

Time- and State-Dependent Pricing: A Unified Framework*

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Abstract

We develop a tractable unified framework for solving optimal time- and state-dependent price setting problems. We use it to study models with adjustment costs and infrequent information, entertaining various alternatives for the source and nature of infrequent information. In particular, we consider both models where information is infrequent for exogenous reasons and models where firms face a lump-sum cost to gather and process information. Our choice of state variables is key to make the problems tractable. Specifically, we replace the usual state variable in state-dependent pricing problems - the discrepancy between the firm's price and its frictionless optimal level - with its expectation conditional on the firm's information set, and augment the state space with the time elapsed since the last date when information was fully factored into the pricing decision ("information date"). This allows us to formulate all price-setting problems as two-dimensional optimal stopping problems. Our analysis uncovers new insights about price setting. Despite the presence of menu costs, firms may choose to change prices in the absence of new information. Time dependency in pricing rules arises as a consequence of the build up of unobserved information. In these circumstances, the inaction region changes as a function of the time elapsed since the last information date. When the next information date is known, the presence of menu costs produces an extreme form of inaction: irrespective of the size of the expected price discrepancy, it is never optimal to adjust just prior to the information date.

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1 Introduction

The recent availability of vast amounts of micro price data has generated renewed interest in price setting among macroeconomists, especially since the seminal work of Bilal and Klenow (2004). This has led to noteworthy developments in this field - in particular in terms of microfounded models with explicit price-setting frictions. Many papers have expanded the frontier of so-called menu-cost models.¹ In addition, recent work has analyzed the implications of explicit informational frictions for price setting behavior.² Despite this progress, the price setting literature still appears to be guided by a dichotomy between time-dependent and state-dependent pricing rules,³ and these two branches have developed essentially in parallel.

In this paper we develop a tractable unified framework for solving optimal time- and state-dependent price-setting problems. We use it to study models with adjustment costs and infrequent information, and also illustrate how the framework can be applied to most price-setting problems analyzed previously in the literature. We entertain various alternatives for the source and nature of infrequent information. In particular, we consider both models where information is infrequent for exogenous reasons and models where firms face a lump-sum cost to gather and process information. The relevance of a model with separate adjustment and information costs has been acknowledged for at least twenty years (Blanchard and Fischer 1989 page 413, Caballero 1989 page 29). However, problems of this kind are usually hard to solve.

The key to making our approach tractable is our choice of state variables. We rely on a commonly used second-order approximation to the profit loss due to price-setting frictions, which implies that these losses are proportional to the square of the discrepancy between the firm's (log) price and its frictionless optimal level (henceforth the *price discrepancy*). Because of the intertemporal nature of the problem, the price setter must forecast the path of these squared discrepancies. Given the stochastic processes used to model the frictionless optimal price, these forecasts turn out to be a very simple function of two conveniently chosen state variables: the conditional expectation of the price discrepancy and the time elapsed since the last date when information was fully factored into the pricing decision (henceforth *information dates*). This choice of state variables allows us to cast each pricing problem as a two-dimensional optimal stopping problem. For each case that we analyze, we rewrite the Bellman equation that characterizes the

¹Some examples are Golosov and Lucas (2007), Gertler and Leahy (2008), Midrigan (2011), and Nakamura and Steinsson (2010).

²For instance, Reis (2006), Woodford (2009), Maćkowiak and Wiederholt (2009).

³The titles of the following papers, among others, are illustrative of this dichotomy: "Time- or state-dependent price setting rules? Evidence from micro data," (Dias, Marques and Silva 2007), "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?" (Klenow and Kryvtsov 2008), "Is Firm Pricing State or Time Dependent? Evidence from U.S. Manufacturing" (Midrigan 2010).

firm's value function in the inaction region as either an ordinary differential equation in one of those two state variables, or as a partial differential equation in both variables. Boundary conditions dictated by the nature of the problem pin down the solution, which in some cases can be written in (almost) closed form, or is otherwise obtained numerically through algorithms that make use of finite-difference methods.

Our unified framework makes it clear that pure time- and pure state-dependent pricing policies are special cases of more general time-and-state-dependent rules that turn out to be optimal in the presence of both adjustment and information frictions. Time dependency in pricing rules arises as a consequence of the build up of unobserved information. In these circumstances, the inaction region widens with the time elapsed since the last information date. The reason is that the option value of waiting for new information increases with time, due to the accumulation of underlying (unobserved) innovations. When the next information date is known, the presence of menu costs produces an extreme form of inaction: irrespective of the size of the expected price discrepancy, it is never optimal to adjust just prior to the information date. Finally, our results show that despite the presence of menu costs, firms may choose to change prices in the absence of new information (henceforth uninformed adjustments). This may be optimal in environments in which the drift in the frictionless optimal price is high relative to its volatility. These uninformed adjustments resemble indexation by trend inflation, a feature that is sometimes assumed in DSGE models (an early example is Yun 1996).

Section 2 introduces our framework, with a focus on our choice of state variables. In Section 3 we examine the connection between information and time dependency of pricing rules. We start with a setting where innovations are infrequent, but become known immediately so that the price setter's information is always up-to-date, as in Danzinger (1999) and Gertler and Leahy (2008). In the presence of menu costs, this assumption implies a purely state-dependent pricing rule, whereby price adjustments take place whenever shocks drive the price discrepancy outside an optimally chosen inaction region. We contrast this problem with cases in which unobserved information builds up over time and is revealed infrequently to the price setter, for reasons that are outside of her control. In these cases the inaction region depends on the time elapsed since the last information date, widening over time.

While in Section 3 we analyze cases where infrequent information arises for exogenous reasons, in Section 4 we tackle problems where it results from a lump-sum cost of gathering and processing information. We start by revisiting a benchmark case with costly information, but without adjustment costs, as in Caballero (1989) and Reis (2006).⁴ The optimal pricing rule implies infre-

⁴Moscarini (2004) obtains infrequent information sampling as the optimal policy under limited information-

quent information gathering and processing, but (potentially) continuous price adjustments.⁵ We then consider the simplest setting with both information and adjustment costs, following Bonomo and Carvalho (2004, 2010).⁶ When these two costs are borne out together, price setters optimally choose to fix prices between optimally chosen information-adjustment dates.⁷ This rule can be seen as a Taylor-Phelps pricing model with endogenous “contract lengths”.

In Section 4 we also analyze the case where information and adjustment costs are dissociated, i.e. firms have the option to adjust without information and to gather and process information but then choose not to adjust. In this case the optimal pricing rule features both infrequent price changes and infrequent incorporation of information into prices. It is characterized by an inaction region for price adjustment and information gathering/processing, which is defined by the intersection between an adjustment inaction region and an information inaction region. The borders inherited from the adjustment inaction region trigger uninformed adjustments, while the border inherited from the information inaction region triggers information gathering/processing. As in the case of exogenous known information dates, it is never optimal to make an uninformed price adjustment just prior to an information date. Rather than incurring the menu cost to make such an adjustment and then immediately incurring the information gathering/processing cost, it is always better to reverse the order of these actions and keep the option to adjust, to be exercised or not depending on the new information. Gorodnichenko (2008), Abel, Eberly and Panageas (2010), Alvarez, Guiso and Lippi (2010), and Alvarez, Lippi and Paciello (2010) also analyze models in which agents face information and adjustment costs. The main difference between our work and theirs is that our approach to solving the problem with information and adjustment costs encompasses cases in which uninformed adjustments are optimal.⁸

processing capacity.

⁵In contrast, Burstein (2006) derives state-dependent pricing plans in a framework in which firms face a fixed cost of changing their price paths.

⁶The assumption of a single adjustment/information cost is also consistent with Woodford (2009), who in addition assumes that firms are subject to an information-processing constraint in periods in which they choose not to incur the lump-sum adjustment/information cost.

⁷In earlier work, Ball, Mankiw, and Romer (1988) solve for such a fixed-price time-dependent rule as an approximation to the optimal pricing policy in a menu-cost model.

⁸In Gorodnichenko’s (2008) model, firms always have some (imperfect) information about the frictionless optimal price. Abel, Eberly and Panageas (2010) show that their asymptotic result of convergence to a purely time-dependent portfolio management policy survives if one allows for what they refer to as “automatic transfers” between the agent’s investment portfolio and the transactions account. However, they do not investigate the optimality of automatic transfer plans. Alvarez, Lippi and Paciello (2010, section 7.3) and Alvarez, Guiso and Lippi (2010, Appendix AA-3) discuss the case of adjustment without information. They provide sufficient conditions under which such uninformed adjustments are not optimal. For the price-setting problem, Alvarez, Lippi and Paciello (2010) show that this is the case for a sufficiently small rate of inflation. For the problem of asset management with consumption of durables, Alvarez, Guiso and Lippi (2010) show that this is the case when there is no uncertainty in asset returns. These two papers then focus on parameterizations that satisfy those sufficient conditions, and otherwise prevent agents from making uninformed adjustments by imposing the restriction that adjustment requires observation. For a more extensive discussion see Bonomo, Carvalho and Garcia (2010).

Finally, in Section 5 we analyze a setup in which part of the relevant information is continuously observed and processed by firms at no cost, while another part is only entertained infrequently.⁹ In this situation, some adjustments may occur given the partially available information. If such information is the aggregate price level, this pricing rule might lead to partially informed adjustments based on realized aggregate inflation. This form of indexation is arguably a realistic representation of price and wage setting rules during periods of very high inflation, as witnessed in Brazil, Israel and Chile in the 80s.¹⁰

The rest of the paper is organized as follows. Section 2 formulates the basic firm problem under infrequent information. Section 3 presents two benchmark cases with continuously available information, which help build intuition for the nature of the optimal pricing policies analyzed subsequently. Section 4 analyzes price-setting under exogenously infrequent information, whereas in Section 5 infrequent information arises endogenously, due to the existence of information gathering and processing costs. Section 6 extends the analyses to an environment with continuous information about one of two components of firms' frictionless optimal prices, and infrequent information about the other component. The last section concludes with a discussion of other applications of our framework, and of directions for future research.

2 The framework

We start by setting up the firm's problem under infrequent information. To help build intuition we rely on an heuristic characterization of its essential features. In Appendices A, B, and C, we present results that are not essential for the exposition of the paper, but which formalize arguments and provide microfoundations for assumptions made in the main text.

The general idea behind the price-setting problems we analyze is that, in the absence of frictions (and thus under full information), a firm would set its price equal to the so-called frictionless optimal price - which is its instantaneously profit-maximizing price. In the presence of impediments to such "ideal" price setting, firms choose the optimal pricing policy subject to adjustment costs and exogenously infrequent or costly information gathering and processing, in order to maximize intertemporal profits.

In Appendix A we present a simple general equilibrium model that yields an expression for the (logarithm of the) frictionless optimal price for a firm, p_t^* , as the sum of two components - an aggregate (nominal aggregate demand) and an idiosyncratic (productivity) component. For

⁹Klenow and Willis (2007) and Knotek (2010), also allow for continuous incorporation of partial information into pricing decisions, in the context of menu-cost models.

¹⁰This type of price-setting policies have also been assumed in the literature on the consequences of indexation for the cost of disinflation (see Bonomo and Garcia, 1994 for price setting, and Jadresic, 2002, for wage setting).

Sections 2-5, the distinction between the sources of variation in the frictionless optimal price is a potential distraction, and for expositional simplicity we refer to a single source of uncertainty. In contrast, in Section 6 we allow for partial information about the frictionless optimal price, and thus distinguish between those two components.¹¹

Any deviation between a firm’s actual price, p_t , and p_t^* - what we refer to as a price discrepancy or price gap - entails an instantaneous flow “cost” in the form of foregone (potential) profits. In Appendix B we use the same simple model from which p_t^* is derived to show that these “discrepancy” or “deviation costs” can be taken as being approximately equal to the square of the price discrepancy: $(p_t - p_t^*)^2$. The objective of firms is to minimize the present discounted value of expected total costs, which comprise the (integral of) flow deviation costs plus any other costs that prevent the firm from charging the frictionless optimal price continuously. Changing the price to reduce the gap relative to the frictionless optimal price entails a lump-sum adjustment cost. In subsequent sections, whenever gathering and processing information entails a cost, the latter is also factored into the intertemporal price-setting problem.

Under infrequent information about p_t^* , in order to evaluate the expected flow cost due to price discrepancies the firm must form a probabilistic assessment of p_t^* given its information. Let $t_0 < t$ denote the last time when the firm had access to and processed full information about p_t^* . We can then decompose the instantaneous expected flow cost due to a price gap at time t as:

$$\begin{aligned} E_{t_0}(p_t - p_t^*)^2 &= (p_t - E_{t_0}p_t^*)^2 + E_{t_0}(p_t^* - E_{t_0}p_t^*)^2 \\ &= (p_t - E_{t_0}p_t^*)^2 + Var_{t_0}(p_t^*), \end{aligned} \tag{1}$$

where E_{t_0} and Var_{t_0} denote, respectively, the conditional expectation and conditional variance given time t_0 information. The first term in the right-hand side of (1) represents the flow cost of deviating from the *expected* level of the frictionless optimal price, and the second term represents the expected flow cost from not continuously entertaining information about the latter. In the absence of adjustment costs, p_t would be set equal to $E_{t_0}p_t^*$, reducing the first part of the discrepancy cost to zero. Otherwise the firm must optimally solve the trade-off between letting p_t drift away from $E_{t_0}p_t^*$, and paying the cost to adjust.

