Welfare |
|---|---|---|---|
| T1 | −1.92% | −2.68% | −2.56% |
| T2 | −2.54% | −3.02% | −2.85% |
| T3 | −2.92% | −3.46% | −3.17% |

Note: T1: baseline; T2: ignore insurance effect; T3: ignore insurance effect and composition effect.

### 7.3 Wage decomposition

Now I would like to examine the mechanisms of the persistent wage loss by quantifying different channels of wage change. From now on, without loss of generality, the analysis will be focus on high school graduates who plan to enter the labor market by the time the recession started.

Individuals face a tradeoff when they enter the labor market during a recession. On the one hand, they can start with a lower-paid job and catch up later through on-the-job search. However, it may take time to climb up the wage ladder due to search frictions. On the other hand, they can go to school or stay at home and wait until the recession is over. In this case, at the later time of labor market entry, they will lose experience and tenure. Remember that worker's wage is a share of his current period output. Therefore, the wage loss can be decomposed into the loss from the share and loss from the output. In the first case, workers start in a job that offers a lower share while in the second case, the loss in experience and tenure will be reflected in the loss in the output. Now I explore the impact of a recession on experience, tenure and share of worker's output.

Figure 8 presents the change in experience and tenure from a recession occurring at potential entry. There is a five-quarter decrease in experience and the loss in experience slowly recovers over time. This is because the employment probability increases after the recession, as shown in Figure 6. Similarly for tenure, the recession causes a drop of six quarters and it recovers in 18 years. There is a larger drop in tenure compared to experience because workers switch jobs more often after the recession in order to climb up the wage ladder.

The above analysis of experience and tenure captures the impact of a recession on human capital accumulation. At the same time, a recession also affects job mobility. In the model, workers search for a higher $\mu$ when they are doing on-the-job search, therefore, $\mu$ captures the wage gain from moving up the wage ladder. Figure 9 presents the change in $\mu$ from a recession occurring at potential entry and from a recession occurring 10 years after entry. There is an initial drop of 10-15 ppt during the recession. It only takes three years after the recession to catch up. Furthermore, $\mu$ is 1.5 ppt higher than the control group ten years after the recession. This is because those who increase their schooling during a recession can get a higher share of output from firms.

I conduct a wage decomposition analysis by exploring different channels of wage change. There

