

A Glass half full: Contrarian trading in the flash crash¹

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Abstract

Stocks with better past returns crash more than other stocks on May 6, 2010. I find evidence that this is related to such stocks being unattractive to contrarian buyers. This suggests the importance of contrarian investors in stabilizing price fluctuations. However, the glass is half full---that the contrarian investors shun certain types of stocks limits the extent of price stability that relies heavily on this and other similar types of trading strategies. Stocks with better past returns exhibit more negative co-skewness, which holds in almost every month since the 1960s and for past return horizons ranging from one month to three years. This has interesting implications for risk premia associated with short-term reversal, medium-term momentum, and long-run reversal portfolios.

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1 Introduction

On May 6, 2010, the U.S. equity market experienced a flash crash. The S&P 500 index e-mini futures price dropped from 1130.5 at 2:30pm Eastern Time (ET) to an intraday low of 1056 at 2:45pm. This loss of -6.59% occurred in a mere 15 minutes. Interestingly, another 15 minutes later at 3pm, the price is back at 1112.75, largely erasing the loss. The loss was completely erased and the futures price hit a post-crash high of 1136 by day end. Perhaps more dramatic were the price swings in the individual stocks. As an example, the price Accenture (NYSE ticker: ACN) crashed from \$41.52 before 2:30pm to a penny at 2:47pm before recovering all the loss and hit a post-crash high of \$42.06 by day end. Even a big stock like Apple suffered a loss of about -20% in the flash crash before bouncing back. The fast crash and reversal attracted regulatory scrutiny and media attention, particularly on the roles played by various financial institutions and their effects on the flash crash. CFTC and SEC (2010) provide a rich analysis of the market events during the flash crash.

This paper explores the cross section of individual-stock crash sizes --- i.e. why some stocks crash more than the others on May 6, 2010. The goal is to understand: (1) is there any pattern in the cross section of crashes? Are there any types of stocks that are especially “fragile” in an extreme event like the flash crash; (2) Test potential explanations to any pattern uncovered; and (3) study the implications for price stability and risk premium.

In the cross section, there is a big difference in crash sizes, which is largely unexplained by the market beta. Figure 1 shows that those ten percent of the stocks with the worst crash sizes experience an average crash size of -22% before subsequently recover almost all of the losses later in the day. In contrast, those ten percent of the stocks with the smallest crash sizes experience an average crash size about -1%. Figure 2 shows that the cross-sectional crash size in excess of market beta exposure is essentially as disperse as the raw crash size. For example, the difference between the 5th quantile and

the median of the raw crash size is -10% , about the same as the difference between the 5th quantile and the median of the market beta adjusted crash size.

Next, this paper examines the relation between crash size and a number of widely used risk and characteristics measures, including the market beta, Fama and French (1993) size beta (SMB) and value beta (HML), momentum beta, short-term reversal beta, size, book-to-market, past one-month return, past six-month return, past twelve-month return, past three-year return, volatility, coskewness, and turnover. Past return emerge as the variable that has the strongest relation to the cross-sectional crash size, among the variables examined. Stocks with higher past return crash more on average, and the effect becomes stronger for more extreme crash sizes. For example, the quintile of stocks with the highest past six-month returns' 5th quantile and median of crash sizes exceed the quintile of stocks with the lowest past six-month return by -8.2% and -1.7% , respectively. The effect of past return on individual-stock crash size is stronger than the effect from the market beta. The other risk and characteristics measures have substantially less effect than past return and market beta on the crash size on May 6, 2010.

Why do stocks with high past returns crash more in the flash crash? After considering several potential explanations, this paper finds evidence consistent with the hypothesis that certain types of potential buyers shun stocks with high past returns unless such stocks crash more than other stocks. The intuition can be seen from the following hypothetical but likely plausible reasoning by some investors: “Apple stock (ticker: AAPL) is down 20% in the last few minutes. Perhaps this is a good buying opportunity. But wait, Apple was up 21% year-to-date before the crash. Is Apple under- or over-valued? Perhaps let me wait until Apple drops further or find another more attractive stock.”

This intuition is a bit simplistic, since investors presumably compare price relative to fundamental. The fundamental value of a stock is difficult to measure. How would a potential buyer decide if the crash

is excessive relative to the fundamental during the flash crash? This paper begins by exploring relative-value trading, which is known to be popular among hedge funds such as the Long-Term Capital Management (for example Lowenstein (2000), and page 41 in Patterson (2010)). A relative trader proxies the fundamental of a security' fundamental by the price of another similar security, and will bet on mean reversion when the prices of the two similar securities diverge. Relative-value trading is one way to operationalize the law of one price.

To study how a relative-value investor might have behaved during the flash crash, this paper takes the approach of imitating relative-value strategies, and relating the strategies' buy and sell signals to the cross-sectional crash sizes. Which strategies? This paper will start with the pairs trading strategy in Gatev, Goetzmann and Rouwenhorst (2006), and then extend and construct proxies for the trading signals of other proprietary strategies. The pairs trading strategy in Gatev, Goetzmann and Rouwenhorst (2006) has a formation period of twelve months, in which pairs of similar stocks are identified by minimizing the sum of squared daily price deviations (the price is normalized to 1 at the beginning of the formation period). The pairs are held fixed in the subsequent six months, which is the trading period. During the trading period, if the pair prices diverge beyond a threshold, the algorithm sells the expensive stock and buys the cheaper stock in the pair, and closes the position when the price gap converges back to zero or at the end of the trading period.

This paper first hypothesizes that the relative return of a pair prior to the flash crash is negatively correlated to their relative crash size. The intuition is that bigger past return difference implies a pairs-trading algorithm perceives the better performing stock in a pair is more over-valued and is reluctant to buy it without a bigger crash (relative to the other stock in the pair). This intuition works even if an investor who follows the Gatev, Goetzmann and Rouwenhorst (2006) algorithm has already shorted the stock with better past return. This is because the algorithm closes its position when the price gap converges back to zero, which also implies more room to crash if a stock's past return is higher than the

other stock in the pair. Linear regression confirms the hypothesis that the lagged relative stock return of a pair is negatively related to the relative crash size of a pair. More importantly, the effect of lagged return on crash size is stronger for extreme crashes. Ex ante, one might conjecture that small shocks (such as tick-size fluctuations) might be absorbed by other liquidity providers, while relative-value investors might be attracted to bigger price fluctuations that cross certain thresholds. This conjecture is supported by the data. For example, among pairs formed at the end of 2009, if the lagged relative return prior to the flash crash is no more than 20% (a pair is ordered so that the lagged relative return is non-negative, i.e. the first stock in a pair has better past return than the other stock), the 5th quantile and the median of the relative crash size are -10% and 0%, respectively (Figure 5). I.e. 5% of the pairs saw the previously better performing stock crash -10% more than the other stock, and half of the pairs saw the previously better performing stock crash more than the oversold stock. In contrast, if the lagged relative return is above 40%, the 5th quantile and median of relative crash size are -18% and -2%, respectively, which are -8% and -2% worse than if the lagged relative return is less than 20%. Quantile regressions show the effect of lagged relative return on relative crash size is statistically significant and holds across different pairs-formation periods.

There are many different proprietary trading strategies other than the pairs-trading strategy. To shed some light into these black boxes, this paper uses a simple model to show that two conditions are sufficient for past return to correlate positively with the over-valuation perceived by a black box. The first condition is that such a black box is a contrarian, in that its signal for fundamental moves slower than the stock price. This captures the behavior of a large number of investors, such as investors who compute fundamental from accounting variables (e.g. Daniel and Titman (2006) on market-to-book), from averaging or smoothing (e.g. Brock, Lakonishok and Lebaron (1992), Zhu and Zhou (2009) on moving averages, Campbell and Shiller (1988) on smoothed price-earnings multiples). The second condition is that such algorithms' profit target is bigger when the perceived mis-pricing is bigger. This

condition implies that, for an algorithm that has already shorted the stock perceived more over-valued before the flash crash, the algorithm will aim for a bigger price drop before covering the short position (hence buying the stock).

These conditions on the proprietary algorithms imply the following hypothesis: the return of a stock prior to the flash crash is negatively correlated to its crash size. The previous finding that stocks with high past return crash more is consistent with this hypothesis. Quantile regressions further confirm statistically significant negative relation between a stock's past return and its crash size, controlling for a host of alternative hypotheses. The effect is again stronger for the extreme crash sizes, and is statistically significant for past return horizons ranging from one month up to about 30 months.

Is contrarian trading stabilizing? Contrarians here can include a hedge fund running a relative-value algorithm, but potentially also capture a much larger group of investors who informally contemplate whether a stock is under-valued if it has a severe crash on May 6, 2010 but has a stellar past return. The answer based on results in this paper is a glass half full. Interestingly, the finding of bigger crash size for those stocks less attractive to contrarian buyers in fact points to the importance of such investors in stabilizing stocks. However, the glass is half full because the analysis simultaneously finds a group of stocks (the previously high flying ones) a typical contrarian buyer is unlikely interested in, at least not with a big crash size. This imposes a limit on the extent of price stability that relies solely on this and other similar types of trading strategies. These results are suggestive that more contrarian trading (perhaps more in terms of diversity of strategies than in terms of overall asset under management) might mitigate extreme liquidity shocks. On the other hand, there are limits on the diversity of strategies. For example, data snooping concerns might preclude an algorithm to trade too aggressively on strategies involving too many conditioning variables --- such as those stocks that had stellar past returns but poor recent returns. A number of the known strategies tend to focus on monotonic patterns in

expected return (e.g. the monotonic increase in expected return for book-to-market sorted portfolios, as in Fama and French (1993)).

The finding that stocks with high past return crash more in an extreme event like the flash crash leads to the question of whether such stocks tend to do badly when the market experiences big shocks in other times. This relates to coskewness, which is shown by Kraus and Litzenberger (1976) and Harvey and Siddique (2000) to be important for the cross section of stock returns. This paper then runs Fama-Macbeth regressions and finds that higher past return forecasts more negative coskewness in the subsequent month. This finding holds in essentially every month since the 1960s and for all past return horizons from one month up to three years. This confirms Harvey and Siddique (2000) by providing additional evidence that momentum is related to coskewness. On the other hand, this finding makes the short-term reversal (Jegadeesh (1990)) and long-term reversal (Debondt and Thaler (1985)) more interesting because the higher expected return of the past loser stocks comes with more positive coskewness.

This paper is organized as follows. Section 2 provides literature review. Section 3 documents the cross-sectional crash size for different beta- or characteristics-sorted portfolios. Section 4 studies relative-value trading, while Section 5 analyzes contrarian trading in general. Section 6 examines alternative hypotheses. Section 7 and 8 discuss implications of the empirical findings, and Section 9 concludes.