As for the second term in (1), it is zero when information can be freely and continuously incorporated into the pricing decision. If information gathering and processing is costly, the firm can reduce the second term at the expense of incurring the information cost. Finally, if information is exogenously infrequent, the firm cannot take interim actions to reduce the second term, but it

¹¹ While in Appendices A, and B we differentiate among firms and index them by i , throughout the paper we focus on the problem of an individual firm, and omit the i subscripts for notational simplicity.

will affect the price adjustment decision, as will become clear. In what follows we refer to times when the firm gathers and processes relevant information about p_t^* as *information dates*.

We assume throughout that p_t^* follows a Markovian stochastic process, and that for any $\Delta t > 0$ the distribution of $p_{t+\Delta t}^* - p_t^*$ depends only on Δt . From this assumption and the structure of the firm's problem, given an information date t_0 , the value function at a time $t > t_0$ - the optimized value of the firm's dynamic cost-minimization problem, denoted by V - is determined by two state variables: the time elapsed since the last information date, denoted by $\tau \equiv t - t_0$, and the deviation of p_t from its expected frictionless optimal level (which we refer to as the *expected discrepancy*), defined as:

$$z_t \equiv p_t - E_{t-\tau} p_t^*. \quad (2)$$

We thus write the firm's normalized expected profit loss as a function of τ and z :

$$\begin{aligned} E_{t-\tau}(p_t - p_t^*)^2 &= z_t^2 + \text{Var}_{t_0}(p_{it_0+\tau}^*) \\ &\equiv f(z_t, \tau). \end{aligned} \quad (3)$$

With lump-sum menu costs, price changes will be infrequent. In the absence of price changes and information updates, the value function V obeys the following Bellman equation:

$$V(z_t, \tau) = f(z_t, \tau)dt + e^{-\rho dt} E_t V(z_{t+dt}, \tau + dt), \quad (4)$$

where ρ is the time discount rate. Equation (4) is valid for all environments considered in this paper, including the standard full-information case. For each pricing problem that we consider, this Bellman equation is rewritten as either an ordinary differential equation in one of the two aforementioned state variables, or as a partial differential equation in both of them. The boundary conditions that pin down the solution to the firm's pricing problem vary depending on the nature of informational frictions, and will be introduced subsequently for each of the cases that we analyze.

3 Time dependency and information

In this section we examine the link between information and time dependency of pricing rules. We start with a setting where innovations are infrequent, but become known immediately so that the price setter's information is always up-to-date, as in Danzinger (1999) and Gertler and Leahy (2008). In the presence of menu costs, this assumption implies a purely state-dependent pricing rule, whereby price adjustments take place whenever the price discrepancy moves outside of an optimally chosen inaction region. We contrast this problem with cases in which unobserved information builds

up over time and is revealed infrequently to the price setter. In these cases the inaction region depends on the time elapsed since the last information date, widening over time.

3.1 Continuous information, infrequent innovations

When information is continuous, in the sense that firms can costlessly entertain news as they arrive, the problem becomes a particular case of our general framework. The last information date t_0 is always the current date. Hence, $E_{t_0} p_t^* = p_t^*$ and the problem features only one state variable: the perfectly and continuously observed price discrepancy $z_t = p_t - p_t^*$. The flow-cost function simplifies to:

$$f(z_t, 0) = z_t^2,$$

and the general Bellman equation (4) reduces to:

$$V(z_t) = z_t^2 dt + e^{-\rho dt} E_t V(z_{t+dt}). \quad (5)$$

As a result, optimal pricing rules turn out to be purely state dependent.¹²

Here we study a setting with infrequent shocks to p_t^* . Specifically, we assume that innovations occur according to the realization of a Poisson process with constant arrival rate λ , and that, conditional on an arrival, shocks are Gaussian with zero mean and constant variance:

$$dp_t^* = \mu dt - \sigma \varepsilon dq_t,$$

where q_t is a Poisson arrival process with intensity λ , and ε is a standard normal random variable. In the absence of price changes, the control z_t evolves according to the following stochastic differential equation:

$$dz_t = -\mu dt + \varepsilon \sigma dq_t. \quad (6)$$

Given the law of motion for z (equation 6), the differential form of the Bellman equation (5) implies the following equation for V :

$$-V_z(z)\mu - (\rho + \lambda)V(z) + \lambda E[V(z + \sigma\varepsilon)] + z^2 = 0. \quad (7)$$

The general solution to (7) is:

$$V(z) = Ae^{\beta_1 z} + Be^{\beta_2 z} + V_p(z), \quad (8)$$

¹²By purely state dependent we mean that time (or any function thereof) is not among the set of state variables for this problem. Of course pricing rules are always state dependent relative to the full set of relevant state variables.

where $V_p(z)$ denotes a particular solution, and $e^{\beta z}$ solves the homogeneous equation - which implies that the βs are the solutions to:

$$\frac{1}{2}\beta^2\sigma^2 = \ln\left(1 + \frac{\rho}{\lambda} + \frac{\mu}{\lambda}\beta\right).$$

One particular solution corresponds to the case in which the price setter never adjusts its price:

$$V_p(z) = \int_0^\infty \lambda e^{-\lambda\tau} \left(\int_0^\tau e^{-\rho s} (z^2 - 2z\mu s + \mu^2 s^2) ds + e^{-\rho\tau} E[V_p(z - \mu\tau + \sigma\varepsilon)] \right) d\tau. \quad (9)$$

The solution to the above equation can be obtained by the method of undetermined coefficients. We guess (and verify) that $V_p(z) = az^2 + bz + g$, and substitute it into equation (9) to find:¹³

$$a = \frac{1}{\rho}, \quad (10)$$

$$b = \frac{-2\mu}{\rho^2}. \quad (11)$$

The optimal pricing rule is characterized by a triple (l, c, u) . The parameters l and u are, respectively, the lower and upper levels of the price discrepancy which trigger price adjustments - i.e. they are the bounds of the inaction region - and c is the target discrepancy. The value function should satisfy several conditions. An optimality condition for the target discrepancy:

$$V_z(c) = 0, \quad (12)$$

two indifference (“value-matching”) conditions between l and c and between u and c , which express that the difference between the value function at the bounds of the inaction region and at the optimal target discrepancy should be equal to the adjustment cost K :¹⁴

$$V(l) = V(c) + K, \quad (13)$$

$$V(u) = V(c) + K, \quad (14)$$

and the so-called “smooth-pasting” conditions for the optimality of the inaction region:

$$V_z(l) = 0, \quad (15)$$

$$V_z(u) = 0. \quad (16)$$

¹³The constant g is not used in the solution for the optimal pricing policy.

¹⁴As shown in Appendix B, this parameter can be interpreted as the cost of a price change as a fraction of steady-state profits.

Conditions (12)-(16) allow us to determine the constants A , B and the policy parameters l , u and c numerically.

In order to provide an example of the optimal pricing rule, we assign arbitrary values to the model parameters: $K = 0.01$, $\sigma = 0.1$, $\mu = 0.1$, $\lambda = 1$, $\rho = 0.025$. Figure 1 illustrates the optimal policy, characterized by the inaction region between l and u and the target point c . We also show various possibilities for adjustment of the price discrepancy. First, a jump (denoted by a small circle in the figure) caused by the arrival of a shock brings the price discrepancy outside the lower barrier l , triggering an adjustment to c . In the second arrival of a shock the price discrepancy jumps but stays within the barriers: there is no adjustment. There is also the possibility of uninformed adjustments in the absence of innovations, whenever the trend of the frictionless optimal price causes the price discrepancy to reach the lower boundary of the inaction region.

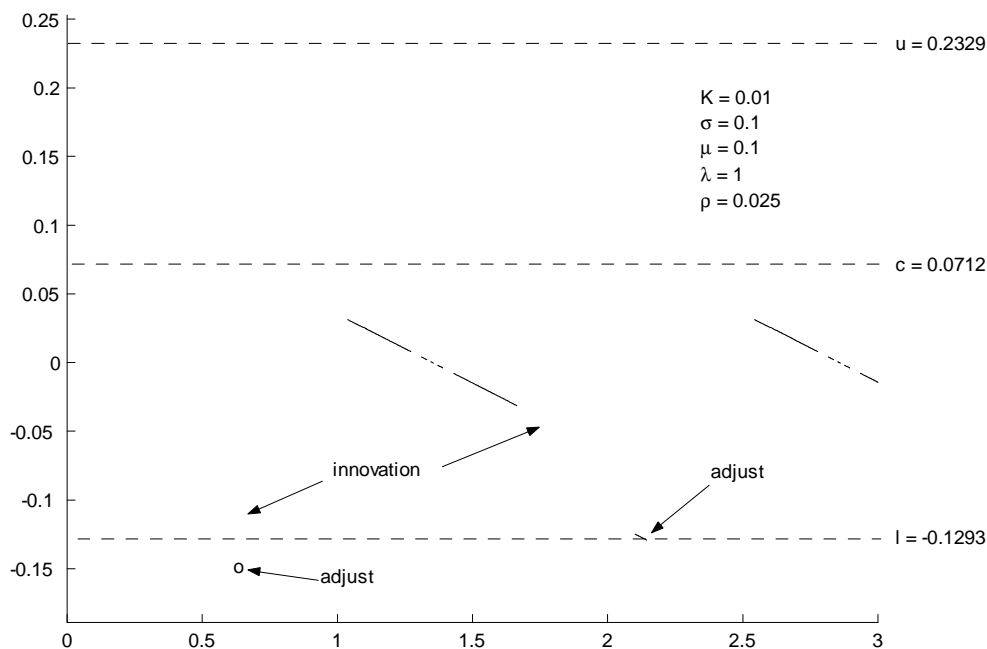


Figure 1: State-dependent pricing policy under infrequent innovations

This infrequent-innovation state-dependent pricing model has some implications that differ from those of the standard menu-cost model with continuous innovations.¹⁵ The first is the possibility of uninformed adjustments - in the case with continuous innovations, all adjustments embed the effects of shocks to p_t^* . A second implication pertains to the size of price changes. While in the standard case price changes in a given direction always have the same size, with infrequent shocks there is a non-degenerate distribution for the size of price increases and decreases. Finally, this

¹⁵See, for example, Almeida and Bonomo (2002), and Golosov and Lucas (2007).

model predicts that uninformed price adjustments are always smaller in magnitude than informed price adjustments. In general, informed price increases are strictly larger than $c - l$, and price decreases are strictly greater in magnitude than $u - c$. Only uninformed price increases (decreases) caused by a positive (negative) trend in the frictionless optimal price have size $c - l$ ($u - c$).

3.2 Build up of unobserved information

In this subsection we tackle pricing problems in which information arrives infrequently for reasons that are outside the control of firms. These cases are analytically simpler than problems with costly information gathering and processing, which we analyze in Section 4. Nevertheless they highlight the importance of unobserved information for the nature of optimal pricing policies. Examples of exogenously infrequent information are pervasive. Economic developments often become news after having evolved unnoticed for some time. Data releases on prespecified dates usually reflect cumulative past information about the state of the economy.

We start with the case in which information about the state of the economy arrives at random dates. It differs from the continuous-information infrequent-innovation specification of the last subsection in that, between information dates, innovations to the frictionless optimal price do take place, but are unobserved. Thus, they accumulate over time until they are fully revealed and incorporated into the price-setting decision on the subsequent information date. In this setting the boundaries of the inaction region depend on the time elapsed since the last information date, so the pricing policy is both time- and state dependent. Specifically, the inaction range widens as time elapses. We then consider the case of deterministic information dates and show that it leads to an extreme form of inaction: irrespective of the price gap, adjustment just prior to information dates is never optimal.

3.2.1 Random information dates

We assume that p_t^* follows a Brownian motion with drift μ :

$$dp_t^* = \mu dt - \sigma dW_t, \tag{17}$$

where W_t is a standard Wiener process. However, it is only observed at a random time, which has a negative exponential distribution.¹⁶ In the absence of price changes the expected price discrepancy z_t has a trend given by $-\mu$. Upon an information arrival, the observed innovation has zero mean, and variance proportional to the time elapsed since the last information date. Formally, while there is no price change, the expected discrepancy evolves according to the following stochastic

¹⁶This is essentially the sticky-information assumption of Mankiw and Reis (2002).

differential equation:

$$dz_t = -\mu dt + \varepsilon \sigma \sqrt{\tau} dq_t, \quad (18)$$

where q_t is a Poisson arrival process with intensity λ , and ε is a standard normal random variable.

Notice the similarity of this case with the continuous-information infrequent-innovation case of the previous subsection. In both cases the state variable z_t jumps upon information arrival. However, in the two cases the jumps have different sources: under continuous information, $z_t = p_t - p_t^*$ jumps whenever p_t^* jumps, while under infrequent information $z_t = p_t - E_{t_0} p_t^*$ jumps when a new information date arrives, bringing updated information about p_t^* . In the latter case the time since the last information date, τ , matters. The reason is that, although the probability of an information arrival does not depend on τ , the amount of information at each arrival (as measured by the variance of the accumulated innovation in p_t^* during this period) is proportional to τ . Thus, the flow deviation costs of being uninformed about the optimal level p_t^* are also increasing in τ :

$$f(z_t, \tau) = z_t^2 + \sigma^2 \tau. \quad (19)$$

As a result, the barriers that determine the inaction region now depend on τ .