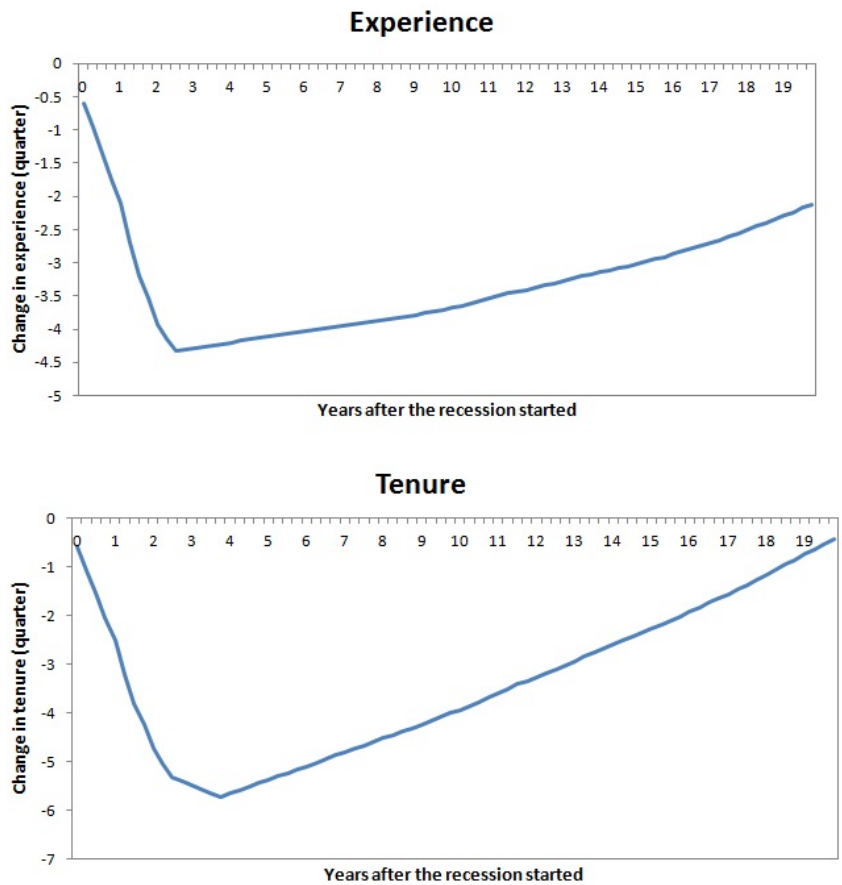



Figure 8: Impact on experience and tenure

are four channels of wage change. First is through shocks to aggregate productivity and match productivity, which are exogenous to agents. The rest of three channels are endogenous. First, individuals can accumulate their human capital by acquiring more schooling. Second, they can also accumulate their human capital through learning-by-doing, that is, by increasing their experience and tenure while working. Lastly, workers can also move between jobs to search for a better wage offer that offers a larger share of output.

Figure 10 presents the wage decomposition for students who just completed high school when the 1981-82 recession started. During the 1981-82 recession, negative aggregate productivity shocks and match productivity shocks lead to a 10.5% decrease (on average) in the productivity of new matches, which accounts for 40.7% of total wage loss. After the recession, the aggregate productivity recovers

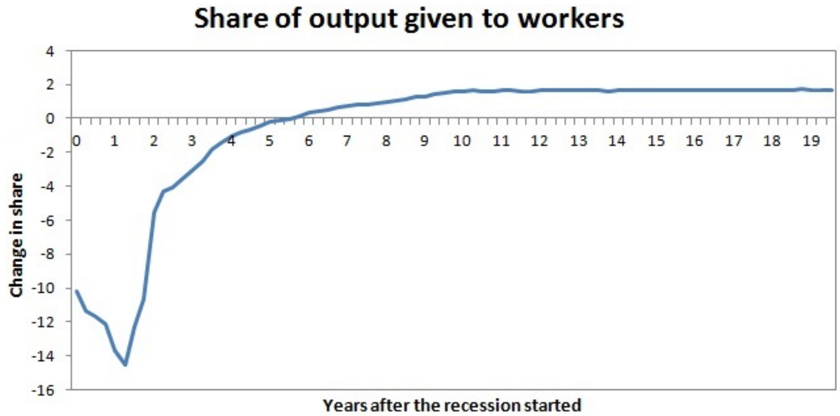



Figure 9: Impact on $\mu$

while the negative match productivity shock is persistent within a match. If workers switch to a new job, they will get a new draw of match productivity. Therefore, workers have strong incentives to switch jobs and the wage loss from match productivity quickly diminishes after the recession.

Besides the exogenous productivity shocks, the majority of the wage loss comes from the loss from $\mu$. Wage loss from $\mu$ accounts for about 44.0% of the total wage loss during the recession, while the loss from experience and tenure only accounts for 19.1% of the wage loss. After the recession, the loss from $\mu$ recovers quickly, which only takes three years, while it takes 15 years for the wage loss from the decreased experience and tenure to fade out. This suggests that the loss in worker's share of output is the major cause of the huge wage loss during the recession and the loss in experience and tenure is the major reason for the persistent wage loss after the recession.

At the same time, the increase in schooling leads to a direct increase in worker's output, which contributes to a 2.5% wage growth. Moreover, the increase in schooling helps workers to get a large fraction of their output from firms. Therefore, the increase in schooling also has an indirect impact on wage growth by improving $\mu$, which contributes to another 1.5% wage growth. Therefore, 15 years after the recession, when the negative impact from the loss in experience and tenure disappears, there is a 4% wage increase because the increase in schooling not only directly increases worker output, but also increases the share of output workers can get from firms.