2 Literature review

CFTC and SEC (2010) provide a rich analysis of the market events during the flash crash. Kirilenko, Kyle, Samadi and Tuzun (2011) study the S&P 500 E-mini futures trading and conclude that high frequency traders were too small to have caused or prevented the flash crash. Easley, Prado and O'Hara

(2011) show that the order toxicity of the S&P 500 E-mini futures increased prior to the flash crash, which can lead to reduced trading of potential liquidity providers. This paper focuses instead on the relative crash sizes of individual stocks, and reaches a glass half full conclusion---the relative-value traders mitigate the crash of some but not all stocks. Consistent with the spirit of the above papers, the contrarian traders examined in this paper are not found to be destabilizing.

This paper relates to the limits to arbitrage literature, and points out that a “strategy vacuum”---those states when fewer trading strategies identify a stock as attractive target---can potentially lead to bigger price deviation from fundamental. Such strategy vacuum adds to the capital constraint (e.g. Shleifer and Vishny (1997)), risk-bearing capacity (e.g. Delong, Shleifer, Summers and Waldmann (1990)), trading restrictions (e.g. Ofek and Richardson (2003)), and others that are previously identified in this literature. The finding in this paper that the relative-value traders are important in mitigating the crash of some stocks is consistent with the risky arbitrageurs’ increasing importance in liquidity provision relative to traditional market makers in recent years.

This paper provides evidence supporting pricing kernel that is nonlinear in market return, such as those in Kraus and Litzenberger (1976), Harvey and Siddique (2000), and Ang, Hodrick, Xing and Zhang (2006).

3 The cross section of crash sizes on May 6, 2010

3.1 Findings

This paper starts by analyzing the cross section of individual stock crashes for portfolios sorted by various beta or characteristics measures. Figure 3 shows the 5th, 10th, 25th, 50th (median), 75th, 90th, and 95th quantiles of crash size within each quintile portfolios sorted by past six-month return before the flash crash (the return includes the three trading days in May 2010 before the flash crash). The stocks

with higher past returns tend to have bigger crash sizes, particularly for the left tail (more extreme crash sizes). For example, the 5th quantile crash size for the quintile with the highest past six-month return is -20.8%, which is -8.2% worse than the quintile with the lowest past six-month return. The median crash size for the quintile with the highest past six-month return is -5.5%, which is -1.7% worse than the quintile with the lowest past six-month return.

Table 1 summarizes the median and the more extreme 5th quantile crash sizes for portfolios sorted by a number of other beta and characteristics measures, including the market beta, Fama and French (1993) size beta (SMB) and value beta (HML), momentum beta, short-term reversal beta, size, book-to-market, past one-month return, past six-month return, past twelve-month return, past three-year return, volatility, coskewness, and turnover. Among these variables, only past returns (for all return horizons examined in this table), market beta (estimated from monthly returns), and turnover have statistically significant explanatory power for the extreme crashes. Past returns generate the biggest difference in the 5th quantile crash sizes between the top and bottom quintiles, ranging from -6.7% to -9.5% when the past return horizons is longer than one month. These differences are bigger than the difference in the 5th quantile crash size between the top and the bottom market-beta quintiles (-6.4%). The difference in the 5th quantile crash size between the top and the bottom turnover quintiles is -5.3%, though the turnover effect appears mostly coming from the lowest turnover quintile having smaller crash size. One potential explanation is that a number of the stocks with the lowest turnover do not trade during the flash crash. The difference in the median crash size between the top and the bottom quintiles is much smaller for all of the variables examined

3.2 Why past return matters for the crash size? Some conjectures

One reason why past return matters is that it correlates with market beta, see for example Grundy and Martin (2001) and Daniel (2011). However, past return is presumably a noisier measure than the

standard market beta estimates used in Table 1. It is unlikely to observe a bigger effect from past return than from market beta, if correlation with the market beta is the only explanation. The subsequent analysis in this paper will control market beta, and the results from past return remain similar.

Could it be the case that the flash crash was caused by momentum traders? This paper cannot rule out this possibility. However, it is unclear why the stocks that the momentum traders want to buy fall more in the flash crash. It is possible that, after the past winner stocks crash, there is an amplification effect if the momentum traders stop loss by selling these stocks. However, amplifying is different from causing the crash.

Could the effect of past return on crash due to potential stock buyers? The intuition can be seen from the following hypothetical but likely plausible reasoning by some investors: “Apple stock (ticker: AAPL) is down 20% in the last few minutes. Perhaps this is a good buying opportunity. But wait, Apple was up 21% year-to-date before the crash. Is Apple under- or over-valued? Perhaps let me wait until Apple drops further or find another more attractive stock.” This intuition is a bit simplistic, since investors presumably compare price relative to fundamental. Section 4 begins by exploring relative-value trading, which proxies the fundamental of a security by the price of another similar security. Section 5 examines the effect of past return on contrarian investors in general.

4 Relative-value trading in the flash crash

4.1 Research approach

This paper will first pretend to be a relative-value hedge fund, generate the trading signals of relative-value strategies, and explore their effects on individual-stock crashes. Which strategies? This paper will start with the pairs trading strategy in Gatev, Goetzmann and Rouwenhorst (2006), and then generalize to contrarian trading in Section 5.

4.2 Pairs trading

What is pairs-trading? Pairs trading algorithms identify pairs of similar securities, and bet on mean reversion when the prices of a pair deviate. Specifically, this paper starts by implementing the pairs trading strategy in Gatev, Goetzmann and Rouwenhorst (2006). Why start with it? Pairs-trading algorithm generates intraday trading signals, which is suitable for an intraday event like the flash crash. The pairs-trading strategy is often mentioned by practitioners (see for example Patterson (2010)) and is known to be used by hedge funds such as the Long Term Capital Management (Lowenstein (2000)). The Gatev, Goetzmann and Rouwenhorst (2006) algorithm is also shown to work out of the sample by Do and Faff (2010).

The Gatev, Goetzmann and Rouwenhorst (2006) algorithm has a formation period of twelve months, in which pairs of similar stocks are identified by minimizing the sum of squared daily price deviations (the price is normalized to 1 at the beginning of the formation period). The pairs are held fixed in the subsequent six months, which is the trading period. During the trading period, if the pair prices diverge beyond a threshold, the algorithm sells the expensive stock and buys the cheaper stock in the pair, and closes the position when the price gap converges back to zero or at the end of the trading period.

To see the intuition of Hypothesis 1 below, Figure 4 shows an example of pairs trading (this figure is from Gatev, Goetzmann and Rouwenhorst (2006)) using a stock pair Kennecot and Uniroyal. As shown in Figure 4, the pairs-trading algorithm will short the expensive stock and long the cheaper stock when the pairs' prices diverge beyond a threshold, and close the long-short positions when the price gap disappears. Assume for now that the flash crash occurs on day 12 in Figure 4 when Kennecot's past return in the trading period is much higher than Uniroyal. As shown in the figure, the pairs-trading algorithm has shorted Kennecot and long Uniroyal. During the flash crash, the pairs-trading algorithm will buy Kennecot if Kennecot crashes sufficiently more than Uniroyal to wipe out the prior return difference. Therefore, the better the prior return of Kennecot is relative to Uniroyal, the more crash the

pairs-trading algorithm will require Kennecot before buying it (by closing the existing long-short position). Another scenario is if the pairs-trading algorithm does not have position on the pair before the flash crash. This scenario can happen if this other pairs-trading algorithm has a different threshold to open position than the algorithm in Figure 4. Using the same assumption that the flash crash occurs on day 12, this other pairs-trading algorithm will buy Kennecot if its crash size is big enough so that it not only wipes out the prior return difference with Uniroyal, but also cross the threshold required by the algorithm to buy Kennecot (and short Uniroyal). In either scenario, the pairs-trading algorithm requires a bigger crash size of the stock with better prior return before buying it.

Hypothesis 1 The relative crash size of a stock pair is negatively correlated to the relative stock return of the pair before the flash crash.

4.3 Empirical results

Using daily US stock return data from the Center for Research in Security Prices (CRSP), this paper constructs pairs using five different formation periods: Apr 1, 2009 to Mar 31, 2010; Mar 1, 2009 to Feb 28, 2010; ...; and Dec 1, 2008 to Nov 30, 2009.³ Table 1 shows the pairs when the formation period is the entire 2009. The top pairs (the most similar pairs) are often competing Exchange-Traded Funds (ETF) tracking the same instrument. For example, pair number 1 is SPDR and iShares Gold ETF, respectively. SPDR and iShares S&P 500 ETF form pair number 4. The first pair of common stocks (pair number 27) is the dual class stocks of Bio-Rad. The first pair of common stocks that are from different companies (pair

³ Earlier formation periods are excluded because the time between the formation period end and the flash crash is more than six months, the trading period in the Gatev, Goetzmann and Rouwenhorst (2006) algorithm. The formation period May 1, 2009 to Apr 30, 2010 is excluded because there are only three trading days between the formation period end and the flash crash, which presumably implies tiny return differences before the crash between stocks in a pair. Nonetheless, result from the formation period ending on Apr 30, 2010 is largely similar to the results for other formation periods.

number 188) is Scana (ticker: SCG) and Westar (ticker: WR), both of which are energy stocks. The last pair (pair number 2857) is the most dissimilar pair and it consists of a coffee company and a pharmaceutical company. Overall, the matching identifies a number of pairs that ex-ante are likely considered similar, and provides another confirmation on the Gatev, Goetzmann and Rouwenhorst (2006) algorithm and this paper's implementation of it. The subsequent analysis in this section uses only pairs of common stocks.

To examine Hypothesis 1, this paper starts by running the following linear regression,

$$PAIRCRASH = a + b \cdot PAIRLAGRET + \varepsilon. \quad (1)$$

PAIRLAGRET is the difference between each pair's cumulative returns from pair formation to before the crash. Each pair is ordered so that $PAIRLAGRET \geq 0$. PAIRCRASH is the difference between a pair's crash sizes. Crash size is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010. The object of interest is the coefficient in front of PAIRLAGRET. The bigger PAIRLAGRET is, the less attractive the stock with better past return is to a relative-value buyer. Hypothesis 1 predicts a negative coefficient for PAIRLAGRET.

Table 3 shows the regression results separately for the five pairs-formation periods. For the formation period ending in Dec 2009, the estimated coefficient for PAIRLAGRET is -0.024 (t-stat=2.42). The coefficient of PAIRLAGRET for other four formation periods is similar and statistically significant. At the estimate -0.024 , every 10% increase in PAIRLAGRET is associated with -0.24% worse PAIRCRASH, which is 1.2% of the 5th-to-95th quantile range of PAIRCRASH observed on May 6, 2010 (Similar to Figure 2, this paper computes that the 5th-to-95th quantile range of PAIRCRASH is 20.7% for the pairs formed at the end of Dec 2009). The 5th-to-95th quantile range of PAIRLAGRET between the pairs formation in Dec 2009 and the flash crash is 84%. This, at the estimate -0.024 , is associated with a -2.0% worse

PAIRCRASH, which is 10% of the 5th-to-95th quantile range of PAIRCRASH observed on May 6, 2010. The regression results support Hypothesis 1.