We look for a policy $\{l(\tau), c(\tau), u(\tau)\}_{0 \leq \tau < \infty}$ where $l(\tau)$, $u(\tau)$, $c(\tau)$ represent, respectively, lower and upper trigger points and target point for the price discrepancy as a function of the time elapsed since the last information date. Using (18) and (19), we can write the differential form of the Bellman equation (4) as:

$$V_\tau(z, \tau) - \mu V_z(z, \tau) - (\rho + \lambda) V(z, \tau) + \lambda E[V(z + \sigma \varepsilon \sqrt{\tau}, 0)] + z^2 + \sigma^2 \tau = 0. \quad (20)$$

Since adjustment costs are lump-sum, any adjustment is made to the point that minimizes the value function:

$$c(\tau) = \arg \min_z V(z, \tau). \quad (21)$$

Since it is always possible to pay the adjustment cost K and adjust to $c(\tau)$, the value function must satisfy:

$$V(z(\tau), \tau) \leq V(c(\tau), \tau) + K. \quad (22)$$

The trigger points $l(\tau)$ and $u(\tau)$ are defined implicitly by:

$$V(l(\tau), \tau) = V(c(\tau), \tau) + K, \quad (23)$$

$$V(u(\tau), \tau) = V(c(\tau), \tau) + K.$$

In Appendix E we provide an algorithm for solving the optimal pricing problem numerically,

using a finite-difference method. With the solution in hand we can study the properties of the optimal rule. Figure 2 depicts the functions $l(\tau), c(\tau), u(\tau)$, which characterize the optimal pricing rule. Notice that the inaction region widens over time. The reason is that the option value of waiting for an information arrival increases with τ , due to the higher amount of information produced by the accumulation of underlying innovations. We also depict a sample trajectory for z_t , with two information arrivals. Observe that every time there is an information arrival τ is reset to zero. On the first information date z_t jumps to a point below the lower barrier triggering adjustment to $c(0)$. In the second information arrival, z_t jumps upwards to a point inside the inaction range - thus, there is no adjustment.

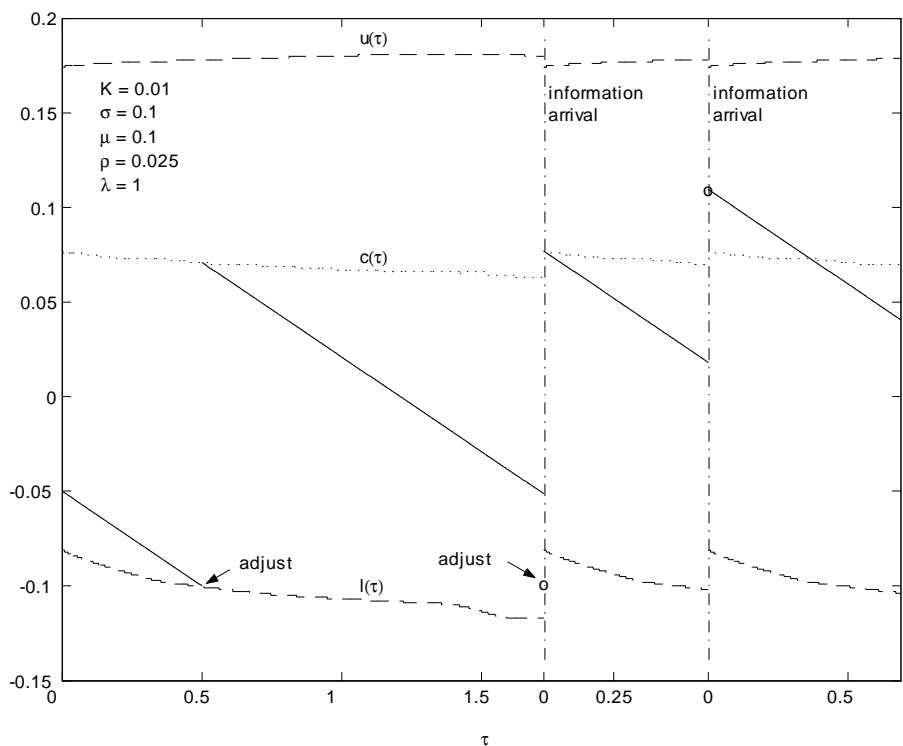


Figure 2: Optimal pricing policy under exogenous and randomly infrequent information

3.2.2 Deterministic information dates

While the timing of some information releases is unknown, important pieces of information become public periodically. The release of economic statistics and monetary policy decisions are noteworthy examples. How does the previous pricing problem change if information arrives at regular intervals of time T ?

In the absence of price changes and between information dates (for $0 < \tau < T$), the expected

discrepancy z_t has a deterministic trend $-\mu$ and no innovation:

$$dz_t = -\mu dt.$$

We continue to assume that the frictionless optimal price evolves according to (17). As a result the flow cost, $f(z_t, \tau)$, is still given by (19) and we can represent the general Bellman equation (4) as the following partial differential equation:

$$-\mu V_z(z, \tau) + V_\tau(z, \tau) - \rho V(z, \tau) + z^2 + \sigma^2 \tau = 0. \quad (24)$$

The general solution to (24) is:

$$V(z, \tau) = \frac{2\mu^2}{\rho^3} - \frac{2z\mu}{\rho^2} + \frac{z^2}{\rho} + \frac{\sigma^2}{\rho^2} + \frac{\sigma^2 \tau}{\rho} + e^{-\frac{\rho z}{\mu}} G\left(\frac{z + \mu \tau}{\mu}\right), \quad (25)$$

where $G(\cdot)$ is a function to be determined by the nature of the firm's optimization problem.

Our goal is to solve for the optimal pricing rule $\{l(\tau), c(\tau), u(\tau)\}_{0 \leq \tau \leq T}$. Conditions (21), (22), and (23) are still valid. However, since information arrives deterministically, we need to tie the value function just before the information release to the value function just after the information date. When information is revealed, the expected discrepancy receives a shock with distribution $N(0, \sigma^2 T)$, and τ is reset to zero. We thus have the following additional value-matching condition:

$$V(z, T) = E\left[V\left(z + \sigma\sqrt{T}\varepsilon, 0\right)\right], \quad (26)$$

where ε is a standard normal random variable. This problem can be solved numerically as described in Appendix E.¹⁷

Figure 3 illustrates the optimal pricing rule under deterministic information arrival assuming $T = 1$. We depict a sample path for z_t . Initially z_t is close to zero, and arrives at time 1 outside the inaction region for $\tau = 0$. Then, the accumulated shock is revealed and z_t jumps to the position marked with o - outside the time-zero inaction range. An immediate adjustment is triggered to $c(0)$. Then, with no information, z_t decreases at a constant rate from $c(0)$, and so on.

As in the case with random information arrival, this specification entails the possibility of uninformed adjustments. Another common feature is that the pricing rule is both time- and state dependent, with an inaction region that widens over time. There is, however, an obvious distinguishing feature. In the case of prespecified information dates the inaction range becomes arbitrarily large just before information releases. The intuition is clear: in this situation the option value of waiting becomes very large. This is a stark testable implication of this specification: one

¹⁷In Appendix D we also provide an alternative solution approach for the no-drift case ($\mu = 0$).

should see fewer adjustments when potentially important information is about to be released.

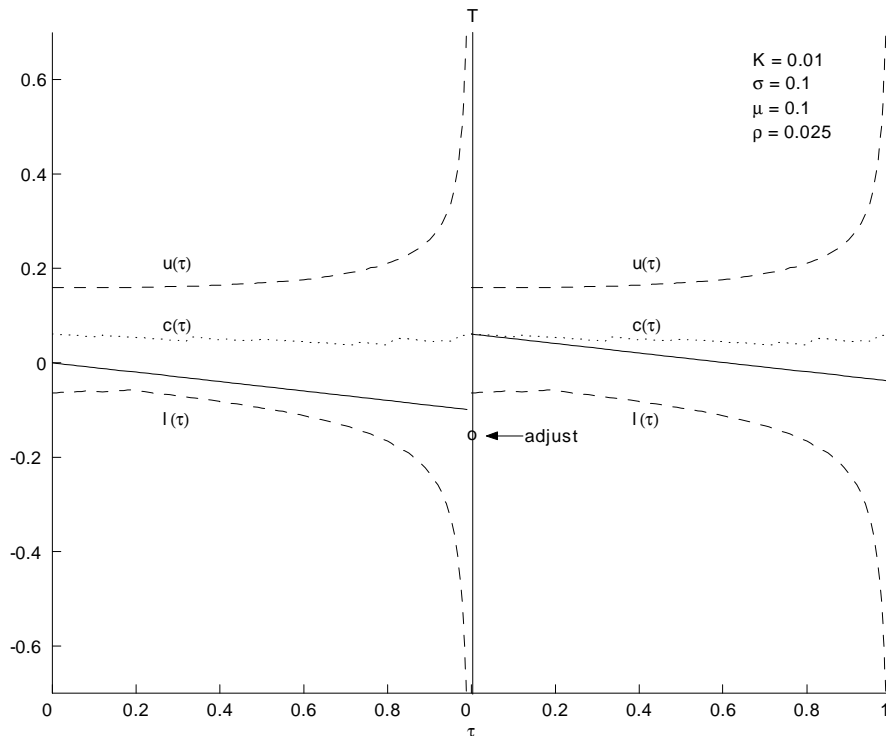


Figure 3: Optimal pricing policy under exogenous and deterministically infrequent information

4 Costly information

In the previous section we analyzed specifications in which infrequent information arises for reasons that are outside the control of price setters. However, even in contexts in which continuous access to information is possible, costs of gathering and processing such information might lead firms to incorporate it into pricing decisions only infrequently.

We start by revisiting a benchmark case with costly information, but without adjustment costs, as in Caballero (1989) and Reis (2006). The optimal pricing rule implies infrequent information gathering and processing, but (potentially) continuous price adjustments. We then consider the simplest framework with both information and adjustment costs, following Bonomo and Carvalho (2004, 2010). When these two costs are borne out together, price setters optimally choose dates to gather and process information and change prices. This implies a fixed-price time-dependent pricing rule with optimally chosen information-adjustment dates. Finally, we analyze the case where information and adjustment costs are dissociated, i.e. firms have the option to adjust without information and to gather and process information but then choose not to adjust. In this case

the optimal pricing rule features both infrequent price changes and infrequent incorporation of information into prices.

4.1 A benchmark model of costly information

The frictionless optimal price evolves according to (17) but there is a lump-sum cost F for information gathering and processing.¹⁸ Since there are no adjustment costs, the expected deviation z_t is always kept equal to zero - i.e., the firm always charges the expected value of its frictionless optimal price given its information. As a result, the flow deviation cost depends only on the time elapsed since the last information date:

$$f(\tau) = \sigma^2 \tau.$$

Thus, between information dates the Bellman equation (4) simplifies to:

$$V(\tau) = \sigma^2 \tau dt + e^{-\rho dt} E_t V(\tau + dt),$$

which is equivalent to the following ordinary differential equation:

$$V'(\tau) - \rho V(\tau) + \sigma^2 \tau = 0.$$

The solution to the homogeneous equation is given by:

$$V_{\text{hom}}(\tau) = A e^{\rho \tau},$$

where A is a constant to be determined. A particular solution is given by:

$$V_{\text{part}}(\tau) = \frac{\sigma^2 \tau}{\rho} + \frac{\sigma^2}{\rho^2},$$

which leads to the general solution:

$$V(\tau) = A e^{\rho \tau} + \frac{\sigma^2 \tau}{\rho} + \frac{\sigma^2}{\rho^2}. \quad (27)$$

For an arbitrary policy that specifies intervals of length τ between information dates, the value function must satisfy the following value-matching condition:

$$V(\tau) = V(0) + F,$$

¹⁸As shown in Appendix C, this parameter can be interpreted as the cost of information gathering and processing as a fraction of steady-state profits.

which implies that:

$$A = \frac{e^{-\rho\tau}}{1 - e^{-\rho\tau}} \left(F - \frac{\sigma^2\tau}{\rho} \right). \quad (28)$$

After plugging (28) into (27), the second condition is an optimality condition for the choice of information dates:

$$V'(\tau^*) = 0,$$

which yields:

$$\rho\tau^* + e^{-\rho\tau^*} = 1 + \rho^2 \frac{F}{\sigma^2}.$$

The latter equation implicitly defines the optimal time interval between information dates. The solution can be expressed in “quasi-closed-form” as:

$$\tau^* = \frac{1}{\rho} + \rho \frac{F}{\sigma^2} + \frac{1}{\rho} h^{-1} \left(-e^{-(1+\rho^2 F/\sigma^2)} \right),$$

where $h(x) = xe^x$.¹⁹

Notice that this pricing rule prescribes continuous uninformed price adjustment between information dates whenever $\mu \neq 0$, and a (possible) jump to the level of the frictionless optimal price on information dates, in order to keep the process z_t always at zero. As a consequence it does not prescribe price inaction, although the adjustment to the frictionless optimal level comes with a lag.

4.2 Information and adjustment costs

4.2.1 The simplest framework: undissociated costs

We follow Bonomo and Carvalho (2004, 2010) and develop an alternative to the previous model that entails not only inattention, but also microeconomic inaction. The key assumption is that the information gathering and processing cost and the menu cost are borne out together. This is arguably the simplest framework with both information and adjustment costs.