I do the same exercise for other education groups and the conclusion is similar: the losses in

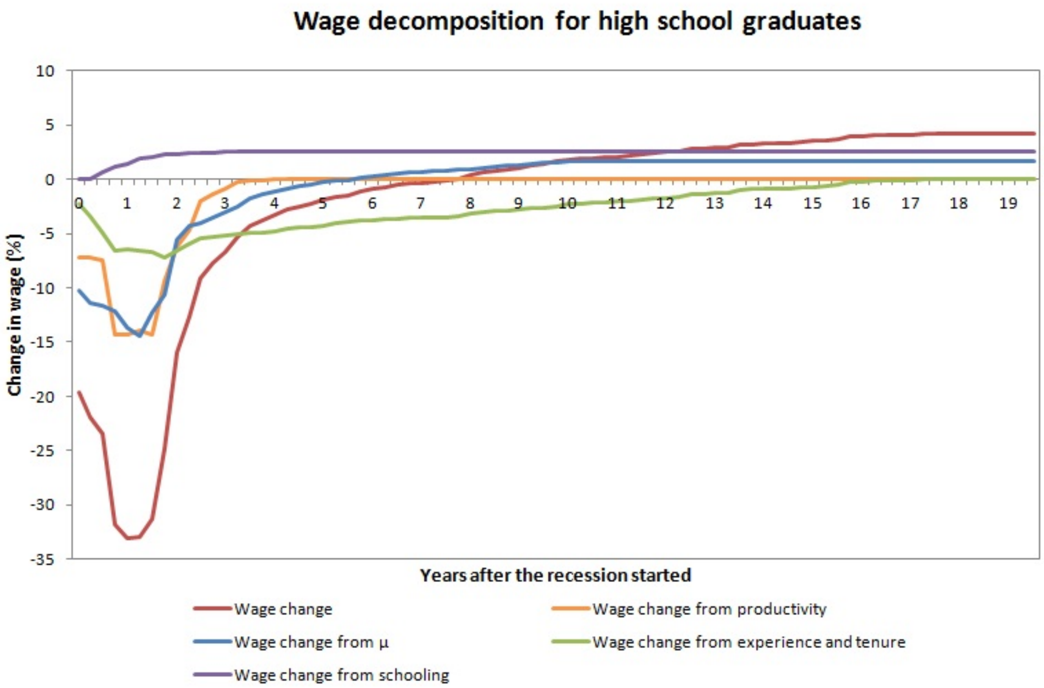



Figure 10: Wage decomposition

productivity and $\mu$ are the two major causes of the wage loss during the recession. The persistency of the wage loss after the recession is mainly explained by the loss in experience and tenure.

# 8 Conclusion

In this paper, I evaluate the lifetime welfare and labor market consequences from an early career recession in a general equilibrium framework. I also explore the mechanisms that account for lifetime wage changes by decomposing those changes into different channels: changes from schooling, from work experience, and from job mobility.

The model is an extension of a directed search model, the Block Recursive Equilibrium (BRE) framework by Menzio and Shi (2010a). Using BRE allows me to solve the model outside of the steady state because the value functions and policy functions depend only on aggregate shocks, and not the endogenous distribution of workers across wages and employment states. Therefore, I can solve the model outside of the steady state. Furthermore, the framework can easily incorporate on-the-job search and endogenous separation. I further extend this framework by endogenizing the schooling decision, introducing job tenure, allowing for unobserved heterogeneity in agents, and proposing a wage contract that facilitates the wage decomposition analysis.

Counterfactual analysis shows that experiencing the 1981-1982 recession as a youth causes a 2.2% to 3.0% lifetime welfare loss. The welfare loss is larger for students with higher schooling level because they are less likely to acquire more schooling during the recession.

I find that endogenizing schooling decision is important for the welfare analysis for two reasons, the insurance effect and the composition effect. The analysis shows that the increase in schooling during a recession increases future wages and employment. I also find that during a recession, there is a change in the workforce composition with respect to initial human capital. Therefore, endogenizing schooling decisions can avoid overestimation of the welfare loss.

Lastly, I also explore the mechanisms of the wage change. Wage decomposition shows that loss from worker's share of output explains the majority of wage loss during the recession, but quickly recovers after the recession. In contrast, loss in experience and tenure explains a small proportion of the wage loss, but takes longer to recover after the recession.