More importantly, the effect of lagged return on crash size is stronger for extreme crashes. Ex ante, one might conjecture that small shocks (such as tick-size fluctuations) might be absorbed by other liquidity providers, while pairs traders might be attracted to big price fluctuations that cross certain thresholds. To begin exploring the effect of PAIRLAGRET on the distribution of PAIRCRASH, Figure 5 shows the 5th, 10th, 25th, 50th, 75th, 90th, 95th quantiles of PAIRCRASH separately for three groups of pairs: pairs whose PAIRLAGRET is no more than 0.2, between 0.2 and 0.4, and above 0.4. These pairs are formed at the end of 2009. For those pairs whose PAIRLAGRET is above 0.4, the stock with better past return appears 40% overvalued (relative to the other stock in the pair) and is unattractive to a relative-value buyer without a big crash. For such pairs, the 5th quantile of PAIRCRASH is -0.18 , i.e. the stock with better year-to-date return crashed 18% more than the other stock in the pair. In contrast, for those pairs whose PAIRLAGRET is less than 0.2, the 5th quantile of PAIRCRASH is -0.10 . The difference in the 5th quantile extreme crash is -8% . At the 10th, 25th, and 50th (median) quantiles, the crash size difference between the two groups (PAIRLAGRET <0.2 , and PAIRLAGRET ≥ 0.4) is -5% , -3% , and -2% , respectively. In contrast, past return does not affect smaller crash sizes. For example, the 90th and 95th quantiles of PAIRCRASH differ by 0 and 1% between the two groups, respectively. The plot is similar when other cutoffs are used to group pairs.

More formally, I study the effect of lagged return on the distribution of crash sizes using quantile regressions, which analyzes the quantiles of crash size conditioning on PAIRLAGRET. More specifically, the q -th quantile regression for $0 < q < 1$ solves

$$\min_{\beta} \left[q \sum_{i \in \{i: y_i \geq x'_i \beta\}} |y_i - x'_i \beta| + (1 - q) \sum_{i \in \{i: y_i < x'_i \beta\}} |y_i - x'_i \beta| \right] \quad (2)$$

Here in (2), variable y is PAIRCASH and vector x includes PAIRLAGRET and a constant.⁴ Figure 6 shows the estimated slope coefficient and the 95% confidence interval of PAIRLAGRET in 91 quantile regressions, ranging from the 5th to the 95th quantiles of PAIRCASH, for pairs formed at the end of 2009.⁵ The results from quantile regressions confirm the finding in Figure 5. For example, the estimated slope coefficient of PAIRLAGRET in the 5th quantile regression for PAIRCASH is -0.078 . This implies that every 10% increase in PAIRLAGRET is associated with -0.78% additional PAIRCASH, more than three times the average effect of PAIRLAGRET on PAIRCASH estimated in (1). The 5th-to-95th quantile range of PAIRLAGRET between the pairs formation in Dec 2009 and the flash crash is 84%. This, at the estimate -0.078 , is associated with a -6.6% worse PAIRCASH, which is 32% of the 5th-to-95th quantile range of PAIRCASH observed on May 6, 2010.

Figure 7 repeats the quantile regression (2) separately for other pairs-formation periods. The results for the other formation periods are similar. Specifically, Figure 7 shows the estimated slope coefficient of PAIRLAGRET in the 5th, 10th, 25th, and 50th (median) quantile regressions for PAIRCASH. All the estimates are statistically significant at the 95% level. The estimates are more negative for more extreme quantiles across the pairs-formation periods. This shows that a lack of interest from relative-value buyers (higher PAIRLAGRET) is associated with more extreme crash sizes, consistent with Hypothesis 1.

5 Contrarian trading in the flash crash

⁴ In comparison, the ordinary least squares regression in (1) solves $\min_{\beta} \sum_{i=1}^n (y_i - x_i' \beta)^2$. The median regression (for the 50th quantile) solves $\min_{\beta} \sum_{i=1}^n |y_i - x_i' \beta|$, which is a special case of regression (2).

⁵ Statistical software such as SAS by default suppresses reporting of the standard errors in plots for those quantiles below the 5th and above the 95th, because of the noise associated with such extreme quantiles. This paper has verified that estimates for these extreme quantiles exhibit big oscillations and are not statistically significant.

5.1 *Cracking a proprietary algorithm's black box*

There are many different proprietary trading strategies other than the pairs-trading strategy. To shed some light into these black boxes, this paper uses a simple model below to study how the mispricing perceived by a black box is related to past return. Suppose the past return r of a stock relates to the change f in the stock's fundamental according to

$$r = a \cdot f$$

for some $a \geq 0$. If $a = 1$, the return accurately reflects the change in fundamental. A proprietary black box may not believe that the change in fundamental is accurately reflected in the price at all times.

Assuming the black box observes a noisy signal s of the change in fundamental according to

$$s = b \cdot f + \varepsilon.$$

Therefore, the black box perceives the stock return overshoots the change in fundamental by $r - s$. A researcher who does not observe the proprietary signal and observe only past return can nonetheless estimate the black box's perception because

$$r - s = \frac{a - b}{a} \cdot r - \varepsilon.$$

If $a > b$, high past return implies that the black box likely perceives that the stock return is too high relative to change in fundamental. I.e. the assumption of $a > b$ implies that the black box is contrarian.

Therefore, the black box will likely perceive stocks with better past returns as more overvalued and unwilling to buy them without a bigger price correction (assuming the black box has not shorted the stocks perceived as overvalued). The condition $a > b$ likely captures the behavior of a large number of investors, such as investors who compute fundamental from accounting variables (e.g. Daniel and Titman (2006) on market-to-book), from averaging or smoothing (e.g. Brock, Lakonishok and Lebaron (1992), Zhu and Zhou (2009) on moving averages, Campbell and Shiller (1988) on smoothed price-

earnings multiples). On the other hand, if the black box has already shorted a stock with better past return, it will also require a bigger price correction to buy and cover the short position assuming the profit target is positively correlated with the perceived overvaluation. I.e. if the black box aims to make a bigger profit if the perceived overvaluation is bigger at the time the short position is entered.

Hypothesis 2. The crash size of a stock is negatively correlated to its return before the flash crash.

5.2 Empirical results

This section runs the quantile regression specification (2), except that here the dependent variable is CRASH and the independent variables include LAGRET and a constant. CRASH for each stock is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010. LAGRET is a stock's past return. Figure 8 shows the quantile regression results where LAGRET for each individual stock is measured by its year-to-date cumulative return in 2010 before the flash crash. Stocks with high LAGRET indeed tend to have more severe CRASH, and the effect is similarly stronger for the extreme crash sizes. For example, the estimated slope coefficient of LAGRET is -0.061 (95% confidence interval: -0.102 to -0.051) in the 5th quantile regression for CRASH. I.e. every 10% increase in past return is associated with -0.61% additional crash size. The 5th-to-95th quantile range of LAGRET is 99%. This, at the estimate -0.061 , is associated with a -6.1% additional CRASH, which is 42% of the 5th-to-95th quantile range of CRASH observed on May 6, 2010.

Figure 9 further shows that the relation between CRASH and LAGRET remains similar and statistically significant when different horizons are used to measure LAGRET (from past one month up to about 30 months), providing further support for Hypothesis 2.

6 Alternative hypotheses

6.1 Risk

The difference in crash size across stocks may be due to their difference in loadings of risk factors such as the market. To account for such difference, each stock's market beta is estimated according to Fama and French (2004) using two to five years (as available) of monthly returns prior to May 2010. The difference in each pair's beta is included as an additional explanatory variable in the quantile regressions of PAIRCRASH on PAIRLAGRET. The effect of PAIRLAGRET on PAIRCRASH remains similar to those in Section 4.3. Similarly, the effect of LAGRET on CRASH in Section 5.2 remains similar when each stock's beta is included as an additional explanatory variable in the quantile regressions. As an additional robustness check against time-varying beta, the market beta is estimated according to Lewellen and Nagel (2006) using daily data in the quarter before the flash crash and the results are still similar. In addition, this paper has controlled betas for the size and value factors in Fama and French (1993), a momentum factor based on the return 2-12 months ago (Jegadeesh and Titman (1993), Carhart (1997)), and the short-term reversal factor based on the previous month return (Jegadeesh (1990)). The momentum and short-term reversal factors are constructed in the same way as the Fama and French (1993) size and value factors. The quantile regression results in Section 4.3 and 5.2 remain similar after controlling betas of these factors. These results are suppressed for brevity.

6.2 Index arbitrage

An index arbitrageur trades an index (such as an index futures contract) and the component stocks of the index, in order to capture potential price discrepancies. On May 6, 2010, the S&P 500 index e-mini futures reached an intraday low at 2:45pm. For individual stocks, the median (average) time of intraday low is 2:48pm (3:04pm) from the Trade and Quote (TAQ) database. This suggests that an index arbitrageur is likely considering selling individual stocks at around 2:45pm. To account for the selling of index arbitrageurs, this paper includes the variable SIZE as an additional explanatory variable in the quantile regressions in Section 5.2, where SIZE is each stock's market capitalization (in millions USD) at

the end of Apr 2010. Because S&P 500 is a value-weighted index, the selling of individual stocks by index arbitrageurs is likely proportional to the market capitalization of the stocks. The effect of LAGRET on CRASH remains similar. In addition, this paper also includes the variable PAIRSIZE as an additional explanatory variable in the quantile regressions in Section 4.3, where PAIRSIZE is the difference between SIZE for the two stocks in a pair. The effect of PAIRLAGRET on PAIRCRAH remains similar. The results are also similar if the logarithm of market capitalization is controlled instead.

6.3 Heterogeneity in pairs

Section 4.3 studies the relative crash sizes of stock pairs. However, some pairs are more similar than others. To control the heterogeneity in pairs, this paper includes the variable PAIRDEVIATION as an additional explanatory variable in the quantile regressions, where PAIRDEVIATION is the root mean squared price deviation in the pair-formation period (beginning prices of both stocks are normalized to 1). The effect of PAIRLAGRET on PAIRCRAH remains similar. The result is also similar if controlling the rank order of PAIRDEVIATION instead of the value of PAIRDEVIATION. This paper has also separated the stock pairs into two halves based on the PAIRDEVIATION and run the quantile regressions of PAIRLAGRET on PAIRCRAH separately for each half of the pairs. The results are similar and statistically significant at the 95% level. The estimated coefficient of PAIRLAGRET is not statistically significantly different across the two halves, though the magnitude is somewhat bigger for those pairs that are more dissimilar.

6.4 Volatility and higher-order moments

Section 5.2 studies the crash sizes of individual stocks, which can be affected by the stock volatility. This paper includes the variable VOLATILITY as an additional explanatory variable in the quantile regressions, where VOLATILITY is a stock's daily return volatility in Apr 2010. The effect of LAGRET on

CRASH remains similar. This paper has also controlled skewness, kurtosis, and the 5th, 10th, 15th, ..., 95th quantiles of a stock's daily return in Apr 2010. The results are similar.