Between information dates (and thus also in the absence of price changes) the expected discrepancy z_t has a deterministic trend $-\mu$ and no innovation:

$$dz_t = -\mu dt,$$

and the value function V satisfies the partial differential equation (24). It is easy to confirm that, along the curves $\tau = t$ and $z = z_0 - \mu t$, the value function satisfies the ordinary differential equation:

$$V'(t) - \rho V(t) + \sigma^2 t + (z_0 - \mu t)^2 = 0.$$

¹⁹In Appendix D we obtain the solution to this price-setting problem through an alternative approach.

A particular solution for the latter equation is:

$$V_p(t) = \frac{\mu^2}{\rho}t^2 + \left(\frac{2\mu^2}{\rho^2} + \frac{\sigma^2 - 2z_0\mu}{\rho} \right)t + \frac{2\mu^2}{\rho^3} + \frac{\sigma^2 - 2z_0\mu}{\rho^2} + \frac{z_0^2}{\rho},$$

which combined with the solution to the homogeneous equation $V'(t) - \rho V(t) = 0$ yields the general solution:

$$V(t) = Ae^{\rho t} + \frac{\mu^2}{\rho}t^2 + \left(\frac{2\mu^2}{\rho^2} + \frac{\sigma^2 - 2z_0\mu}{\rho} \right)t + \frac{2\mu^2}{\rho^3} + \frac{\sigma^2 - 2z_0\mu}{\rho^2} + \frac{z_0^2}{\rho},$$

where A is to be determined from the optimality conditions of the problem. Relying on the previous change of variables, the solution can be rewritten as:

$$V(z, \tau) = Ae^{\rho\tau} + \frac{\mu^2}{\rho}\tau^2 + \left(\frac{2\mu^2}{\rho^2} + \frac{\sigma^2 - 2(z + \mu\tau)\mu}{\rho} \right)\tau + \frac{2\mu^2}{\rho^3} + \frac{\sigma^2 - 2(z + \mu\tau)\mu}{\rho^2} + \frac{(z + \mu\tau)^2}{\rho}. \quad (29)$$

For an arbitrary policy that specifies intervals of length τ between information-adjustment dates, and a discrepancy c on such dates, the value function must satisfy the following value-matching condition:

$$V(c - \mu\tau, \tau) = V(c, 0) + Q,$$

where Q denotes the information-adjustment cost. This yields:

$$A = \frac{Q\rho^2 - 2\mu^2\tau + 2c\mu\rho\tau - \rho\sigma^2\tau - \mu^2\rho\tau^2}{(e^{\rho\tau} - 1)\rho^2}. \quad (30)$$

After plugging (30) into (29), the final conditions are first-order conditions for the choice of the optimal policy τ^* and c^* :

$$V_\tau(c^*, \tau^*) = 0,$$

$$V_z(c^*, \tau^*) = 0,$$

which yield:

$$\begin{aligned} c^* &= \mu \left(\frac{1}{\rho} - \frac{e^{-\rho\tau^*}}{1 - e^{-\rho\tau^*}} \tau^* \right), \\ 0 &= \frac{(\sigma^2 - 2c^*\mu + \mu^2\tau^*)\tau^*e^{\rho\tau^*} + 2\frac{\mu^2}{\rho}\tau^* - \rho Qe^{\rho\tau^*}}{(1 - e^{\rho\tau^*})^2} - 2\frac{\frac{\mu}{\rho}c^* - \frac{\mu^2 + \sigma^2}{\rho^2}}{1 - e^{\rho\tau^*}}. \end{aligned}$$

This system of two equations can be solved numerically for τ^* and c^* .²⁰

Under this pricing policy there are no uninformed price adjustments. If there is a positive drift in the frictionless optimal price, the expected price discrepancy decreases until the next information-

²⁰In Appendix D we obtain the solution to this price-setting problem through an alternative approach.

adjustment date. At that point information is factored into the pricing decision, and z_t jumps and is immediately adjusted to c^* . Notice that adjustment happens independently of the size of the surprise due to information, and that the size of the adjustment is stochastic.

This model can be seen as providing microfoundations for the pricing policy in the seminal work of Phelps (1978) and Taylor (1979, 1980). It implies that “contract lengths”, rather than being exogenously given, are chosen optimally.

4.2.2 Dissociated costs

While the assumption of joint information and adjustment costs makes the previous model quite tractable, it is perhaps more reasonable to assume that these costs are dissociated. The relevance of a model with separate adjustment and information costs has been acknowledged for at least twenty years (e.g. Blanchard and Fischer 1989, page 413). The pricing problem in that case is much more involved. The adjustment cost (K) and the information cost (F) jointly induce an inaction region in which prices are kept fixed, and no information is entertained. This region can be understood as resulting from the intersection between an adjustment inaction region and an information inaction region.²¹ The borders inherited from the adjustment inaction region trigger uninformed price adjustments, while the border inherited from the information inaction region triggers information gathering/processing. It might thus be optimal for a firm to adjust without new information, and to gather and process new information but then choose not to change its price. The likelihood of these events depends on the configuration of parameters.

Inside the inaction region, the differential equation which characterizes the evolution of the value function is still given by (24):

$$-\mu V_z(z, \tau) + V_\tau(z, \tau) - \rho V(z, \tau) + z^2 + \sigma^2 \tau = 0,$$

with general solution given by (25):

$$V(z, \tau) = \frac{2\mu^2}{\rho^3} - \frac{2z\mu}{\rho^2} + \frac{z^2}{\rho} + \frac{\sigma^2}{\rho^2} + \frac{\sigma^2\tau}{\rho} + e^{-\frac{\rho z}{\mu}} G\left(\frac{z + \mu\tau}{\mu}\right),$$

where $G(\cdot)$ is a function to be determined by the nature of the firm’s optimization problem.

As in standard menu-cost models, the option to pay the adjustment cost K and reset z to an optimally chosen expected discrepancy $c(\tau)$ implies that:

$$\forall(z, \tau), \quad V(z, \tau) \leq V(c(\tau), \tau) + K, \tag{31}$$

²¹Notice that these two inaction regions are interrelated, since they are derived jointly from the same pricing problem.

where:

$$c(\tau) = \arg \min_z V(z, \tau).$$

The bounds that define the adjustment inaction region, $l(\tau), u(\tau)$, are a function of the time elapsed since the last information date, and satisfy (31) with equality:

$$\begin{aligned} V(l(\tau), \tau) &= V(c(\tau), \tau) + K, \\ V(u(\tau), \tau) &= V(c(\tau), \tau) + K. \end{aligned}$$

In addition, the option to incur the cost F and entertain information implies that:

$$\forall(z, \tau), \quad V(z, \tau) \leq E[V(z + \sigma\sqrt{\tau}\varepsilon, 0)] + F. \quad (32)$$

When (32) holds with equality, it becomes a value-matching condition that the value function must satisfy on information dates. This condition is analogous to (26) in the case with exogenous deterministic information. On information dates, the expected discrepancy receives a shock with distribution $N(0, \sigma^2\tau)$ - where τ denotes the time elapsed since the previous information date - and τ is reset to zero. The difference relative to that case is that information dates are now determined by firms' decisions to incur the cost F and entertain information:

$$V(z, \tau^*(z)) = E[V(z + \sigma\sqrt{\tau^*(z)}\varepsilon, 0)] + F. \quad (33)$$

In (33), ε is a random variable with distribution $N(0, 1)$, and $\tau^*(z)$ is the function that defines the boundary of the information inaction region.

In Appendix E we provide an algorithm for solving this problem numerically using a finite-difference method, and also an alternative solution approach for the no-drift case ($\mu = 0$). Figure 4 illustrates the optimal pricing rule under adjustment and information gathering/processing costs. The dashed (red) lines $l(\tau), u(\tau)$ are the boundaries of the inaction region that trigger uninformed adjustments, while the solid (blue) line $\tau^*(z)$ is the boundary that triggers information gathering/processing. We illustrate a sample path in which z_t is initially close to zero. Due to the high enough drift μ , the expected discrepancy hits the lower boundary $l(\tau)$, leading to an uninformed adjustment to $c(\tau)$. After that, the expected discrepancy drifts down until it touches the information boundary $\tau^*(z)$ at a point where $z \approx -0.04$ and $\tau \approx 0.73$. At that point the firm incorporates information into the pricing decision as the expected discrepancy receives a shock with distribution $N(0, \sigma^2 \times 0.73)$. The time-elapsed variable τ is reset to zero, and the firm decides whether or not to pay the menu cost and change its price, depending on whether the just-learned price discrepancy

is inside or outside the inaction region defined by $(l(0), u(0))$.

As in the case with deterministic information releases, notice that it is never optimal to make an uninformed price adjustment just prior to an information date. This result can be seen visually in the example depicted in Figure 4. For $\tau > 0$ the (red) dashed $l(\tau)$ and $u(\tau)$ lines trigger adjustment without information. Such an adjustment brings the expected discrepancy to $c(\tau)$, which is always “distant” from the information boundary $\tau^*(z)$ that triggers information gathering/processing. The intuition for this feature of the optimal policy is similar to the case with deterministic information releases, the difference arising from the fact that in the current problem information is controlled by the price setter: rather than incurring the menu cost to make an uninformed adjustment and then immediately incurring the information gathering/processing cost, it is always better to reverse the order of these actions and keep the option to adjust, to be exercised or not depending on the new information.

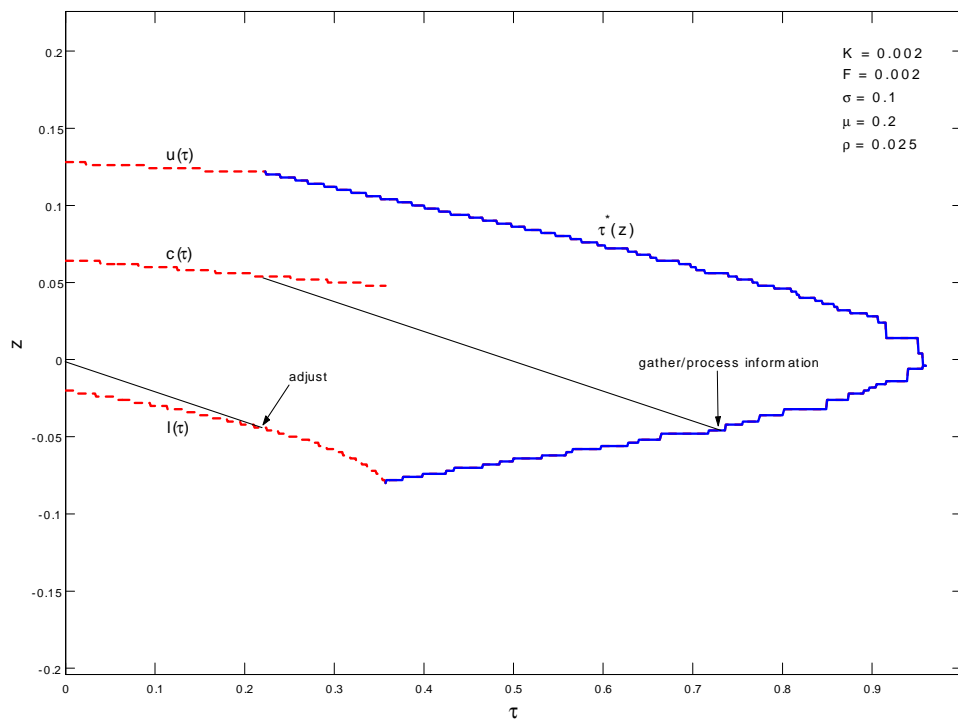


Figure 4: Optimal pricing policy under adjustment and information gathering/processing costs

Alvarez, Lippi, and Paciello (2010) also propose a price-setting model with costly adjustments and information acquisition. Beyond the difference in the solution approach, another key difference is that our approach to solving the problem with dissociated costs entertains cases in which

uninformed adjustments are optimal. In turn, Alvarez, Lippi and Paciello (2010) provide sufficient conditions under which such uninformed adjustments are not optimal and focus on parameterizations that satisfy those conditions. They otherwise prevent agents from making such adjustments by imposing the restriction that adjustment requires observation.²²

5 Partial information

In the previous sections we assumed that information was either continuous or infrequent (for exogenous or endogenous reasons). It is perhaps more reasonable to assume that there is always some continuous flow of information that can be factored into pricing decisions somewhat costlessly, and some information that is only incorporated infrequently - either for exogenous reasons, or due to information gathering and processing costs.²³

In this section we extend our previous results by assuming that one component of the frictionless optimal price is continuously and freely observable, and that processing it entails no costs. In particular, we assume that:²⁴

$$dp_t^* = \mu dt - \sigma_i dW_{it} - \sigma_a dW_{at},$$

where W_{it} and W_{at} are independent standard Wiener processes. Information about W_{it} is continuously and freely available, and costless to process. In contrast, firms face infrequent information about W_{at} . Thus, from here on when we refer to information dates we mean “ W_{at} -information dates.”

In the next subsections we analyze optimal pricing policies in two environments with partial continuous information. In the first one, information about W_{at} arrives infrequently for reasons that are outside the control of the firm. In the second, this component of the frictionless optimal price is subject to costly information gathering and processing.

5.1 Exogenous information dates

For simplicity we present only the case with deterministic arrival of information about W_{at} . Between information dates and in the absence of price adjustments, z_t changes continuously because of both the drift and the W_{it} process:

$$dz_t = -\mu dt + \sigma_i dW_{it}.$$

²²For a more detailed discussion of the differences between our papers see Bonomo, Carvalho and Garcia (2011).