This model incorporates two types of productivity shocks that will affect wages and unemployment. But the model does not have industry-specific productivity shocks and does not distinguish between job-to-job transitions within industry and across industry. Future studies can explore different sources of the business cycle and allow them to have different impacts on workers working in different industries.

In addition, it is important to incorporate heterogeneous firms because firms with different productivity and different sizes may behave differently during a recession. It would be interesting to see whether the impact of a recession varies by workers among different firms. However, incorporating firms' productivity will require employer-employee matched panel data and allowing for heterogeneous firm is beyond the scope of this model, so I will leave it for future research.

## References


Jerome Adda, Christian Dustmann, Costas Meghir, and Jean-Marc Robin. Career progression, economic downturns, and skills. Technical report, National Bureau of Economic Research, 2013.

J.R. Betts and L.L. McFarland. Safe port in a storm: The impact of labor market conditions on community college enrollments. *Journal of Human Resources*, pages 741–765, 1995.

Audra J Bowlus and Haoming Liu. The long-term effects of graduating from high school during a recession: bad luck or forced opportunity? *University of Western Ontario, CIBC Centre for Human Capital and Productivity Working Papers*, 2003.

Steven J Davis and Till M von Wachter. Recessions and the cost of job loss. Technical report, National Bureau of Economic Research, 2011.

H. Dellas and V. Koubi. Business cycles and schooling. *European Journal of Political Economy*, 19 (4):843–859, 2003.

Wouter J den Haan, Garey Ramey, and Joel Watson. Job destruction and propagation of shocks. *The American Economic Review*, 90(3):482–498, 2000.

D.T. Ellwood. Teenage unemployment: Permanent scars or temporary blemishes?, 1982.

Burcu Eyigungor. Specific capital and vintage effects on the dynamics of unemployment and vacancies. *The American Economic Review*, 100(3):1214–1237, 2010.

W. Franz, J. Inkmann, W. Pohlmeier, and V. Zimmermann. Young and out in germany: On the youths' chances of labor market entrance in germany. Technical report, National Bureau of Economic Research, 1997.

Y. Genda, A. Kondo, and S. Ohta. Long-term effects of a recession at labor market entry in japan and the united states. *Journal of Human Resources*, 45(1):157–196, 2010.

P. Gregg. The impact of youth unemployment on adult unemployment in the ncds. *The Economic Journal*, 111(475):626–653, 2001.

P. Gregg and E. Tominey. The wage scar from male youth unemployment. *Labour Economics*, 12 (4):487–509, 2005.

James J Heckman and Paul A LaFontaine. The american high school graduation rate: Trends and levels. *The review of economics and statistics*, 92(2):244–262, 2010.

L.S. Jacobson, R.J. LaLonde, and D.G. Sullivan. Earnings losses of displaced workers. *The American Economic Review*, pages 685–709, 1993.

L.B. Kahn. The long-term labor market consequences of graduating from college in a bad economy. *Labour Economics*, 17(2):303–316, 2010.

Michael P Keane and Kenneth I Wolpin. The career decisions of young men. *Journal of political Economy*, 105(3):473–522, 1997.

G. Menzio and S. Shi. Block recursive equilibria for stochastic models of search on the job. *Journal of Economic Theory*, 145(4):1453–1494, 2010a.

G. Menzio and S. Shi. Efficient search on the job and the business cycle. *Journal of Political Economy*, 119:468–510, 2011.

G. Menzio, I.A. Telyukova, and L. Visschers. Directed search over the life cycle. Technical report, National Bureau of Economic Research, 2012.

Guido Menzio and Shouyong Shi. Directed search on the job, heterogeneity, and aggregate fluctuations. *The American Economic Review*, 100(2):327–332, 2010b.

Philip Oreopoulos, Till von Wachter, and Andrew Heisz. The short-and long-term career effects of graduating in a recession. *American Economic Journal: Applied Economics*, 4(1):1–29, 2012.

P. Oyer. Initial labor market conditions and long-term outcomes for economists. *Journal of Economic Perspectives*, 20(3):143–160, 2006.