6.5 Controlling for the alternative hypotheses together

Figure 10 repeats the quantile regressions in Section 4.3, controlling for the alternative hypotheses in Section 6.1 to 6.4 together. Specifically, it shows the quantile regression results of PAIRCASH on PAIRLAGRET, controlling for PAIRBETA, PAIRSIZE, and PAIRDEVIATION for pairs formed at the end of 2009. PAIRBETA in this figure is the difference in market beta of each pair, where market beta is estimated according to Fama and French (2004) using two to five years (as available) of monthly returns prior to May 2010. The effect of PAIRLAGRET on PAIRCASH is similar to Figure 5. PAIRBETA overall has a negative coefficient, suggesting high beta stocks tend to crash more. PAIRSIZE, the proxy for index arbitrage, has a negative coefficient. This is supportive of the hypothesis that index arbitrage has an effect on individual stock crashes. PAIRDEVIATION has a negative coefficient for the left tail of PAIRCASH, while it has a positive coefficient for the right tail of PAIRCASH. This suggests that for pairs that are more dissimilar, the stock with better past performance in the pair is equally likely to have bigger or smaller crash sizes than the other stock in the pair. The results are similar for other pairs-formation periods.

Figure 11 repeats the quantile regressions in Section 5.2, controlling for the alternative hypotheses in Section 6.1 to 6.4 together. Specifically, it shows the quantile regression results of CRASH on LAGRET, controlling for BETA, SIZE, and VOLATILITY, where LAGRET is each stock's year-to-date return in 2010 before the flash crash. BETA in this figure is the market beta of each stock, where market beta is estimated according to Fama and French (2004) using two to five years (as available) of monthly returns prior to May 2010. The effect of LAGRET on CRASH is similar to Figure 6. BETA has a negative coefficient, suggesting high beta stocks tend to crash more. SIZE, the proxy for index arbitrage, has a negative

coefficient. This is again supportive of the hypothesis that index arbitrage has an effect on individual stock crashes. Overall, the effect of SIZE is similar for different quantiles of CRASH. Therefore, index arbitrage does not appear particularly associated with extreme crashes. VOLATILITY has a negative coefficient for the left tail of CRASH, while it has a positive coefficient for the right tail of CRASH. This suggests that a more volatile stock is equally likely to have a bigger or a smaller crash size than a typical stock.

6.6 Statistical issues

Quantile regression results are similar from two different methods to compute the confidence interval, including the rank method in Gutenbrunner and Jureckova (1992) and the bootstrap method in He and Hu (2002).

To test the presence of nonlinearity, this paper includes a quadratic term of PAIRLAGRET in the regressions in Section 4.3, and includes a quadratic term of LAGRET in the regressions in Section 5.2. The quadratic terms are not statistically significant at the 95% level.

6.7 Stub quotes

The result is similar if trades at stub quotes (\$0.01) are excluded.

7 Is contrarian trading stabilizing? A glass half full.

Is contrarian trading stabilizing? The answer from results in this paper is a glass half full. Interestingly, the finding of bigger crash size for those stocks less likely to attract contrarian buyers in fact points to the importance of contrarian investors in stabilizing stocks. However, the glass is half full because the analysis simultaneously finds a group of stocks (the previously high flying ones) a typical contrarian buyer is unlikely interested in, even with big crash sizes. This imposes a limit on the extent of price stability that relies solely on this and other similar types of trading strategies. These results suggest that

more contrarian trading (both more in terms of diversity of strategies than in terms of total asset under management) might mitigate extreme liquidity shocks. On the other hand, there are limits on the diversity of strategies. For example, data snooping concerns might preclude an algorithm to trade too aggressively on strategies involving too many conditioning variables --- such as those stocks that had stellar past returns but poor recent returns. A number of the known strategies tend to focus on monotonic patterns in expected return (e.g. the monotonic increase in expected return for book-to-market sorted portfolios, as in Fama and French (1993)).

8 Risk premium implications

If investors dislike negative skewness, such risk will command a risk premium, see Kraus and Litzenberger (1976) for more details. Harvey and Siddique (2000) show systematic skewness is priced, and exposure to it (coskewness) helps explain cross-sectional expected stock return such as the medium-term momentum.

This paper next examine whether the finding that stocks with better past returns crash more during the flash crash has any implication on coskewness by testing the following hypothesis.

Hypothesis 3. Stocks with higher past returns have more negative coskewness with the market.

Specifically, this paper runs monthly Fama-Macbeth regression (Fama and Macbeth (1973)). In each month, a cross-sectional regression regresses each stock's coskewness with the market (computed according to equation (11) in Harvey and Siddique (2000) using daily data in the month) on past returns.

Figure 12 shows the time series of the coefficient for past return in the monthly cross-sectional regression of coskewness on past six-month return. Most of the estimates are statistically significantly negative. Higher past return is associated with more negative coskewness in almost every month since the 1960s. The Fama-Macbeth regression measures the effect of past return on coskewness using the

average of the monthly cross-sectional regression coefficients. Figure 13 shows the Fama-Macbeth regression results of coskewness on past return, for past return horizons ranging from one month to thirty-six months. The coefficient of past six-month return is -0.37 . A 100% higher past return implies the coskewness is -0.37 more negative. This is a big magnitude because the standardized coskewness coefficient in Harvey and Siddique (2000) tends to be between -1 and 1 . The effect of past return on coskewness is also statistically significant for all the other return horizons up to three years. In addition, the effect remains statistically significant when controlling two monthly lags of coskewness as in Harvey and Siddique (2000). Higher past return also implies more negative skewness, as shown in Figure 13.

The finding that higher past return is associated with more negative coskewness lends additional support to the finding in Harvey and Siddique (2000) based on monthly data that the medium-term momentum is related to coskewness. On the other hand, the finding in this paper shows the effect applies to horizons as short as one month and as long as three years, too. At these horizons, the literature has documented a reversal effect, i.e. stocks with lower past returns at these return horizons tend to have higher subsequent returns (Jegadeesh (1990), Debondt and Thaler (1985)). The finding in this paper makes such short-term and long-term reversal findings more puzzling, since the stocks with low past returns will have more positive coskewness and skewness in addition to higher expected returns going forward.

9 Conclusion

This paper points out that the individual stocks' crash sizes on May 6, 2010 are largely unexplained by market exposure. Contrarian trading is shown to mitigate individual-stock crash because stocks that are unattractive to contrarian buyers (stocks with higher past returns) crash more. On the other hand, the glass is half full because the study simultaneously finds a group of stocks a contrarian buyer is unlikely interested in stabilizing, even with big crash sizes. More work is needed to better understand

the effect of risky arbitrage strategies on asset price fluctuations. Some of the findings from the flash crash extend to other time periods. Stocks with better past returns exhibit more negative co-skewness, which holds in almost every month since the 1960s and for past return horizons ranging from one month to three years. This has interesting implications for risk premia associated with short-term reversal, medium-term momentum, and long-run reversal portfolios.

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Table 1. Extreme crash sizes, by beta- or characteristics-sorted portfolios

This table shows the 5th quantile and the median of crash size for various beta- or characteristics-sorted portfolios. Crash size is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010. For each beta or characteristics, the stocks are sorted into quintiles (quintile 5 has the highest beta or characteristics). MOM (a-b) denotes cumulative past return from month t-b to t-a inclusive, where t is May 2010. The three trading days in May 2010 before the flash crash is included in MOM(a-b). The various monthly betas are computed in the same way as described on Ken French's website. Daily market beta is computed according to Lewellen and Nagel (2006) using daily data in Feb-Apr 2010. Turnover, volatility (from daily stock return), and size (market capitalization) are based on data from Apr 2010. Coskewness is estimated according to Harvey and Siddique (2000). The sample includes all commons stocks in the US market. Those sorts where the 5-1 difference is statistically significant at the 95% level are highlighted in bold.

	5th quantile crash size						Median crash size					
	1 (low)	2	3	4	5 (high)	5-1	1 (low)	2	3	4	5 (high)	5-1
MOM (2-12)	-12.77	-10.63	-12.49	-17.16	-22.3	-9.53	-3.97	-4.61	-4.64	-5.52	-5.49	-1.52
MOM (1-6)	-12.61	-12.49	-11.79	-16.14	-20.84	-8.23	-3.85	-4.58	-5.06	-5.06	-5.51	-1.66
MOM (1-36)	-14.1	-14.39	-11.33	-13.39	-20.84	-6.74	-4.5	-4.89	-4.79	-4.92	-5	-0.5
β_{Market}	-12.57	-13.78	-13.7	-14.08	-18.93	-6.36	-3.65	-4.31	-5.04	-5.26	-5.76	-2.11
Turnover	-10.4	-15.55	-15.57	-14.51	-15.67	-5.27	-1.97	-3.94	-5	-5.44	-5.6	-3.63
Volatility	-14.8	-11.21	-12.38	-15.08	-16.94	-2.14	-4.57	-4.61	-4.69	-4.83	-5.2	-0.63
MOM (1-1)	-14.46	-14.83	-14.41	-12.87	-16.34	-1.88	-4.38	-4.45	-4.74	-4.73	-5.25	-0.87
Coskewness	-12.61	-13.94	-16.28	-13.29	-14.2	-1.59	-4.45	-4.55	-4.89	-5	-5.18	-0.73
β_{UMD}	-15.14	-13.78	-13.92	-12.5	-16.58	-1.44	-4.94	-4.92	-4.63	-4.73	-4.74	0.2
β_{Market} (daily)	-15.78	-13.89	-14.08	-13.45	-16.58	-0.8	-3.7	-4.44	-4.99	-4.97	-5.52	-1.82
B/M	-15.78	-15.04	-12.89	-13.71	-15.57	0.21	-4.93	-4.92	-4.62	-4.65	-4.42	0.51
$\beta_{\text{Reversal (1-1)}}$	-16.58	-14.01	-12.49	-12.7	-16.25	0.33	-5.35	-4.78	-4.59	-4.61	-4.66	0.69
β_{SMB}	-16.25	-12.5	-14.38	-14.87	-14.56	1.69	-4.58	-4.73	-4.5	-4.83	-5.34	-0.76
β_{HML}	-16.18	-13.61	-13.7	-15.11	-14.44	1.74	-4.91	-4.71	-4.76	-4.33	-5.02	-0.11
Size	-15.55	-15.33	-11.41	-11.71	-12.7	2.85	-3.79	-5.19	-4.93	-5.41	-5.85	-2.06

Table 2. Pairs formed on Dec 31, 2009

This table shows the pairs formed using the algorithm in Gatev, Goetzmann and Rouwenhorst (2006), based on CRSP daily U.S. stock return data from Jan 1, 2009 to Dec 31, 2009.