²³Gorodnichenko (2009), Knotek (2009) and Klenow and Willis (2007) propose menu-cost models in which firms continuously incorporate partial information into pricing decisions.

²⁴As mentioned previously, the two-component representation of p_{it}^* can be justified from first principles, as in Appendix A.

W_{at} -driven uncertainty also impacts the expected costs of deviating from the frictionless optimal price due to the build up of unobserved aggregate innovations. The instantaneous flow-cost function is given by:

$$f(z_t, \tau) = z_t^2 + \sigma_a^2 \tau.$$

Hence, the differential form of the Bellman equation (4) is now written as:

$$\frac{1}{2} \sigma_i^2 V_{zz}(z, \tau) - V_z(z, \tau) \mu + V_\tau(z, \tau) - \rho V(z, \tau) + z^2 + \sigma_a^2 \tau = 0. \quad (34)$$

The condition that determines $c(\tau)$, (21), the adjustment-option condition, (22), and the conditions that determine $l(\tau)$ and $u(\tau)$, (23), remain the same. However, the value-matching condition that applies on information dates changes to incorporate only uncertainty due to the unobserved component of p_t^* :

$$V(z, T) = E \left[V \left(z + \sigma_a \sqrt{T} \varepsilon, 0 \right) \right].$$

The numerical solution algorithm used to solve this problem, described in Appendix E, is essentially the same as the one used for solving the problem with deterministic information arrivals and no interim information about W_{it} .

Figure 5 shows the optimal pricing rule and a sample path for z_t . We use the normalization $T = 1$. For τ between zero and one, adjustment is dictated by the evolution of the expected discrepancy, which depends on μ and on realizations of W_{it} . When z_t reaches the lower barrier, adjustment is triggered to $c(\tau)$. These adjustments take into consideration only the continuously and freely available W_{it} -information. When τ reaches time $T = 1$, W_{at} -information arrives and z_t jumps. If it falls outside the inaction range at zero, an adjustment is triggered to $c(0)$.

In this environment there are no totally-uninformed adjustments. The firm uses the W_{it} -information between information dates, and adjusts if the expected price discrepancy becomes large enough. Despite the continuous flow of W_{it} -information, the inaction range still becomes arbitrarily wide before the deterministic times of W_{at} -information arrival, and the implication that one should not see adjustments before an important information announcement continues to hold.

5.2 Endogenous information dates

The setting is identical to the previous subsection, but now W_{at} -information can be factored into pricing decisions at any time, at a lump-sum cost F . The differential equation for the value function in the absence of adjustment and W_{at} -information is still (34):

$$\frac{1}{2} \sigma_i^2 V_{zz}(z, \tau) - V_z(z, \tau) \mu + V_\tau(z, \tau) - \rho V(z, \tau) + z^2 + \sigma_a^2 \tau = 0,$$

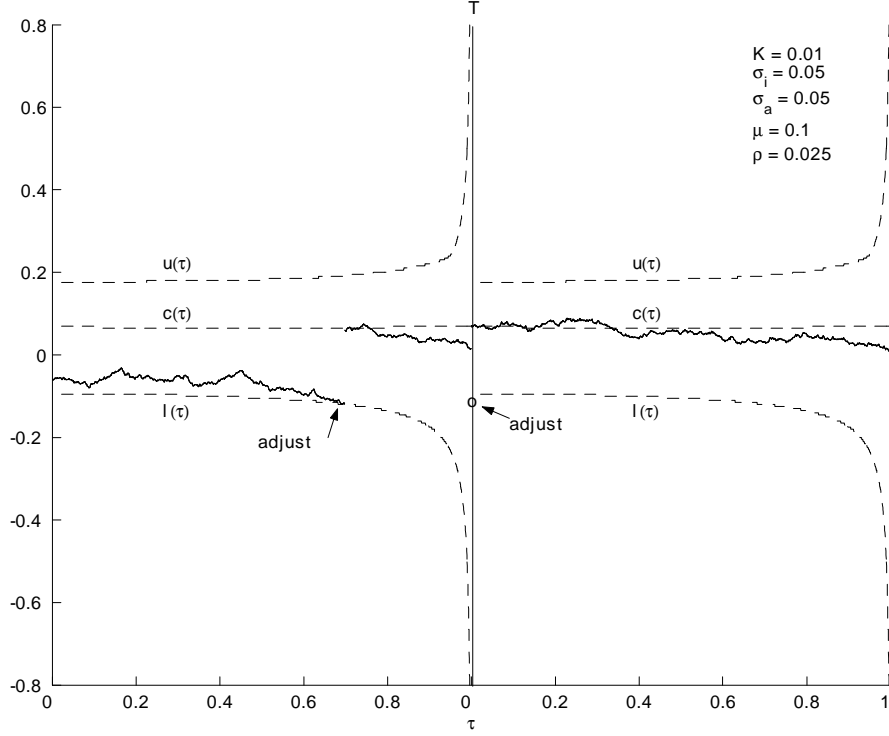


Figure 5: Optimal pricing policy under partial continuous information, and exogenous and deterministically infrequent information

and the optimality conditions are now:

$$\forall (z, \tau), \quad V(z, \tau) \leq E[V(z + \sigma_a \sqrt{\tau} \varepsilon, 0)] + F \text{ and}$$

$$\forall (z, \tau), \quad V(z, \tau) \leq V(c(\tau), \tau) + K,$$

with:

$$c(\tau) = \arg \min_z V(z, \tau).$$

The boundaries of the adjustment inaction region satisfy:

$$V(l(\tau), \tau) = V(c(\tau), \tau) + K,$$

$$V(u(\tau), \tau) = V(c(\tau), \tau) + K,$$

and the boundaries of the information inaction region satisfy:

$$V(z, \tau^*(z)) = E[V(z + \sigma_a \sqrt{\tau^*(z)} \varepsilon, 0)] + F.$$

As in all previous cases, Appendix E provides a numerical solution algorithm for this price-setting problem.

Figure 6 illustrates the optimal pricing rule under adjustment and information gathering/processing costs, and partial information. Again, due to the presence of partial information, in this environment there are no totally-uniformed adjustments. The firm uses the W_{it} -information between information dates, and adjusts if the expected price discrepancy hits the $l(\tau)$ or $u(\tau)$ boundaries of the adjustment inaction region. Those are represented by the outer dashed (red) lines. The inner such line, $c(\tau)$, gives the discrepancy to which firms revert when they choose to adjust. The solid (blue) line is the boundary of the information inaction region.

In the sample path realization for z_t that we depict in Figure 6, there are two partially-informed adjustments before the firm decides to incur the cost to entertain information about W_{at} . At that point the time-elapsd variable τ is reset to zero, and the firm decides whether or not to pay the menu cost and change its price, depending on whether the price discrepancy is inside or outside the inaction region defined by $(l(0), u(0))$. As in the case with no interim information about W_{it} , notice that it is never optimal to make an uninformed price adjustment just before incurring the information gathering and processing cost.

In this example we assume the same parameter values as in Subsection 4.2.2, splitting the sources of variation of the p_t^* process evenly between the W_{it} and W_{at} processes (i.e. $\sigma_i = \sigma_a = \sigma/\sqrt{2}$). This leads to a quite dramatic change in the optimal pricing policy, in that the firm is now willing to wait longer until the subsequent information date than in the case without interim information. This is quite intuitive, since the expected flow deviation cost due to unobserved variation in p_t^* is now smaller.

6 An illustration

In this paper we develop a tractable unified framework for solving optimal time- and state-dependent price-setting problems. While we restrict ourselves to providing the solution to a single firm's optimal pricing problem, once the solution is obtained one can use it to study various questions of interest to macroeconomists. In this section we illustrate one such application, based on the model with dissociated adjustment and information costs of Subsection 4.2.2.

We perform comparative exercises to illustrate how changes in the underlying economic environment affect price-setting statistics such as moments of the distribution of price changes, the frequency of price changes, the frequency of informed and uninformed adjustments, etc. Table 1 provides price-setting statistics for four combinations of parameter values, varying the drift and volatility of the frictionless optimal price process. Specifically, we set $\mu = 0, 0.4$ and $\sigma = 0.1, 0.2$,

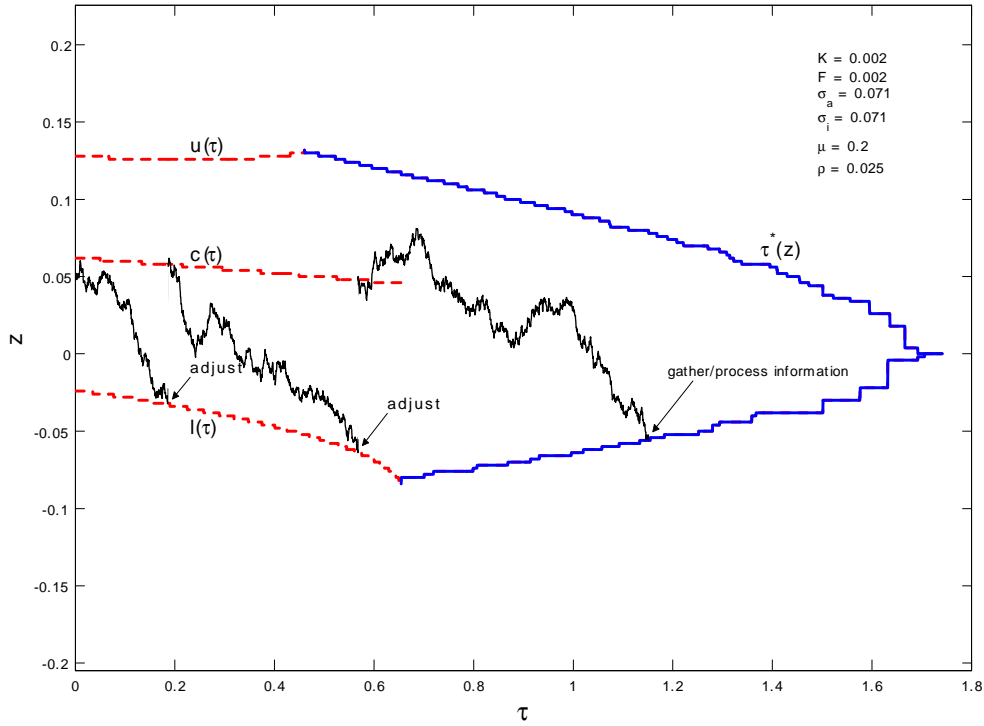


Figure 6: Optimal pricing policy with partial information, and adjustment and information costs

keeping the remaining parameter values from Subsection 4.2.2. Each row corresponds to a combination of these two parameters. From row 1 to 2 and from row 3 to 4 we increase the drift while keeping the same volatility of the frictionless optimal price process. Rows 3 and 4 report results with the higher volatility.

Consider first the effect of an increase in the drift of frictionless optimal price process from zero to 0.4 when volatility is 0.1. Since this change does not affect the degree of uncertainty in the environment, in principle it could be met by an increase only in the frequency of uninformed adjustments. However, as highlighted in the analysis of Subsection 4.2.2, decisions to adjust interact with decisions to gather information. A higher drift increases the frequency of information gathering slightly from 1.32 to 1.41 times per year. The higher rate of information gathering results from a lower frequency of information gathering without price adjustment (which drops from 0.60 to 0.34 times per year) that is more than offset by an increase in the incidence of informed price adjustments (from 0.72 to 1.08 times per year). Not surprisingly, the frequency of price adjustment increases by much more - from 0.72 to 2.56 times per year - with uninformed adjustments representing more than 80% of this increase.

The occurrence of optimal uninformed adjustments does not depend only on the trend of the frictionless optimal price, but also on its volatility. The same increase in trend inflation (from 0 to 0.4) may have very different implications when volatility is higher (0.2 instead of 0.1). Now the increase in the frequency of price changes from 1.46 to 2.08 times per year is mostly due to more informed price adjustments, the frequency of which increases from 1.46 to 2 times per year. This latter increase is not fully matched by a higher frequency of information gathering: a higher proportion of information collection is now followed by a price change (an increase of about 10%), with the incidence of information gathering without price adjustment even decreasing in absolute terms (from 1.23 to 1.10 times per year).

Table 1: Annual frequency of information gathering and price adjustments

Case	σ	μ	Frequ. of inform. gathering	Frequ. of inform. gathering w/o price adjustment	Frequ. of informed price adjustment	Frequ. of uninformed price adjustment	Frequ. of price adjustment
[1]	0.1	0	1.32	0.60	0.72	0	0.72
[2]	0.1	0.4	1.41	0.34	1.08	1.49	2.56
[3]	0.2	0	2.69	1.23	1.46	0	1.46
[4]	0.2	0.4	3.11	1.10	2.00	0.08	2.08

Note: $K = F = 0.002$ and $\rho = 0.025$.

An increase in volatility (from rows 1 to 3 and 2 to 4 in Table 1) has the intuitive effect of increasing the frequency of information gathering. However, this need not lead to more frequent price adjustments. When trend inflation is high (e.g. $\mu = 0.4$ in rows 2 and 4), an increase in volatility may reduce the frequency of uninformed adjustments substantially, more than offsetting the increase in the frequency of informed price adjustments. Underlying this possibility is the fact that, for a given expected negative price discrepancy, gathering information becomes a relatively more attractive option than undertaking uninformed adjustments, when volatility is high. This is apparent from Figure 7, which shows that an increase in volatility from 0.1 to 0.2 shortens the information boundary substantially.²⁵ The reason is that information is more valuable in the latter case. After the information about cumulative innovations to p^* is processed, the price setter may find out that its price is not so out-of-line with the frictionless optimal price. Thus, there is

²⁵ Consider, for example, the intersection between the $l_2(\tau)$ and $\tau_4^*(z)$ curves in Figure 7. At that (z, τ) point one would adjust without gathering information in the lower volatility environment, but would choose to get informed in the higher volatility environment.

some probability that the price setter will choose to leave its price unchanged, whereas under lower volatility she would choose to adjust without information.