P. Oyer. The making of an investment banker: Stock market shocks, career choice, and lifetime income. *The Journal of Finance*, 63(6):2601–2628, 2008.

O. Raaum and K. Røed. Do business cycle conditions at the time of labor market entry affect future employment prospects? *The Review of Economics and Statistics*, 88(2):193–210, 2006.

Jean-Marc Robin. On the dynamics of unemployment and wage distributions. *Econometrica*, 79 (5):1327–1355, 2011.

C.J. Ruhm. Are workers permanently scarred by job displacements? *The American Economic Review*, 81(1):319–324, 1991.

O.N. Skans. *Scarring Effects of the First Labor Market Experience*. IZA, 2010.

A.H. Stevens. Persistent effects of job displacement: The importance of multiple job losses. *Journal of Labor Economics*, pages 165–188, 1997.

Till von Wachter and Stefan Bender. In the right place at the wrong time: The role of firms and luck in young workers' careers. *The American Economic Review*, 96(5):1679–1705, 2006.

# Appendix A Solution to the Equilibrium

Since individuals live a finite life in this model, the equilibrium can be solved using backward induction. At the last period A, the unemployment value function $U_A$, the employment value function $H_A$, and the firm value function $J_A$ satisfy the equilibrium conditions (iii), (iv) and (vi) if and only if

$$\begin{aligned} U_A(s,\psi) &= b^u_{A,k} \\ H_A(\mu,s,\bar{z},\psi) &= \mu y\bar{z}\phi(s) \\ J_A(\mu,s,\bar{z},\psi) &= (1-\mu)y\bar{z}\phi(s) \end{aligned}$$

Notice that in the last period, the value functions of workers and firms depend only on the current period's payoff, not on the continuity values. Therefore, $U_A$ does not depend on the aggregate state of the economy $\psi$ and $H_A$ and $J_A$ depends only on the aggregate state of the economy via the aggregate productivity shock $y$. Hence, without loss of generality, I can write $U_A(s,\psi) = U_A(s,y,z)$, $H_A(\mu,s,\bar{z},\psi) = H_A(\mu,s,\bar{z},y,z)$, and $J_A(\mu,s,\bar{z},\psi) = J_A(\mu,s,\bar{z},y,z)$.

Given the firm's value function in the last period, we can solve the market tightness function $\theta_A$ in the last period, which satisfies the equilibrium conditions (ii) and (vi) if and only if

$$\theta_A(\mu,s,\psi) = \begin{cases} q^{-1}\left(\dfrac{k}{J_A(\mu,s,z,y,z)}\right), & \text{if } J_A(\mu,s,z,y,z) \geq k, \\ 0, & \text{otherwise .} \end{cases} \tag{7}$$

Note that $\theta_A(\mu,s,\psi)$ depends on the fraction of output given to workers, $\mu$, the workers' skill, $s$, and aggregate shocks $y$ and $z$, but not on other components of aggregate state of the economy $(n,u,e,\gamma)$. Hence, we can write $\theta_A(\mu,s,\psi) = \theta_A(\mu,s,y,z)$.

The policy function $x^u_A$ for unemployed workers satisfies the equilibrium condition (iii) if and only if it solves the search problem

$$R_A(U_A(s,\psi),s,\psi) = \max_{\mu}\{p(\theta_A(\mu,s,y,z))[H_A(\mu,s,z,y,z) - U_A(s,y,z)],0\} \tag{8}$$

We can plug (7) into (8) to solve for $\mu$. The solution depends only on $(s,y,z)$. Therefore, we can write $x^u_A(s,\psi) = x^u_A(s,y,z)$.

The policy function $x^e_A$ for employed workers satisfies the equilibrium condition (iv) if and only if it solves the search problem

$$R_A(H_A(\mu,s,\bar{z},\psi),s,\psi) = \max_{\tilde{\mu}}\{p(\theta_A(\tilde{\mu},s,y,z))[H_A(\tilde{\mu},s,z,y,z) - H_A(\mu,s,\bar{z},y,z)],0\} \tag{9}$$

We can plug (7) into (9) to solve for $\tilde{\mu}$, which depends only on $(\mu,s,\bar{z},y,z)$. Therefore, we can write $x^e_A(\mu,s,\bar{z},\psi) = x^e_A(\mu,s,\bar{z},y,z)$.