Rank	Stock 1	Stock 2
1	SPDR Gold	iShares Gold
...		
4	SPDR S&P 500	iShares S&P 500
...		
27	Bio-Rad class A	Bio-Rad class B
...		
188	Scana (SCG, energy)	Westar (WR, energy)
...		
2857	Diedrich coffee	Vanda pharmaceuticals

Table 3. Average effect of pair lagged relative return on relative crash size

This table reports the regression results of the relative crash size of a pair (PAIRCRASH) on the lagged relative return of the same pair (PAIRLAGRET), using all pairs formed at the same formation period. PAIRLAGRET is the difference between each pair's cumulative returns from pair formation to before the crash. Crash size is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010. PAIRCRASH is the difference between a pair's crash sizes. Each pair is ordered so that PAIRLAGRET \geq 0. Five regressions are run, one for each of the five one-year formation periods.

Pair formation period end	Intercept	t	PAIRLAGRET	t	Adjusted R ²
Mar 31, 2010	-0.002	0.51	-0.039	2.50	0.003
Feb 28, 2010	0.004	0.80	-0.052	4.03	0.013
Jan 31, 2010	-0.002	0.51	-0.031	2.90	0.005
Dec 31, 2009	-0.001	0.19	-0.024	2.42	0.005
Nov 30, 2009	0.002	0.57	-0.028	3.28	0.007

Figure 1. Crash and rebound in the cross section

This figure shows the average crash size and average rebound size for portfolios sorted on crash size on May 6, 2010. Crash size is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010. Rebound size is measured by the minimum transaction price after 2:30pm ET relative to the closing price on May 6, 2010.

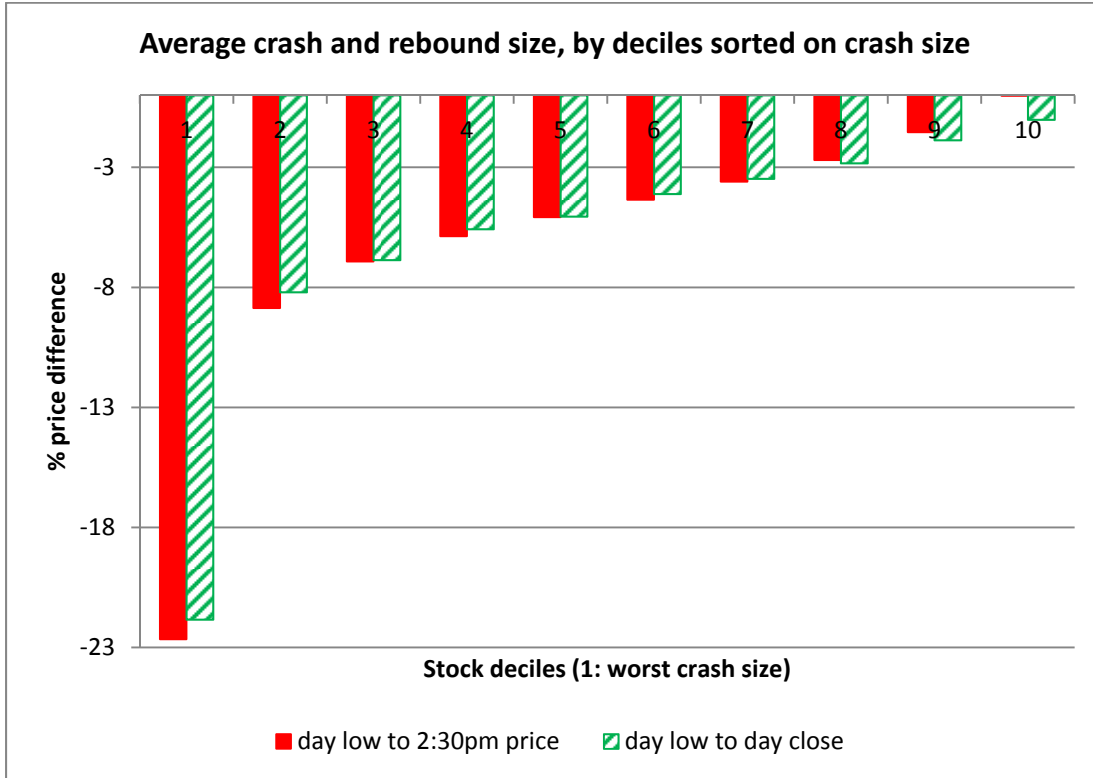


Figure 2. Distribution of individual-stock crash size

This figure shows the 5th to 95th quantiles of individual-stock crash size and individual-stock beta-adjusted crash size during the flash crash. Crash size is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010. Beta-adjusted crash size = crash size – market beta × S&P500 E-mini crash size. The S&P500 E-mini crash size is –6.59%. Each stock’s market beta is estimated according to Fama and French (2004) using two to five years (as available) of monthly returns prior to May 2010.

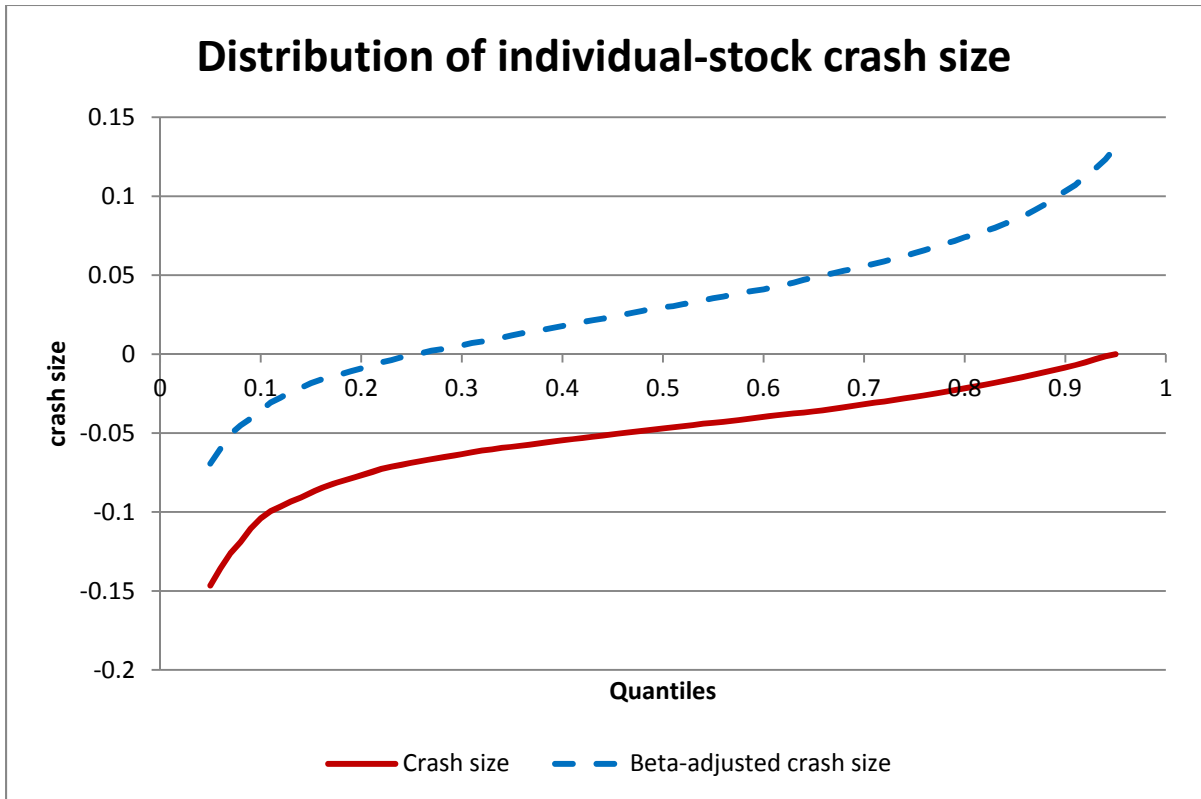


Figure 3. Crash size and past six-month return

This figure shows the 5th, 10th, 25th, 50th, 75th, 90th, and 95th quantile crash size separately for stocks sorted by past six-month returns from Nov 2009 to Apr 2010 (including the three trading days in May 2010 prior to the flash crash). Crash size is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010.

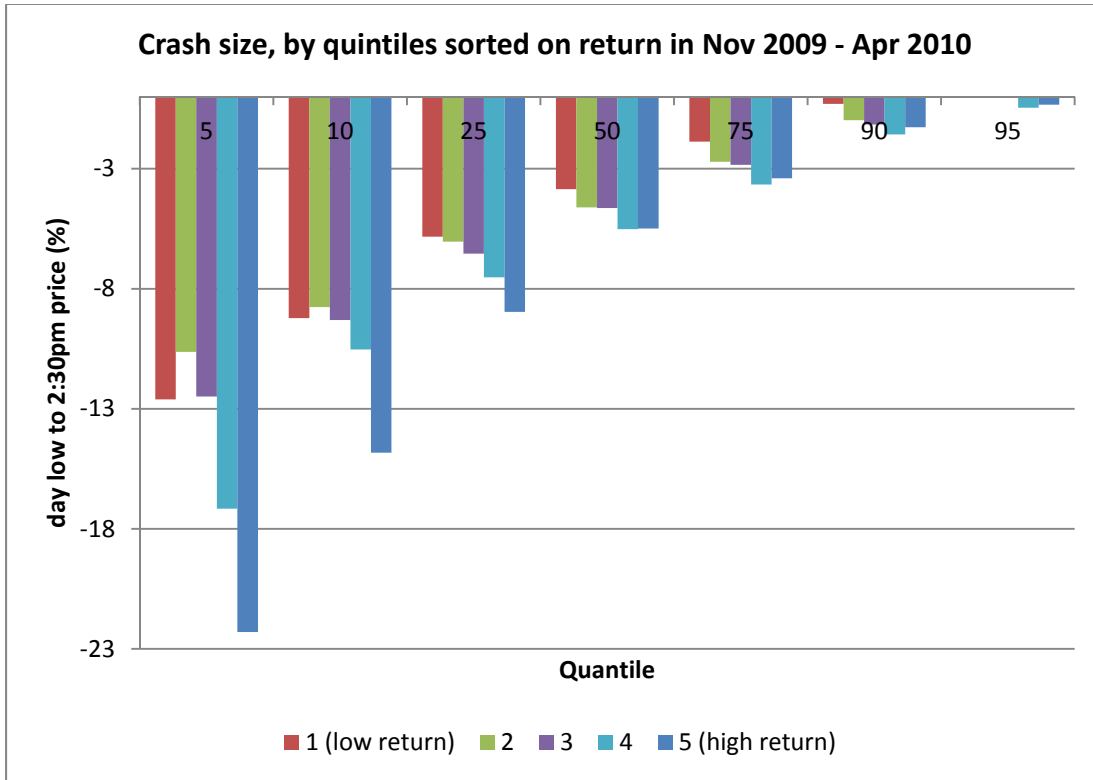


Figure 4. Example of pairs trading

This figure is from Figure 1 in Gatev, Goetzmann and Rouwenhorst (2006) and illustrates pairs trading for a stock pair Kennecot and Uniroyal. The figure shows the daily prices of the two stocks in the trading period August 1963 to January 1964. The beginning stock prices are normalized to 1 for both stocks in the trading period.