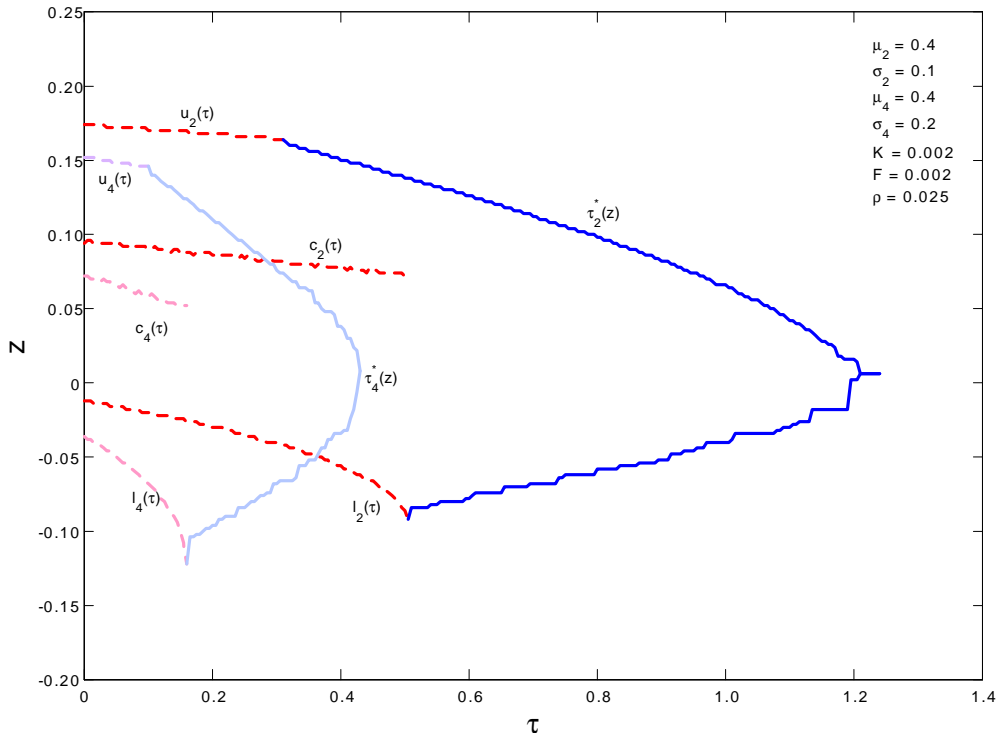


Figure 7: Optimal pricing rules under adjustment and information costs for two different volatilities

The model also has potentially interesting implications for the distribution of price changes. Figure 8 illustrates that distribution under a parameterization with no trend inflation. In this case, all adjustments are informed, and the distribution is symmetric and bimodal.

This pattern may change substantially when the frictionless optimal price has a drift. The distribution of informed adjustments becomes asymmetric, as illustrated in Figure 9 for $\mu = 0.4$ and $\sigma = 0.2$. In addition, the possibility of uninformed adjustments may further alter this pattern, since this distribution differs markedly from the distribution of informed adjustments. Figure 10 shows that the distribution of uninformed adjustment is very concentrated, as the size of uninformed adjustment does not depend much on the time elapsed since the last information time ($c(\tau) - l(\tau)$ varies little with τ). When both types of adjustments are pooled, the distribution of adjustment sizes, although more spread out, still has a spike due to the presence of uninformed adjustment. As illustrated in Figure 11, the spike is apparent even in the present case, where the proportion of uninformed adjustments is less than 4%. This is a stark implication of this model, where both

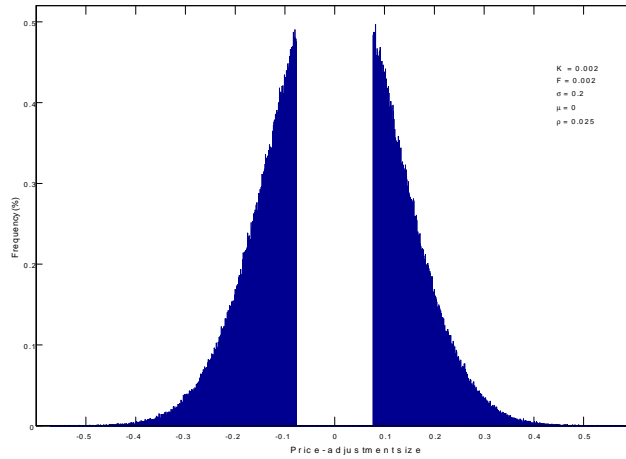


Figure 8: Distribution of price changes under zero inflation

uninformed and informed adjustments may occur, as long as trend inflation is sufficiently high compared to the volatility of the frictionless optimal price.

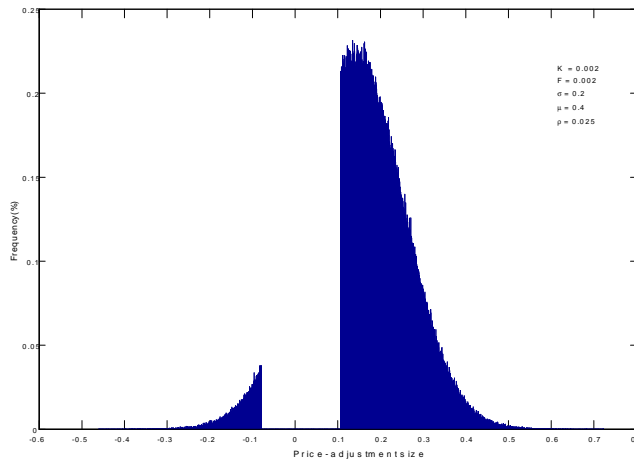


Figure 9: Distribution of informed price changes under high trend inflation

7 Conclusion

In this paper we study optimal price setting under adjustment costs and infrequent information arising from various sources. Pricing rules are more complex than the usual purely state-dependent strategies. In general, the inaction regions depend on the time elapsed since the last information

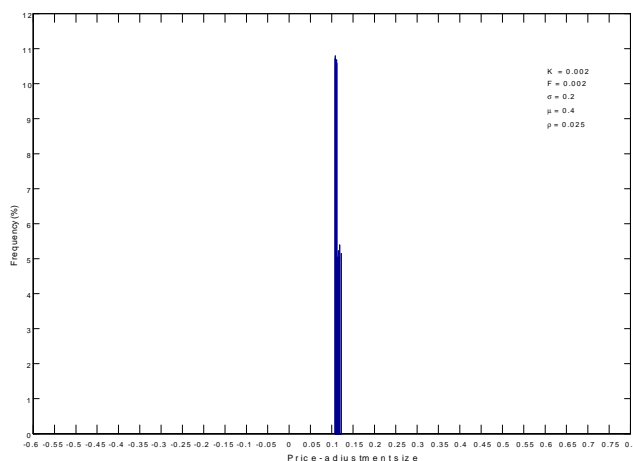


Figure 10: Distribution of uninformed price changes under high trend inflation

date. There is scope for uninformed adjustments. When some important determinant of the frictionless optimal price can be freely and continuously factored into pricing decisions, there can be partially-informed adjustments. There should be no adjustment just prior to the release of important information, if the release date is known. Likewise, it is never optimal to make an uninformed price adjustment just before incurring the information gathering and processing cost.

While in this paper we focus on price setting, as emphasized in the early Bonomo and Garcia (2003) paper, our framework is, more generally, suitable for studying optimal decision-making under adjustment costs and infrequent information. For instance, our results might be of interest in the context of employment adjustment, inventory management and investment problems.

In its current form, our framework has the big advantage that the optimal policies can be solved for independently of equilibrium considerations. This makes the various models that we entertain relatively cheap to solve computationally, and thus allows for a relatively straightforward attack on their quantitative micro and macro implications. Of course such simplicity, which is afforded by the nature of the underlying economic environment, also has some costs. Importantly, it precludes interactions of agents' decisions through general-equilibrium effects. While our formulations can be extended to allow for such interactions, they have to be handled with methods for solving (infinitely-dimensional) heterogeneous agents models, which typically make computational solutions much more costly. Despite this complication, solutions are feasible, and should open the possibility of addressing important research questions.

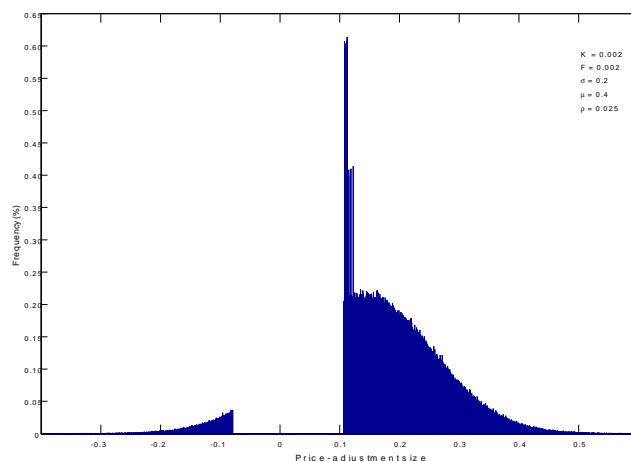


Figure 11: Distribution of price changes (both informed and uninformed) under high trend inflation

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8 Appendix A

We derive the frictionless optimal price in a simple general equilibrium framework. A representative consumer maximizes expected discounted utility:

$$E_{t_0} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} [\log(C_t) - H_t] dt,$$

subject to the budget constraints:

$$B_t = B_0 + \int_0^t W_r H_r dr - \int_0^t \left(\int_0^1 P_{ir} C_{ir} di \right) dr + \int_0^t T_r dr + \int_0^t \Lambda_r dQ_r + \int_0^t \Lambda_r dD_r, \text{ for } t \geq 0.$$

Utility is defined over the composite consumption good $C_t \equiv \left[\int_0^1 (C_{it}/A_{it})^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ with $\theta > 1$, where C_{it} is the consumption of variety i , and A_{it} is a relative-preference shock. P_{it} is the price of variety i , H_t is the supply of labor, which commands a wage W_t , B_t is total financial wealth, T_t are total net transfers, including any lump-sum flow transfer from the government, and profits received from the firms owned by the representative consumer. Q_r is the vector of prices of traded assets, D_r is the corresponding vector of cumulative dividend processes, and Λ_r is the trading strategy, which satisfies conditions that preclude Ponzi schemes. The associated consumption price index, P_t , is given by:

$$P_t = \left[\int_0^1 P_{it}^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \quad (35)$$

The demand for an individual variety is:

$$C_{it} = A_{it}^{1-\theta} \left(\frac{P_{it}}{P_t} \right)^{-\theta} C_t. \quad (36)$$

Firms hire labor to produce according to the following production function:

$$Y_{it} = A_{it} H_{it}.$$

Note that we assume that the productivity shock is perfectly correlated with the relative-preference shock in the consumption aggregator. This has precedence in the sticky-price literature (for instance, King and Wolman 1999 and Woodford 2009). Our specific assumption follows Woodford (2009), and aims to produce a tractable profit-maximization problem that can be written as a price-setting “tracking problem” in which the firm only cares about the ratio of the two stochastic processes driving profits, which will be specified below.²⁶

²⁶More generally, assumptions relating preference and technology processes have been used previously in the literature on “balanced growth” in multi-sector models (e.g. Kongsamut et al. 2001).

The static profit-maximizing price for firm i , P_{it}^* (also referred to as its *frictionless optimal price*), is given by the usual markup rule:

$$P_{it}^* = \frac{\theta}{\theta - 1} \frac{W_t}{A_{it}}. \quad (37)$$

From the representative household's labor supply:

$$\frac{W_t}{P_t} = C_t,$$

which leads to:

$$P_{it}^* = \frac{\theta}{\theta - 1} \frac{P_t C_t}{A_{it}}.$$

In logarithms (lowercase variables denote logarithms throughout), this reads:

$$p_{it}^* = \log\left(\frac{\theta}{\theta - 1}\right) + \log(P_t C_t) - \log(A_{it}).$$

Ignoring the unimportant constant and assuming appropriate exogenous stochastic processes for nominal aggregate demand and for idiosyncratic productivity yields the specifications used throughout the main text.

9 Appendix B

Here we derive the quadratic approximation to the static profit-maximization problem used in the main text. Write real flow profits as:

$$\Pi\left(\frac{P_i}{P}, C, A_i\right) = A_i^{1-\theta} \frac{P_i}{P} \left(\frac{P_i}{P}\right)^{-\theta} C - \frac{W}{PA_i} A_i^{1-\theta} \left(\frac{P_i}{P}\right)^{-\theta} C,$$

where P_i is the price *charged* by firm i . We can use the labor supply equation to express the real wage as a function of aggregate consumption ($\frac{W}{P} = C$), and rewrite the expression for real flow profits as:

$$\Pi\left(\frac{P_i}{P}, C, A_i\right) = A_i^{1-\theta} \left(\frac{P_i}{P}\right)^{1-\theta} C - C^2 A_i^{-\theta} \left(\frac{P_i}{P}\right)^{-\theta}.$$

Let $\bar{\Pi}$ be the steady-state level of real profits in a frictionless economy (upper bars denote steady-state values):²⁷

$$\bar{\Pi} \equiv \Pi\left(\frac{P_i^*}{P}, \bar{C}, A_i\right).$$

²⁷A constant level of aggregate consumption requires the restriction $\left[\int_0^1 A_{it}^{\theta-1} di\right]^{\frac{1}{1-\theta}} = 1$, which we assume holds throughout the paper.