The separation policy function $d_A$ satisfies the equilibrium condition (iv) if and only if it solve the separation problem

$$\max_{d\in[\delta_h,1]} dU_A(s,y,z) + (1-d)[H_A(\mu,s,\bar{z},y,z) + \lambda^e_{h,A}R_A(H_A(\mu,s,\bar{z},y,z),s,y,z)] \tag{10}$$

We can rewrite the employment decision $d_A(\mu,s,\bar{z},\psi)$ as

$$d_A(\mu,s,\bar{z},\psi) = \begin{cases} 1, & \text{if } U_A(s,y,z) > H_A(\mu,s,\bar{z},y,z) + \lambda^e_{h,A}R_A(H_A(\mu,s,\bar{z},y,z),s,y,z), \\ \delta_h, & \text{otherwise} \end{cases}$$

The objective function in (11) depends only on the workers' skill, $s$, on the piece-rate, $\mu$, and on the aggregate shocks $y$ and $z$. Hence, $d_A(\mu, s, \bar{z}, \psi) = d_A(\mu, s, \bar{z}, y, z)$.

In period $A-1$, the unemployment value function $U_{A-1}$ satisfies the equilibrium condition (iii) if and only if

$$U_{A-1}(s, \psi) = b^u_{A-1,k} + \beta E_{\hat{\psi}|\psi}[U_A(s, \hat{y}, \hat{z}) + \lambda^u_{h,A} R_A(U_A(s, \hat{y}, \hat{z}), s, \hat{y}, \hat{z})]$$

The employment value function $H_{A-1}$ satisfies the equilibrium condition (iv) if and only if

$$\begin{aligned} H_{A-1}(\mu, s, \bar{z}, \psi) =& \mu y \bar{z} \phi(s) + \beta E_{\hat{\psi}|\psi} \max_{d \in [\delta_h, 1]} \{ d U_A(\tilde{s}, \hat{y}, \hat{z}) \\ &+ (1-d)[H_A(\mu, \hat{s}, \bar{z}, \hat{y}, \hat{z}) + \lambda^e_{h,A} R_A(H_A(\mu, \hat{s}, \bar{z}, \hat{y}, \hat{z}), \tilde{s}, \hat{y}, \hat{z})]\} \end{aligned}$$

The employment value function $J_{A-1}$ for firms is equal to

$$\begin{aligned} J_{A-1}(\mu, s, \bar{z}, \psi) =& (1-\mu) y \bar{z} \phi(s) \\ &+ \beta E_{\hat{\psi}|\psi}[(1 - d_A(\mu, s, \bar{z}, \hat{y}, \hat{z}))(1 - \lambda^e_{h,A} \tilde{p}(H_A(\mu, \hat{s}, \bar{z}, \hat{y}, \hat{z}), \tilde{s}, \hat{y}, \hat{z})) J_A(\mu, \hat{s}, \bar{z}, \hat{y}, \hat{z})] \end{aligned}$$

Notice that $U_{A-1}$, $H_{A-1}$ and $J_{A-1}$ depend on the aggregate state of the economy $\psi$ only via productivity shocks $y$ and $z$. Using backward induction, we can show that policy functions and value functions for employed workers and unemployed workers, as well as value functions for firms, depend only on the aggregate state of the economy $\psi$ via the aggregate shocks $y$ and $z$, for all ages $a \in \{A, A-1, ..., 1\}$ .

Now let's look at the student's problem. At termination age $\tilde{A}$, a student must enter the labor market in the next period. The schooling value function $W_{\tilde{A}}$ is:

$$W_{\tilde{A}}(s, \psi) = b^s_k - c_h + \beta E_{\hat{\psi}|\psi}[U_{\tilde{A}+1}(s, \hat{y}, \hat{z}) + \lambda^n_h R_{\tilde{A}+1}(U_{\tilde{A}+1}(s, \hat{y}, \hat{z}), s, \hat{y}, \hat{z})]$$

At age $\tilde{A}+1$, students' value depends only on the unemployment value function and the search function, both of which only depends on $y$ and $z$. Therefore, students' value function at age $\tilde{A}$ depends only on $y$: $W_{\tilde{A}}(s, \psi) = W_{\tilde{A}}(s, y, z)$.