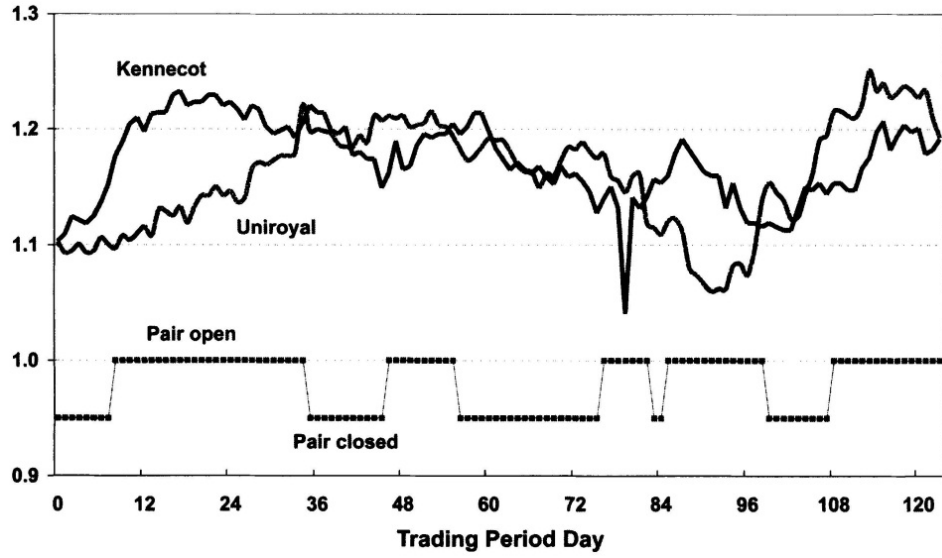


Figure 5. Pair relative lagged return on the distribution of relative crash size, sort

This figure shows the 5th, 10th, 25th, 50th, 75th, 90th, and 95th quantile relative crash size (PAIRCRASH) separately for stock pairs sorted by PAIRLAGRET less than 0.2, between 0.2 and 0.4, and above 0.4. PAIRLAGRET is the difference between each pair's cumulative returns from pair formation to before the crash. Crash size is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010. PAIRCRASH is the difference between a pair's crash sizes. Each pair is ordered so that PAIRLAGRET ≥ 0. The pairs formation period in this figure is Jan 1, 2009 to Dec 31, 2009.

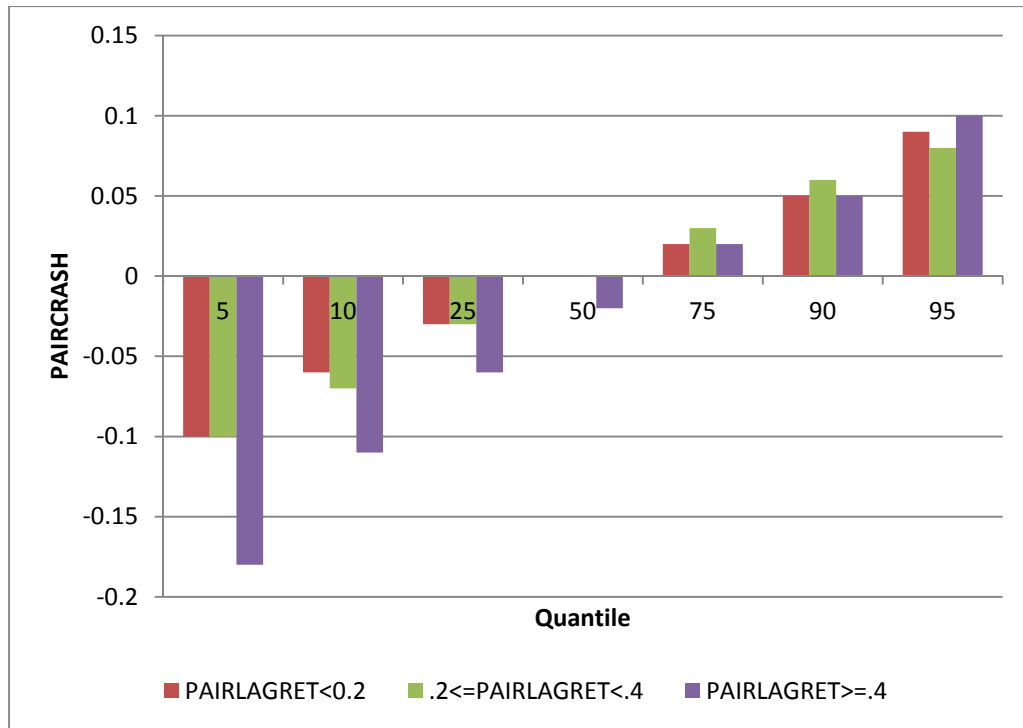


Figure 6. Pair lagged relative return on the distribution of relative crash size, quantile regressions

This figure reports the results of 91 separate quantile regressions. For each quantile $q=5, 6, \dots, 95$, the q -th quantile of relative crash size (PAIRCRASH) is regressed on a constant and PAIRLAGRET. PAIRLAGRET is the difference between each pair's cumulative returns from pair formation to before the crash. Crash size is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010. PAIRCRASH is the difference between a pair's crash sizes. Each pair is ordered so that PAIRLAGRET ≥ 0 . The pairs formation period in this figure is Jan 1, 2009 to Dec 31, 2009. The figure shows the estimated slope coefficient of PAIRLAGRET for each quantile, along with the 95% confidence interval computed from the rank method in Gutenbrunner and Jureckova (1992).

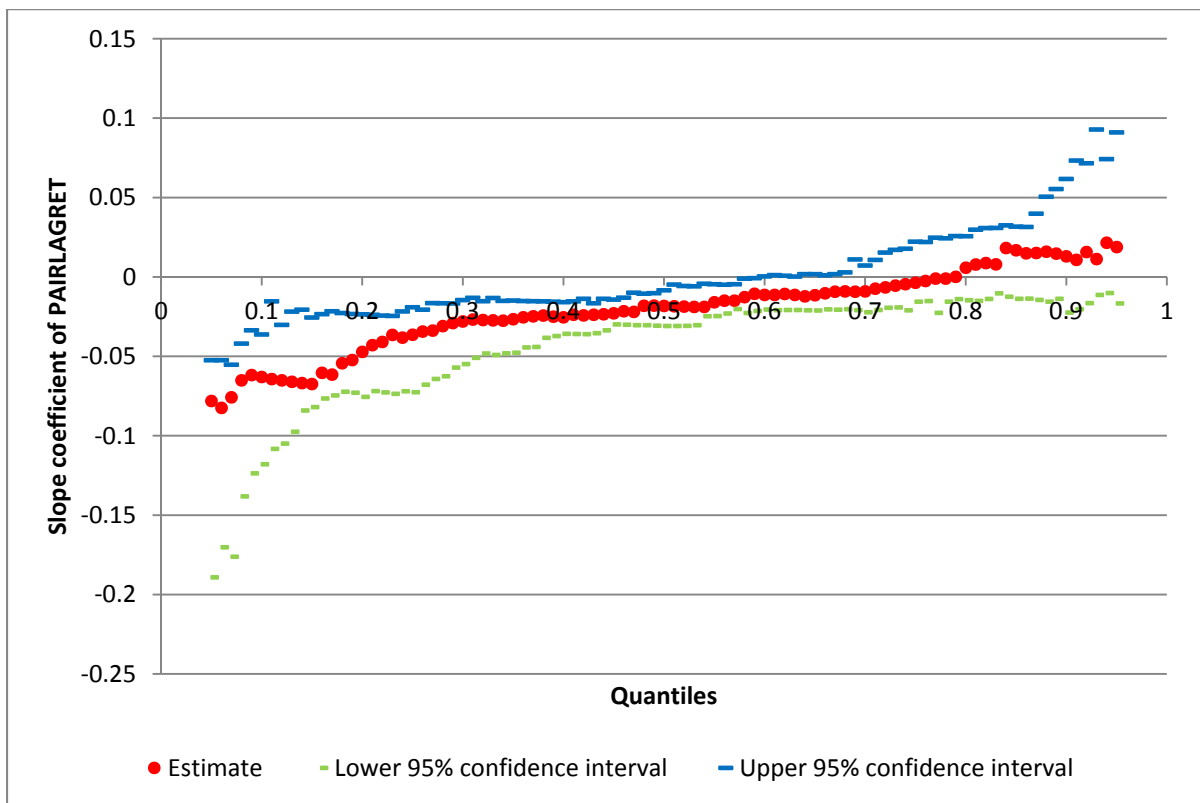


Figure 7. Pair lagged relative return on the distribution of relative crash size, by formation periods

This figure repeats the quantile regressions in Figure 6 for different pairs formation periods. The formation-period end ranges from Mar 31, 2010 (one month before flash crash) to Nov 30, 2009 (five months before flash crash). For each formation period, the figure shows the estimated slope coefficient of PAIRLAGRET for the 5th, 10th, 25th, and median relative crash size. All the estimates are statistically significant at the 95% level according to Gutenbrunner and Jureckova (1992).

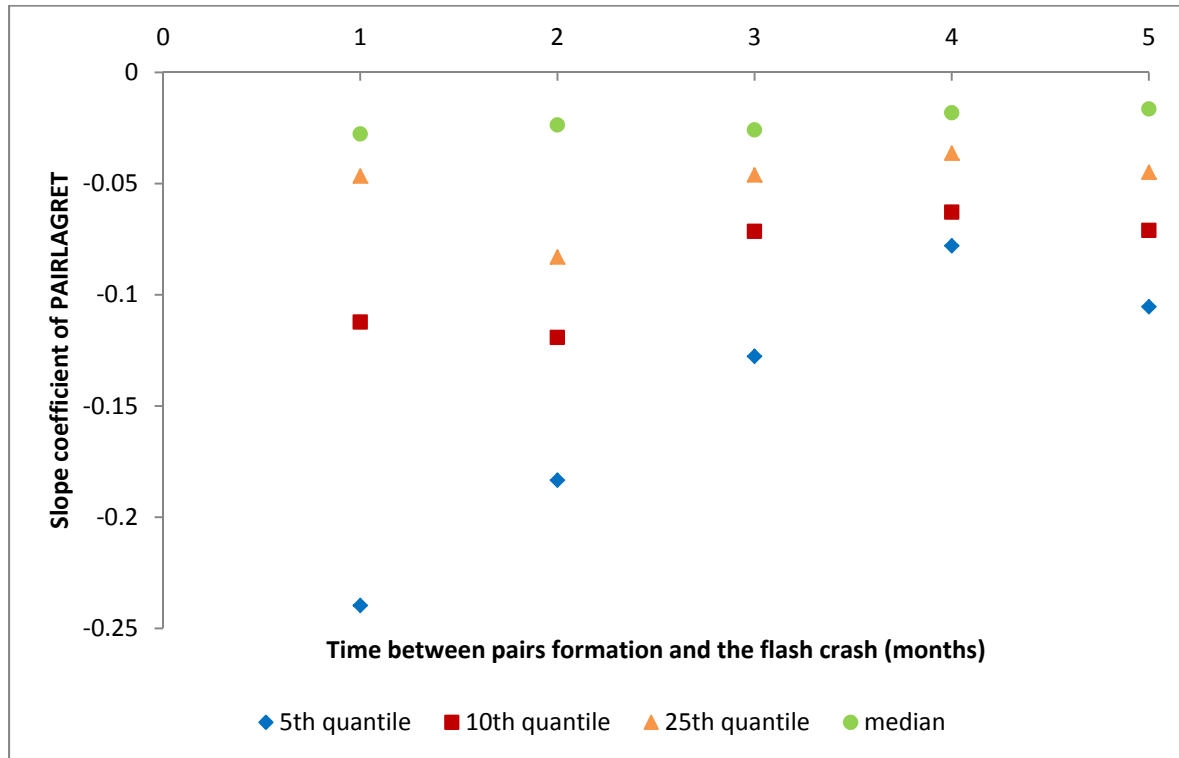


Figure 8. Effect of lagged stock return on the distribution of crash size, quantile regressions