We want to approximate the loss function \bar{L} defined as:

$$\begin{aligned}\bar{L}\left(\frac{P_i^*}{P}, \frac{P_i}{P}, C, A_i\right) &= \frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right) - \Pi\left(\frac{P_i}{P}, C, A_i\right)}{\bar{\Pi}} \\ &= \frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right) - \Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, \bar{C}, A_i\right)} \frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)}{\bar{\Pi}},\end{aligned}\quad (38)$$

The second ratio in (38) can be written as:

$$\begin{aligned}\frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)}{\bar{\Pi}} &= \frac{A_i^{1-\theta} \left(\frac{P_i^*}{P}\right)^{1-\theta} C - C^2 A_i^{-\theta} \left(\frac{P_i^*}{P}\right)^{-\theta}}{\bar{C} - \bar{C}^2} \\ &= \frac{A_i^{1-\theta} \left(\frac{P_i^*}{P}\right)^{1-\theta} C - \frac{\theta-1}{\theta} C A_i^{1-\theta} \left(\frac{P_i^*}{P}\right)^{1-\theta}}{\bar{C} - \frac{\theta-1}{\theta} \bar{C}} \\ &= A_i^{1-\theta} \frac{C}{\bar{C}} \left(\frac{P_i^*}{P}\right)^{1-\theta} \\ &= \left(\frac{C}{\bar{C}}\right)^{2-\theta},\end{aligned}\quad (39)$$

where we use the facts that $\frac{P_i^*}{P} = \frac{\theta}{\theta-1} \frac{C}{A_i}$ and $\bar{C} = \frac{\theta-1}{\theta}$. Note how the link between preferences and technology makes the idiosyncratic shock drop from the expression for maximized profits.

The first ratio in (38) is the proportional profit loss due to the ‘‘suboptimal’’ price P_i . It is convenient to rewrite it as:

$$\frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right) - \Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)} = 1 - \frac{\Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)}.$$

The profit ratio in the above expression can be written as:

$$\begin{aligned}\frac{\Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)} &= \frac{A_i^{1-\theta} \left(\frac{P_i}{P}\right)^{1-\theta} C - C^2 A_i^{-\theta} \left(\frac{P_i}{P}\right)^{-\theta}}{A_i^{1-\theta} \left(\frac{P_i^*}{P}\right)^{1-\theta} C - C^2 A_i^{-\theta} \left(\frac{P_i^*}{P}\right)^{-\theta}} \\ &= \frac{A_i^{1-\theta} \left(\frac{P_i}{P}\right)^{1-\theta} - \frac{\theta-1}{\theta} \frac{P_i^*}{P} A_i^{1-\theta} \left(\frac{P_i}{P}\right)^{-\theta}}{A_i^{1-\theta} \left(\frac{P_i^*}{P}\right)^{1-\theta} - \frac{\theta-1}{\theta} \frac{P_i^*}{P} A_i^{1-\theta} \left(\frac{P_i^*}{P}\right)^{-\theta}} \\ &= \theta \frac{\left(\frac{P_i}{P}\right)^{1-\theta} - \frac{\theta-1}{\theta} \frac{P_i^*}{P} \left(\frac{P_i}{P}\right)^{-\theta}}{\left(\frac{P_i^*}{P}\right)^{1-\theta}} \\ &= \theta \left(\frac{P_i^*}{P_i}\right)^{\theta-1} - (\theta-1) \left(\frac{P_i^*}{P_i}\right)^\theta,\end{aligned}$$

so that:

$$\frac{\Pi\left(\frac{P_i^*}{P}, C, A_i\right) - \Pi\left(\frac{P_i}{P}, C, A_i\right)}{\Pi\left(\frac{P_i^*}{P}, C, A_i\right)} = 1 - \theta \left(\frac{P_i^*}{P_i}\right)^{\theta-1} + (\theta - 1) \left(\frac{P_i^*}{P_i}\right)^\theta. \quad (40)$$

As before, note how the link between preference and technology makes the idiosyncratic shock drop from the expression above.

Combining (39) and (40), and keeping the relevant arguments of the loss function, we obtain:

$$\bar{L}\left(\frac{P_i^*}{P}, \frac{P_i}{P}, C, A_i\right) = \bar{L}\left(\frac{P_i^*}{P_i}, C\right) = \left(\frac{C}{\bar{C}}\right)^{2-\theta} \left[1 - \theta \left(\frac{P_i^*}{P_i}\right)^{\theta-1} + (\theta - 1) \left(\frac{P_i^*}{P_i}\right)^\theta\right]. \quad (41)$$

We can rewrite the loss function \bar{L} in terms of logarithms:

$$G(p_i^* - p_i, c) = e^{(2-\theta)c} \left[\left(1 - \theta e^{(\theta-1)(p_i^* - p_i)}\right) + (\theta - 1) e^{\theta(p_i^* - p_i)} \right].$$

The exact loss function $G(p_i^* - p_i, c)$ can be used in the optimal price-setting problems. However, the presence of aggregate consumption in the expression implies that solving for the optimal pricing rule in the presence of pricing frictions involves a fixed-point problem, even in the absence of strategic complementarity or substitutability in price setting. To make the optimal pricing problem more tractable, we eliminate the effect of aggregate output by assuming $\theta = 2$ (as in Danziger 1999 and Bonomo and Carvalho 2010). In addition, for analytical convenience we take a second-order Taylor expansion of flow profit losses around the frictionless optimal price, based on which we analyze the price-setting problems discussed in the paper:

$$\text{flow profit losses}(p_{it}) \propto (p_{it} - p_{it}^*)^2.$$

10 Appendix C

In this appendix we show how we formalize firms' intertemporal optimization problems in the presence of frictions. For brevity we focus on the case with dissociated information gathering/processing and adjustment costs, which is more involved. The other cases are simpler and can be formalized analogously. Let \hat{F} and \hat{K} denote the levels of, respectively, the information and adjustment costs.

Formally, the pricing problem of a firm may be written as:²⁸

$$\tilde{V}(s_{t_0}) = \max_{\left\{ \{t_j, t_{j_n}, X_{t_{j_n}}\}_{n=0}^{N_j} \right\}_{j=1}^{\infty}} E_{t_0} \sum_{j=0}^{\infty} e^{-\rho(t_j-t_0)} E_{t_j} \left[\begin{array}{l} -e^{-\rho(t_{j+1}-t_j)} \widehat{F} + \int_{t_j}^{t_{j_0}} e^{-\rho r} \Pi \left(\frac{X_r}{P_r}, C_r, A_r \right) dr \\ + e^{-\rho(t_{j_0}-t_j)} \sum_{n=0}^{N_j} \left[\begin{array}{l} \int_{t_{j_n}}^{t_{j_{n+1}}} e^{-\rho r} \Pi \left(\frac{X_r}{P_r}, C_r, A_r \right) dr \\ - e^{-\rho(t_{j_n}-t_{j_0})} \widehat{K} \end{array} \right] \end{array} \right],$$

where for all $r \notin \{t_{0_0}, \dots, t_{0_1}, t_{0_{N_0}}, \dots, t_{1_0}, \dots\}$ $X_r = X_{r-}$. $\tilde{V}(s_{t_0})$ denotes the expected present value of real profits Π , net of adjustment and information costs, when the state of the economy is s_{t_0} . The sequence t_j denotes the information dates - dates in which the firm incurs the information gathering and processing cost. The sequence t_{j_n} denotes the dates in which the firm incurs the menu cost and changes its price, and the sequence $X_{t_{j_n}}$ denotes the prices chosen on these dates.

Let $V^*(s_{t_0})$ denote the expected present value of profits of a hypothetical identical firm in the same economy that does not face any pricing friction. Then,

$$V^*(s_{t_0}) = E_{t_0} \left[\int_{t_0}^{\infty} e^{-\rho r} \Pi \left(\frac{P_r^*}{P_r}, C_r, A_r \right) dr \right],$$

where P_r^* is the individual price that maximizes real profits at time r , i.e. the frictionless optimal price of the firm. With this auxiliary value function, $\widehat{V}(s_{t_0}) \equiv V^*(s_{t_0}) - \tilde{V}(s_{t_0})$ is the minimized expected present value of the real profit losses due to the existence of information and adjustment costs, and our problem can be equivalently rewritten in terms of $\widehat{V}(s_{t_0})$.

Defining $\widehat{L} \left(\frac{P_r^*}{P}, \frac{X}{P}, C, A \right) \equiv \Pi \left(\frac{P_r^*}{P}, C, A \right) - \Pi \left(\frac{X}{P}, C, A \right)$ to be the instantaneous real profit loss due to a “suboptimal” price X , and normalizing the pricing problem by the steady-state level of real profits in a frictionless economy, $\bar{\Pi}$, we can rewrite the firm’s program as:

$$\bar{V}(s_{t_0}) = \min_{\left\{ \{t_j, t_{j_n}, X_{t_{j_n}}\}_{n=0}^{N_j} \right\}_{j=1}^{\infty}} E_{t_0} \sum_{j=0}^{\infty} e^{-\rho(t_j-t_0)} E_{t_j} \left[\begin{array}{l} e^{-\rho(t_{j+1}-t_j)} \bar{F} + \int_{t_j}^{t_{j_0}} e^{-\rho r} \bar{L} \left(\frac{P_r^*}{X_r}, C_r \right) dr \\ + e^{-\rho(t_{j_0}-t_j)} \sum_{n=0}^{N_j} \left[\begin{array}{l} \int_{t_{j_n}}^{t_{j_{n+1}}} e^{-\rho r} \bar{L} \left(\frac{P_r^*}{X_r}, C_r \right) dr \\ + e^{-\rho(t_{j_n}-t_{j_0})} \bar{K} \end{array} \right] \end{array} \right],$$

where $\bar{V}(s_{t_0}) \equiv \frac{\widehat{V}(s_{t_0})}{\bar{\Pi}}$, $\bar{L} \left(\frac{P_r^*}{X_r}, C_r \right) \equiv \frac{\widehat{L} \left(\frac{P_r^*}{P_r}, \frac{X_r}{P_r}, C_r, A_r \right)}{\bar{\Pi}}$, $\bar{F} \equiv \frac{\widehat{F}}{\bar{\Pi}}$, $\bar{K} \equiv \frac{\widehat{K}}{\bar{\Pi}}$. Note that from (41), $\bar{L}(\cdot, \cdot)$ only depends on the ratio of the frictionless optimal price to the charged price, and on aggregate consumption.

²⁸We drop the i subscripts in order to simplify the notation.

Finally, assuming $\theta = 2$, and relying on the same second-order Taylor approximation of flow profit losses used in Appendix B, the firm's (approximate) pricing problem can be written as:

$$V(s_{t_0}) = \min_{\left\{ \left\{ t_j, t_{j_n}, x_{t_{j_n}} \right\}_{n=0}^{N_j} \right\}_{j=1}^{\infty}} E_{t_0} \sum_{j=0}^{\infty} e^{-\rho(t_j - t_0)} E_{t_j} \left[\begin{array}{l} e^{-\rho(t_{j+1} - t_j)} F + \int_{t_j}^{t_{j_0}} e^{-\rho r} (x_r - p_r^*)^2 dr \\ + e^{-\rho(t_{j_0} - t_j)} \sum_{n=0}^{N_j} \left[\begin{array}{l} \int_{t_{j_n}}^{t_{j_{n+1}}} e^{-\rho r} (x_r - p_r^*)^2 dr \\ + e^{-\rho(t_{j_n} - t_{j_0})} K \end{array} \right] \end{array} \right],$$

where $V(s_{t_0}) \equiv \frac{\bar{V}(s_{t_0})}{2}$, $F \equiv \frac{\bar{F}}{2}$ and $K \equiv \frac{\bar{K}}{2}$.

11 Appendix D

Here we provide alternative solution approaches to some of the problems analyzed in the main text.

11.1 Deterministic information arrival, no drift

We provide an alternative solution for the case of $\mu = 0$, which has been analyzed by Bonomo and Garcia (2001). Then, there are no uninformed adjustments: a firm either adjusts on an information date or waits for the next such date. Suppose the firm has just learned the discrepancy z_{t_0} on an information date t_0 . It has the option of incurring the menu cost K and adjusting the discrepancy to 0. As a result, the optimal policy can be characterized by the barrier parameter S , which gives the (symmetric) trigger points for the discrepancy on information dates. After that, the firm will incur the expected flow deviation cost starting from the appropriate initial discrepancy (z_{t_0} or 0) until the next information date.