Next, I solve the searching problem for students at age $\tilde{A}$.

$$R_{\tilde{A}}(U_{\tilde{A}}(s, \psi), s, \psi) = \max_{\mu} \{ p(\theta_{\tilde{A}}(\mu, s, y, z))[H_{\tilde{A}}(\mu, s, z, y, z) - U_{\tilde{A}}(s, y, z)], 0\}$$

As a result, the solution to the student search problem $x^n_{\tilde{A}}$ depends only on $\psi$ via productivity shocks, where $x^n_{\tilde{A}}(s, \psi) = x^n_{\tilde{A}}(s, y, z)$.

The schooling policy function $I^s_{\tilde{A}}$ satisfies the equilibrium condition (v) if and only if it solves the separation problem

$$\max_{I^s \in \{0,1\}} \{ I^s [W_{\tilde{A}}(s, y, z) + \lambda^n_h R_{\tilde{A}}(W_{\tilde{A}}(s, y, z), s, y, z)] + (1 - I^s)[U_{\tilde{A}}(s, y, z) + \lambda^u_{h,\tilde{A}} R_{\tilde{A}}(U_{\tilde{A}}(s, y, z), s, y, z)]\}$$

We can rewrite the employment decision $I^s_{\tilde{A}}(s, \psi)$ as

$$I^s_{\tilde{A}}(s, \psi) \quad = \quad \begin{cases} 1, & \text{if } W_{\tilde{A}}(s, y, z) + \lambda^n_h R_{\tilde{A}}(W_{\tilde{A}}(s, y, z), s, y, z) > U_{\tilde{A}}(s, y, z) + \lambda^u_{h,\tilde{A}} R_{\tilde{A}}(U_{\tilde{A}}(s, y, z), s, y, z), \\ 0, & \text{otherwise} \end{cases}$$

The objective function in (11) depends only on the workers' skill and productivity shocks, where $I^s_{\tilde{A}}(s,\psi) = I^s_{\tilde{A}}(s,y,z)$. Similarly, it's easy to show that the policy functions for returning to school for unemployed workers ($I^u_{\tilde{A}}(s,\psi)$) and employed workers ($I^e_{\tilde{A}}(\mu,s,\bar{z},\psi)$) depend only on $\psi$ via $y$ and $z$.

The schooling value function $W_{\tilde{A}-1}$ satisfies the equilibrium condition (v) if and only if

$$\begin{aligned} W_{\tilde{A}-1}(s,\psi) =& b^s_k - c_h + \beta E_{\hat{\psi}|\psi} \max_{I^s\in\{0,1\}} \{I^s[W_{\tilde{A}}(\hat{s},\hat{y},\hat{z}) + \lambda^n_h R_{\tilde{A}}(W_{\tilde{A}}(\hat{s},\hat{y},\hat{z}),\hat{s},\hat{y},\hat{z})] \\ &+ (1-I^s)[U_{\tilde{A}}(\hat{s},\hat{y},\hat{z}) + \lambda^u_{h,\tilde{A}} R_{\tilde{A}}(U_{\tilde{A}}(\hat{s},\hat{y},\hat{z}),\hat{s},\hat{y},\hat{z})]\} \end{aligned}$$

Note that $W_{\tilde{A}-1}$ does not depend on the aggregate state of the economy $\psi$, except for the aggregate shocks. Using backward induction, the remaining equilibrium value and policy functions for students and policy functions for returning to school depend only on the aggregate state of the economy $\psi$ via the productivity shocks $y$ and $z$, for all ages below the maximum schooling age $\tilde{A}$ ($a \in \{\tilde{A}, \tilde{A}-1, ..., 1\}$) . Hence, an equilibrium exists, and it is block recursive.