This figure reports the results of 91 separate quantile regressions. For each quantile $q=5, 6, \dots, 95$, the q -th quantile of crash size (CRASH) is regressed on a constant and LAGRET. LAGRET for each individual stock is measured by its year-to-date cumulative return in 2010 before the flash crash. CRASH is measured by the minimum transaction price after 2:30pm ET relative to the last price before 2:30pm on May 6, 2010. The figure shows the estimated slope coefficient of LAGRET for each quantile, along with the 95% confidence interval computed from the bootstrap method in He and Hu (2002).

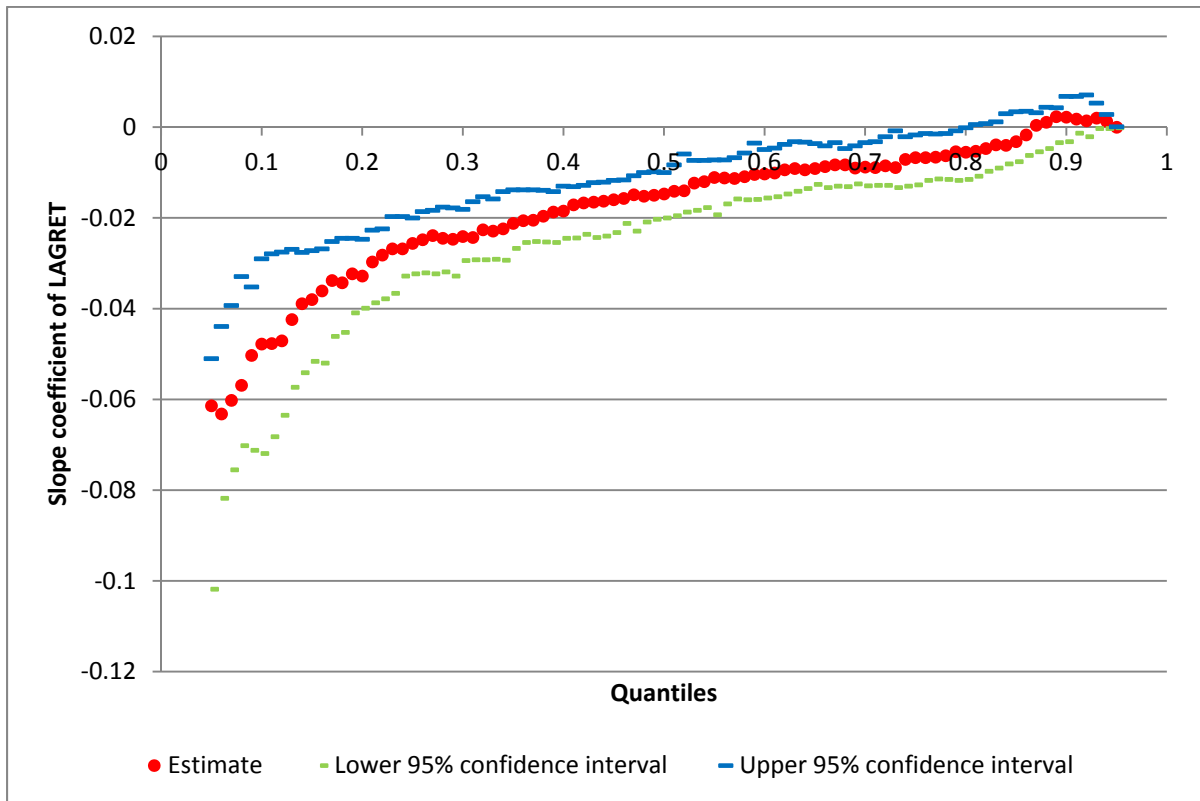


Figure 9. Effect of lagged stock return on the distribution of crash size, by lagged return horizon

This figure repeats the quantile regressions in Figure 8, varying the horizon of LAGRET. Specifically, LAGRET for each individual stock is measured by its past one month return (from Apr 1, 2010 to the day before flash crash), past two month return (from Mar 1, 2010 to the day before flash crash), ..., up to past three-year return, respectively. For each return horizon, this figure shows the estimated slope coefficient of LAGRET for the 5th, 10th, 25th, and median CRASH. Statistical significance at the 95% level according to He and Hu (2002) are indicated by a solidly filled marker. An unfilled marker indicates an insignificant estimate.

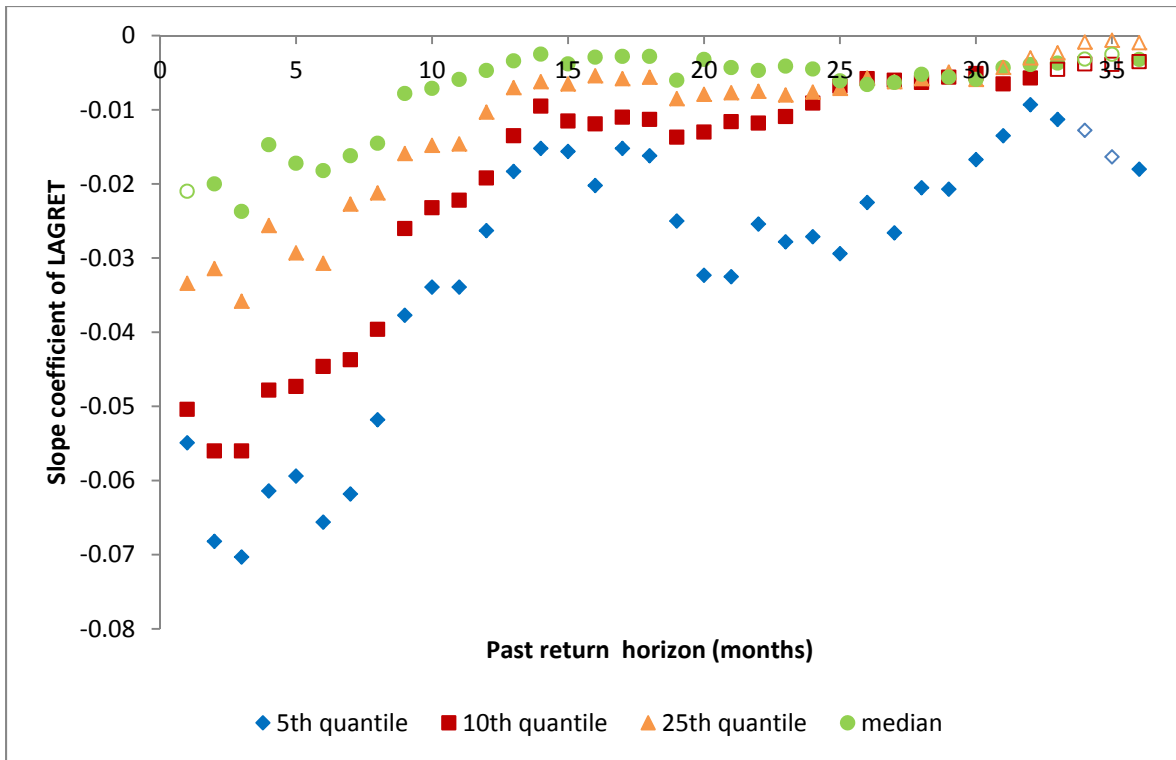


Figure 10. Controlling alternative hypotheses, for pairs

This figure repeats the quantile regressions of PAIRCASH on PAIRLAGRET in Figure 6, controlling for PAIRBETA, PAIRSIZE, and PAIRDEVIATION. PAIRBETA is the difference in market beta, estimated as in Fama and French (2004), between a stock pair. PAIRSIZE is the pair's difference in market capitalization (in millions USD), at the end of Apr 2010. PAIRDEVIATION is the root mean squared price deviation in the pair formation period (beginning price normalized to 1). Each pair is ordered so that PAIRLAGRET ≥ 0 . The figure shows the estimated slope coefficient of PAIRLAGRET and the other three control variables for each quantile, along with the 95% confidence interval computed from the rank method in Gutenbrunner and Jureckova (1992).