To introduce the appropriate notation, let S be such that:

$$V(S, 0) = V(0, 0) + K. \quad (42)$$

For $z_{t_0} > S$ or $z_{t_0} < -S$:

$$V(z_{t_0}, 0) = V(0, 0) + K. \quad (43)$$

For $-S < z_{t_0} < S$:

$$V(z_{t_0}, 0) = B(z_{t_0}, T) + e^{-\rho T} V(z_{t_0}, T), \quad (44)$$

where the function $B(z, \tau)$ corresponds to the expected flow deviation cost over a period of length τ starting from the initial discrepancy z , and is given by:

$$B(z, \tau) = \int_0^\tau e^{-\rho s} (z^2 + \sigma^2 s) ds = z^2 \left(\frac{1 - e^{-\rho \tau}}{\rho} \right) + \sigma^2 \left(\frac{-\tau e^{-\rho \tau}}{\rho} + \frac{1 - e^{-\rho \tau}}{\rho^2} \right). \quad (45)$$

As in the general case analyzed in the main text, the solution must also satisfy condition (26), which can be used to rewrite equation (44):

$$V(z_{t_0}, 0) = B(z_{t_0}, T) + e^{-\rho T} E \left[V \left(z_{t_0} + \sigma \sqrt{T} \varepsilon, 0 \right) \right]. \quad (46)$$

This problem can be solved numerically as described in the next appendix.

11.2 No adjustment cost

This solution is similar in spirit to Reis (2006). Since the control problem between information dates is trivial (changing p_i to keep $z_i = 0$), we can use an alternative Bellman equation to solve for the optimal choice of information dates:

$$V(0) = \min_{\tau} \{ B(\tau) + e^{-\rho\tau} (V(0) + F) \}, \quad (47)$$

where $B(\tau) = B(0, \tau)$ represents the expected cost due to lack of information over a time interval of length τ , and is given by:

$$B(\tau) = \int_0^{\tau} e^{-\rho s} (\sigma^2 s) ds = \sigma^2 \left(\frac{1 - e^{-\rho\tau}}{\rho^2} - \frac{\tau e^{-\rho\tau}}{\rho} \right).$$

The first-order condition with respect to τ yields:

$$\sigma^2 \tau^* = \rho (V(0) + F), \quad (48)$$

which states that the marginal cost of postponing an information date ($\sigma^2 \tau^*$) must equal the marginal benefit of doing so ($\rho (V(0) + F)$). Solving for $V(0)$ in (47) and combining with (48) delivers the solution to the problem.

11.3 Single information/adjustment cost

This solution approach is the one employed by Bonomo and Carvalho (2004, 2010). Given that there is no price adjustment between information dates, let us concentrate on these dates. We can characterize the optimal pricing rule with the following Bellman equation:

$$\begin{aligned} V^* &= \min_{\tau, z} \int_0^{\tau} e^{-\rho s} f(z - \mu s, s) ds + e^{-\rho\tau} (Q + V^*) \\ &= \min_{\tau, z} \int_0^{\tau} e^{-\rho s} ((z - \mu s)^2 + \sigma^2 s) ds + e^{-\rho\tau} (Q + V^*), \end{aligned} \quad (49)$$

where $V^* = V(c, 0)$. The first-order conditions are:

$$\int_0^{\tau^*} e^{-\rho s} (c - \mu s) ds = 0,$$

$$(c - \mu \tau^*)^2 + \sigma^2 \tau^* - \rho(V^* + Q) = 0.$$

Rearranging the first condition yields:

$$c = \mu \left(\frac{1}{\rho} - \frac{e^{-\rho \tau^*}}{1 - e^{-\rho \tau^*}} \tau^* \right).$$

To get the second equation, solve the Bellman equation (49) for V^* to obtain:

$$V^* = \frac{\int_0^{\tau^*} ((c - \mu s)^2 + \sigma^2 s) ds + e^{-\rho \tau^*}}{1 - e^{-\rho \tau^*}}.$$

Plugging this expression for V^* into the second first-order condition and simplifying yields the solution to the problem.

11.4 Dissociated information and adjustment costs, no drift

In the particular case of $\mu = 0$, the logic behind the solution is similar to the case of exogenous deterministic information and no drift. There are no uninformed adjustments, and the optimal price-change decision can be characterized by a barrier parameter S , which gives the (symmetric) trigger-points for the discrepancy on information dates. In addition, and differently from the case with exogenous information arrival, the optimal policy specifies the time until the next information date as a function of the discrepancy as of the previous such date (here denoted $\tau^*(z_{t_0})$).

Suppose the firm has just learned the discrepancy z_{t_0} . It has the option of paying the menu cost K and adjusting the discrepancy to zero:

$$V(z_{t_0}, 0) \leq K + V(0, 0). \quad (50)$$

As in the case of deterministic information and no drift, $V(S, 0) = V(0, 0) + K$. After choosing whether or not to adjust the firm will incur the expected deviation cost of being away from the frictionless optimal price starting from the appropriate initial discrepancy (z or 0) until the next optimally chosen information date, and the discounted value of the information cost F plus the value function at that time:

$$V(z, 0) = \min_{\tau} [B(z, \tau) + e^{-\rho \tau} (F + V(z, \tau))], \quad (51)$$

where the function $B(\cdot, \cdot)$ is again given by (45). Using the notation for the function which gives the optimal time until the next information date for each initial discrepancy z , (51) can be rewritten as:

$$V(z, 0) = B(z, \tau^*(z)) + e^{-\rho\tau^*(z)} (F + V(z, \tau^*(z))). \quad (52)$$

Condition (33) still holds, and as a result (51) and (52) become, respectively:

$$V(z, 0) = \min_{\tau} [B(z, \tau) + e^{-\rho\tau} E[V(z + \sigma\sqrt{\tau}\varepsilon, 0)]], \text{ and} \quad (53)$$

$$V(z, 0) = B(z, \tau^*(z)) + e^{-\rho\tau^*(z)} E[V(z + \sigma\sqrt{\tau^*(z)}\varepsilon, 0)]. \quad (54)$$

This problem can be solved numerically as described in the next appendix.

12 Appendix E

In this appendix we provide numerical solution algorithms for the optimal price-setting problems.

12.1 Random information arrival

In order to find the optimal rule $\{l(\tau), c(\tau), u(\tau)\}$, we need to find the value function. We start by discretizing the partial differential equation (20) over a grid with time-increments Δt and discrepancy-increments Δz , using an explicit finite-difference method. We make the following approximations:

$$\begin{aligned} z &\approx n \Delta z, \\ \tau &\approx m \Delta t, \\ V_{\tau} &\approx \frac{v_{n,m+1} - v_{n,m}}{\Delta t}, \\ V_z &\approx \frac{v_{n,m+1} - v_{n-1,m+1}}{\Delta z}, \end{aligned} \quad (55)$$

and obtain:

$$v_{n,m} = p^0 v_{n,m+1} + p^- v_{n-1,m+1} + \frac{\lambda}{\rho + \frac{1}{\Delta t}} \sum_{j=-J}^J \pi(j) v_{n+j,0} + \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) [(n \Delta z)^2 + \sigma^2 m \Delta t], \quad (56)$$

where $\pi(\cdot)$ is a discretization of the normal distribution,²⁹ and:

$$p^0 = \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \left(\frac{-\mu}{\Delta z} + \frac{1}{\Delta t} \right),$$

$$p^- = \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \frac{\mu}{\Delta z}.$$

If we have the value function for all states at time $m + 1$, we can use equation (56) to find the value function at time m . We start with an arbitrary value function for very large τ and proceed backwards using the difference equation until arriving at $\tau = 0$. If τ is large enough, the resulting value function should be a good approximation for small τ , even though the initial guess for the value function is arbitrary. The reason is that if τ is large enough (relative to λ and ρ) distant points in the grid have little importance for the value function evaluated at a small τ . In the end we use conditions (21), and (23) for each τ to find $c(\tau)$, $u(\tau)$, and $l(\tau)$.

12.2 Deterministic information arrival

12.2.1 No-drift case

Start with a guess for $V(\cdot, 0)$. Impose (43) to get a new V , replacing $V(z, 0)$ by $V(0, 0) + K$ whenever $V(z, 0) > V(0, 0) + K$. Use the right-hand-side of (46) to obtain a new value for $V(z, 0)$, and iterate to convergence. At the end, find S according to (42).

An alternative solution method explores the structure of the general solution of the differential equation (24) when $\mu = 0$. In this case, the value function takes the form:

$$V(z, \tau) = \frac{z^2}{\rho} + \frac{\sigma^2 \tau}{\rho} + \frac{\sigma^2}{\rho^2} + e^{\rho \tau} H(z),$$

with the $H(z)$ function being determined by the specific boundary conditions of the problem. It turns out that the previous method, which iterates directly on V , is more stable than methods that use the above equation and iteration on $H(z)$. The reason is that while V must necessarily be constant at the boundaries of the z -grid, H must not be constant, and thus the unavoidable truncation imposed by the grid generates more problems for the numerical integration required for taking the expectation under the Gaussian distribution in (46).

²⁹In all solutions presented in this paper we use a 120-mass-point discretization of the normal distribution.

12.2.2 General case

Making the same approximations as in (55) we discretize (24) as:

$$v_{n,m} = p^0 v_{n,m+1} + p^- v_{n-1,m+1} + \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \left[(n \Delta z)^2 + \sigma^2 m \Delta t \right], \quad (57)$$

where:

$$p^0 = \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \left(\frac{-\mu}{\Delta z} + \frac{1}{\Delta t} \right),$$

$$p^- = \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \frac{\mu}{\Delta z}.$$

As before, if we have the value function for all states at time $m + 1$, we can use equation (57) to find the value function at time m . We use the following algorithm. We guess a value function at time zero and impose condition (22) as follows: find $c(0)$ and impose $V(z, 0)$ by $V(c(0), 0) + K$ whenever $V(z, 0) > V(c(0), 0) + K$. We then compute the expectation in (26) to find the value function at time T , and the discrepancy z that minimizes the function at T , $c(T)$. Imposing condition (22) determines the new value at T . We then use the difference equation (57) to find the value function at time $T - \Delta t$, and solve for the value of z that minimizes the function at $T - \Delta t$, and so on, until we arrive at time zero. At that point we test if the value function at time zero is close enough (according to a convergence criterion set a priori) to the value we had at the previous iteration; otherwise we continue until convergence. In the end we use conditions (21), and (23) for each τ to find $c(\tau)$, $u(\tau)$, and $l(\tau)$.

12.3 Dissociated information and adjustment costs

12.3.1 No-drift case

Start with a guess for $\tau^*(.)$ and $V(., 0)$. Impose (50) to get the new V , replacing $V(z, 0)$ by $V(0, 0) + K$ whenever $V(z, 0) > V(0, 0) + K$. Use the right-hand-side of (54) to obtain a new value for $V(z, 0)$, and then use this new $V(., 0)$ to find a new $\tau^*(.)$ by solving for τ in (53). Repeat the process until convergence, and at the end find S such that $V(S, 0) = V(0, 0) + K$.

12.3.2 General case

Because we start from the same partial differential equation (24), we use the same finite-difference discretization scheme as in Subsection 12.2.2:

$$v_{n,m} = p^0 v_{n,m+1} + p^- v_{n-1,m+1} + \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \left[(n \Delta z)^2 + \sigma^2 m \Delta t \right], \quad (58)$$

where:

$$p^0 = \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \left(\frac{-\mu}{\Delta z} + \frac{1}{\Delta t} \right)$$

$$p^- = \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \frac{\mu}{\Delta z}.$$

We then apply the following algorithm. We guess values for the function $v_{n,m}$ for a large grid of times and discrepancies. It is important to impose conditions (32) and (31), which state that at any time and discrepancy the price setter will incur the information and/or the adjustment cost, if it is advantageous for her to do so. We then choose a time T large enough to exceed the optimal time interval between information dates for any initial discrepancy z_{t_0} . For such time T , find the z that minimizes $V(z, T)$, denoted $c(T)$, and impose conditions (32) and (31) to determine the new value at T . Then use the difference equation (58) to find the value function at time $T - \Delta t$. Next, impose conditions (32) and (31) to determine the new value at $T - \Delta t$ and so on, until time zero. At that point test if the value function at each time and discrepancy is close enough (according to some convergence criterion set a priori) to the value function at the previous iteration. Otherwise begin another iteration. After convergence, use conditions (21), and (23) for each τ to find $c(\tau)$, $u(\tau)$, and $l(\tau)$ and condition (33) to determine $\tau^*(z)$ for any given discrepancy.

12.4 Exogenous infrequent W_{at} -information

Discretizing the partial differential equation (34) using the explicit difference method, and making the same approximations as in (55), we arrive at:

$$v_{n,m} = p^0 v_{n,m+1} + p^- v_{n-1,m+1} + p^+ v_{n+1,m+1} + \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \left[(n \Delta z)^2 + \sigma_a^2 m \Delta t \right], \quad (59)$$

where:

$$\begin{aligned}
 p^0 &= \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \left(- \left(\frac{\sigma_i}{\Delta z} \right)^2 + \frac{1}{\Delta t} \right) \\
 p^- &= 0.5 \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \left(\left(\frac{\sigma_i}{\Delta z} \right)^2 + \frac{\mu}{\Delta z} \right) \\
 p^+ &= 0.5 \left(\frac{1}{\rho + \frac{1}{\Delta t}} \right) \left(\left(\frac{\sigma_i}{\Delta z} \right)^2 - \frac{\mu}{\Delta z} \right)
 \end{aligned}$$

To find the solution we apply the same algorithm as in Subsection 12.2.2.

12.5 Costly W_{at} -information gathering and processing

The numerical solution for this case can be obtained by applying the algorithm described in Subsection 12.3.2, but with the finite-difference scheme of Subsection 12.4 that provides the discretization (59) for the partial differential equation (34).