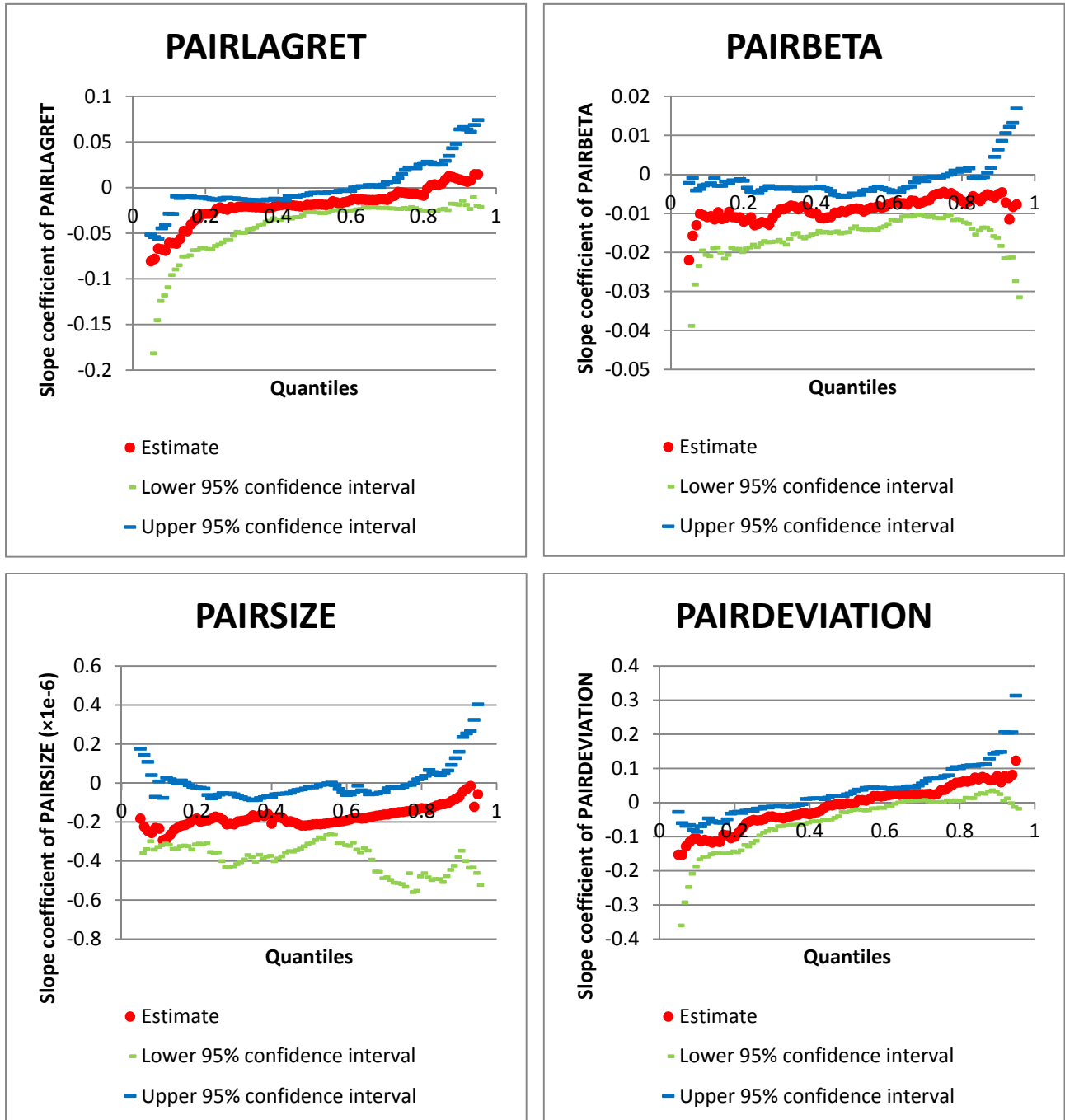


Figure 11. Controlling alternative hypotheses, for individual stocks

This figure repeats the quantile regressions of CRASH on LAGRET in Figure 8, controlling for BETA, SIZE, and VOLATILITY. BETA is a stock's market beta, estimated as in Fama and French (2004). SIZE the market capitalization (in millions USD) at the end of Apr 2010. VOLATILITY is a stock's daily return volatility in Apr 2010. The figure shows the estimated slope coefficient of LAGRET and the other three control variables for each quantile, along with the 95% confidence interval computed from the bootstrap method in He and Hu (2002).

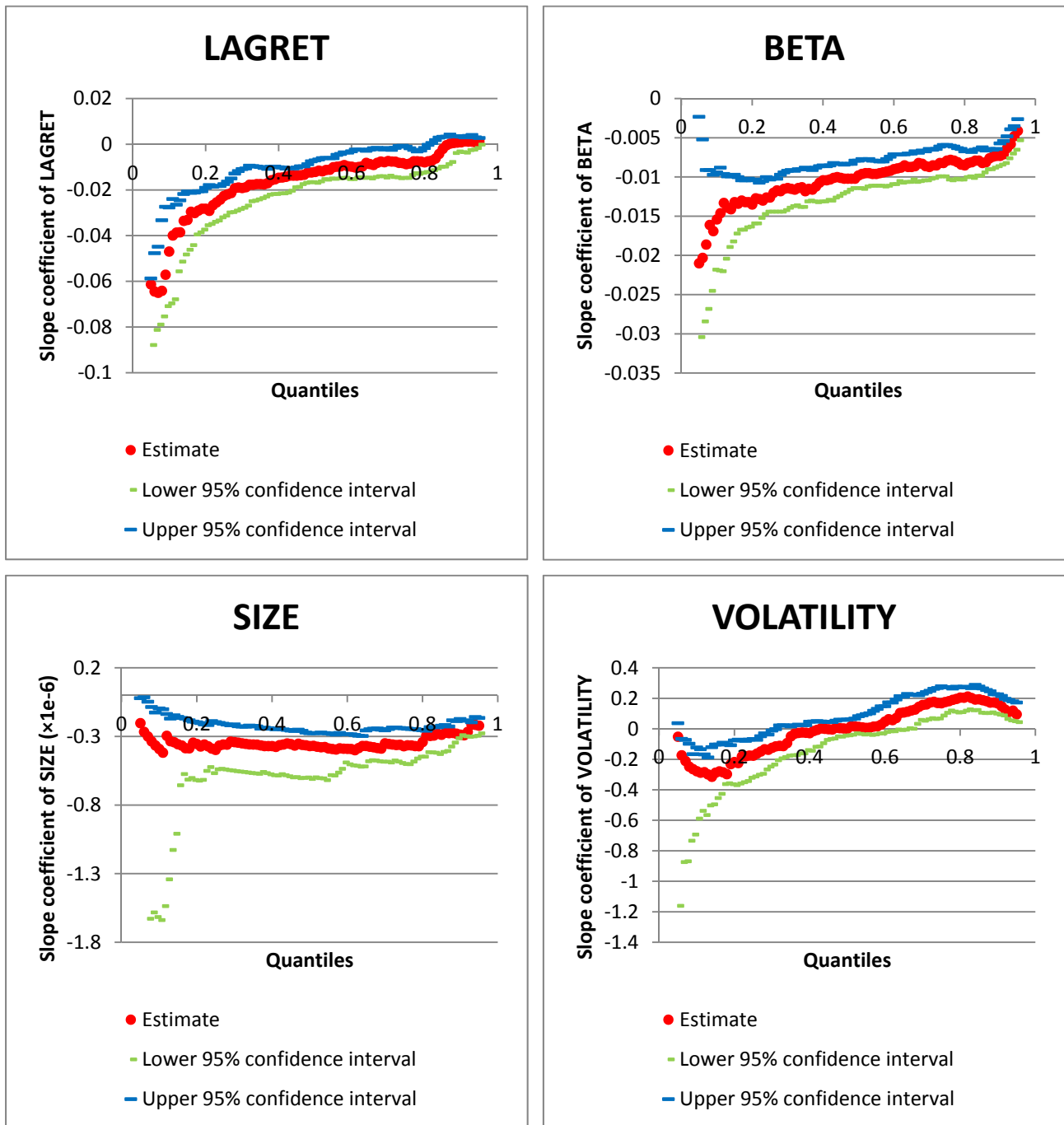


Figure 12. Time-series coefficients of monthly regressions of coskewness on lagged six-month return

This figure shows the time series of the coefficients for lagged six-month return in the monthly cross-sectional regressions of individual-stock coskewness on lagged six-month return. Coskewness for each stock in a month is estimated from daily data according to equation (11) in Harvey and Siddique (2000). A solidly filled marker indicates statistical significance at the 95% level according to the ordinary least squares standard error. An unfilled marker indicates that the coefficient for lagged six-month return is statistically insignificant at the 95% level.

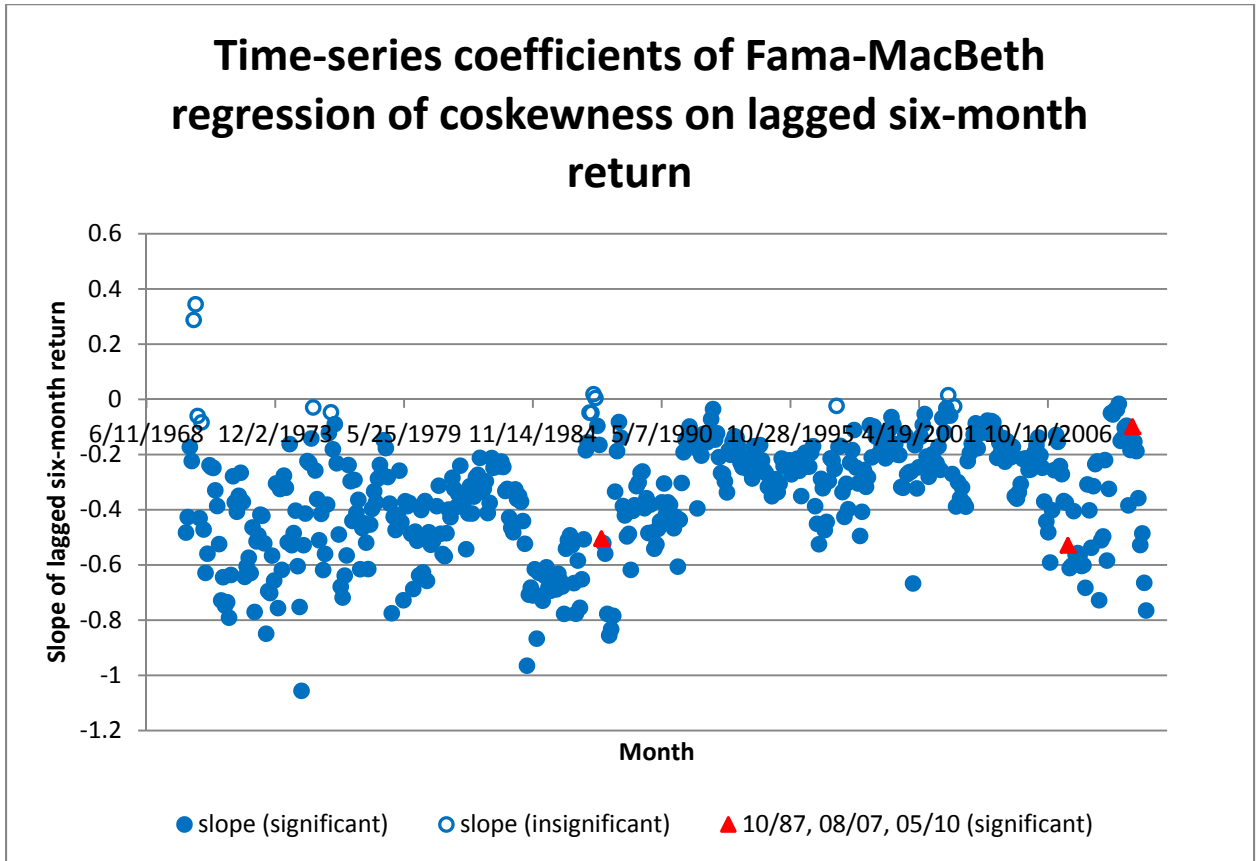


Figure 13. Fama-Macbeth regression of coskewness on lagged return, by horizon of lagged return

This figure shows the coefficient for lagged return along with its 95% confidence interval in the Fama-Macbeth regressions of individual-stock coskewness (or skewness) on lagged return. The lagged return horizon ranges from one month up to three years. Coskewness and skewness for each stock in a month are estimated from daily data (coskewness is computed according to equation (11) in Harvey and Siddique (2000)). Also shown is the coefficient for lagged return in the Fama-Macbeth regressions of individual-stock coskewness on lagged return, controlling for two monthly lags of coskewness as in Harvey and Siddique (2000).

